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The Leakage of Air Through Radial Labyrinth Glands*

— On the Straight Through Type —

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A theoretical formula for calculating the leakage is derived taking the carry-over of kinetic energy of the fluid from the preceding throttling into consideration.

The carry-over factor in the formula is evaluated from the experimental results obtained by the radial labyrinth glands for compressed air. A consideration for the carry-over is presented assuming that the flow issuing from the throttling fin broadens out proportionally with the radial distance in the expansion chamber, and it is shown that the carry-over factor is a function of the ratio s/δ and the resistance coefficient of the flow that depends on the ratios h/s and δ/h , where s is the width of the chamber, h is the height of the fin and δ is the tip clearance of the fin.

The results of the experiment performed to determine the discharge coefficient of a single radial fin are also presented for three distinct types of fin.

1. Introduction

At a place where it is required to prevent undesirable gas leakage from one space to another due to the pressure difference, it is important to keep its gas tightness effectively. Labyrinth glands are commonly used for this purpose in steam turbines and certain other types of rotary machine. With increasing operating pressure of gas, the leakage through labyrinth gland is raised due to the increase of the specific weight of the gas.

Methods which are considered to decrease the gas leakage through labyrinth gland are to make

(1) the area of the throttling opening smaller,

ler,

- (2) the number of fins larger,
- (3) the carry-over effect smaller and
- (4) the discharge coefficient of fin lower.

In the first method, it is necessary to keep some clearance of throttling to avoid contact between the stationary part and the moving one of labyrinth gland. In the second method, a large number of fins in gland is required to make its shaft length longer and its shaft diameter larger from the stand point of design, consequently the area of the throttling opening becomes larger, and causes the increase of leakage through the labyrinth gland. The third method is important in a straight through type labyrinth gland and the last one has to be considered on shape of fin.

Axial flow type labyrinth glands have been studied experimentally and theoretically by many

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researchers⁽¹⁾. But on the radial flow type there are only a few studies^{(2) (3)}, especially on the straight through type, and much of its characteristics is still unknown.

In this paper, a theoretical formula for calculating the leakage is derived taking the carry-over of kinetic energy of the fluid from the preceding throttling into consideration. The carry-over factor in the formula is evaluated from the experimental results obtained by the radial labyrinth glands for compressed air and a consideration for the carry-over is presented. The results of the experiment made to determine the discharge coefficient of single radial fin and the flow patterns at the tip of fin are also presented for three distinct types of fin.

2. Experimental equipment

2.1 General arrangement of the experimental equipment

Fig. 1 shows the schematic diagram of the experimental equipment. Air compressed by the air compressor ① is supplied to the two tanks ②, ③ with its pulsation of pressure damped. The air discharged from the tanks is preheated to about 45°C by the electric heater ④ in order to eliminate the error which may be caused by condensation at the throttlings, and flows into the labyrinth gland ⑦ after passing through the filter ⑤ and the calibrated orifice flow meter ⑥. In the labyrinth gland the air flows in radial direction being throttled by successive fins, and finally goes into the atmosphere.

For measurement of the air temperature at the inlet and outlet of labyrinth gland and at the flow meter, the copper-constantan thermocouples ⑧ are used. The inlet pressure of labyrinth gland being varied with the drain valves fitted on the tanks is below 2 kg/cm² in gauge. Pressures along the labyrinth gland and at the flow meter are measured

by the mercury manometers ⑨.

The labyrinth gland unit consists of the labyrinth gland (7₁) which can be interchanged with other types and the circular plate (7₂) coupled with the shaft of the variable speed motor by rubber coupling.

Some preliminary tests performed prior to this experiment showed that the influence by rotation of the circular plate was negligible, so this experiment was made under static condition. For setting the clearance between the fin tip and the circular plate, three thickness gauges and the clearance adjuster ⑩ to fix those gauges in the clearance were used.

Table 1 Dimensions of labyrinth gland

Gland No.	Fin height h mm	Pitch of fin Δr mm	Fin thickness t mm	Width of chamber s mm	Number of fins n	Outer dia. D_n mm
No. 1	12	12	2	10	5	140
No. 2	9	12	2	10	5	140
No. 3	6	12	2	10	5	140
No. 4	3	12	2	10	5	140
No. 5	1.5	12	2	10	5	140
No. 6	12	10	2	8	6	144
No. 7	9	10	2	8	6	144
No. 8	6	10	2	8	6	144
No. 9	3	10	2	8	6	144
No. 10	1.5	10	2	8	6	144
No. 11	15	8	2	6	7	140
No. 12	12	8	2	6	7	140
No. 13	15	6	2	4	8	128
No. 14	12	6	2	4	8	128
No. 15	15	5	2	3	9	124
No. 16	12	5	2	3	9	124

Fig. 1 Schematic diagram of experimental equipment

(a) Labyrinth gland

(b) Circular plate

Fig. 2 Shape of labyrinth gland

$$\nu^2 = \bar{c}_{i-1}^2 / c_i^2 = \Delta p_i' / \Delta p_i^*$$

$$\Delta p_i = \Delta p_i^* - \Delta p_i' = (1 - \nu^2) \Delta p_i^*$$

$$\therefore \Delta p_i^* = \Delta p_i / (1 - \nu^2)$$

$$p_{i-1}^* = p_{i-1} (1 - \Delta p_i' / p_{i-1}) = p_{i-1} \{1 - \nu^2 (\Delta p_i^* / p_{i-1})\} = p_{i-1} \left(1 - \frac{\nu^2}{1 - \nu^2} \frac{\Delta p_i}{p_{i-1}}\right)$$

By substituting the mean value of fins $\left(1 - \frac{\nu^2}{1 - \nu^2} \frac{\Delta p_i}{p_{i-1}}\right)_m = \zeta_m (> 1)$ in $\left(1 - \frac{\nu^2}{1 - \nu^2} \frac{\Delta p_i}{p_{i-1}}\right)$, $p_{i-1}^* = \zeta_m p_{i-1}$

Eq. (8) is given by

$$\left(\frac{G}{\alpha_i F_i}\right)^2 \frac{1}{\Delta r} - \left(\frac{G}{\alpha_i F_i}\right)^2 \frac{2}{\kappa} \frac{1}{\zeta_m p_{i-1}} \left(\frac{1}{1 - \nu^2}\right) \left(\frac{\Delta p_i}{\Delta r}\right) = -\frac{2g}{p_0 v_0} \zeta_m p_{i-1} \left(\frac{\Delta p_i}{\Delta r}\right) \left(\frac{1}{1 - \nu^2}\right) \dots\dots\dots (9)$$

which can be integrated if the number of throttlings is large enough and successive pressure drops are fairly small so that $\Delta p / \Delta x$ can be replaced by dp / dx . As there is no carry-over to the first throttling, we integrate Eq. (8) from the 2nd to the n th throttling on the assumption that α and ν are constant.

$$\frac{1}{\Delta r} \left(\frac{G}{\alpha}\right)^2 \int_{r_1}^{r_n} \frac{1}{F^2} dr - \left(\frac{G}{\alpha}\right)^2 \frac{2}{\kappa} \left(\frac{1}{\zeta_m}\right) \left(\frac{1}{1 - \nu^2}\right) \int_{p_1}^{p_n} \frac{1}{F^2} \frac{dp}{p} = -\frac{2g}{p_0 v_0} \zeta_m \left(\frac{1}{1 - \nu^2}\right) \int_{p_1}^{p_n} p dp \dots\dots\dots (10)$$

The 1st term of the left-hand side of Eq. (10) is

$$\begin{aligned} \frac{1}{\Delta r} \left(\frac{G}{\alpha}\right)^2 \int_{r_1}^{r_n} \frac{1}{F^2} dr &= \frac{1}{\Delta r} \left(\frac{G}{\alpha}\right)^2 \frac{1}{2\pi\delta} \int_{F_1}^{F_n} \frac{dF}{F^2} = \frac{1}{\Delta r} \left(\frac{G}{\alpha}\right)^2 \frac{1}{2\pi\delta} \frac{F_n - F_1}{F_1 F_n} \\ &= \left(\frac{G}{\alpha}\right)^2 \frac{r_n - r_1}{\Delta r} \frac{1}{F_1 F_n} = \left(\frac{G}{\alpha}\right)^2 \frac{n-1}{F_1 F_n} \dots\dots\dots (11) \end{aligned}$$

where $F = 2\pi r\delta$ is the area of the throttling opening and δ is the clearance of the tip.

In the 2nd term of the left-hand side, for mean value of $1/F^2$, $(1/F_1 F_n)$ may be given by means of the Martin's analysis⁽²⁾. Then the 2nd term of the left-hand side is

$$-\left(\frac{G}{\alpha}\right)^2 \frac{2}{\kappa} \left(\frac{1}{\zeta_m}\right) \left(\frac{1}{1 - \nu^2}\right) \int_{p_1}^{p_n} \frac{1}{F^2} \frac{dp}{p} = \left(\frac{G}{\alpha}\right)^2 \frac{2}{\kappa} \left(\frac{1}{\zeta_m}\right) \left(\frac{1}{1 - \nu^2}\right) \frac{1}{F_1 F_n} \log_e \frac{p_1}{p_n} \dots\dots\dots (12)$$

The right-hand side of Eq. (10) is

$$-\frac{2g}{p_0 v_0} \zeta_m \left(\frac{1}{1 - \nu^2}\right) \int_{p_1}^{p_n} p dp = -\frac{g}{p_0 v_0} \zeta_m \left(\frac{1}{1 - \nu^2}\right) p_1^2 \left[1 - \left(\frac{p_n}{p_1}\right)^2\right] \dots\dots\dots (13)$$

Combining Eqs. (11), (12), and (13), Eq. (10) is

$$\left(\frac{G}{\alpha}\right)^2 \frac{n-1}{F_1 F_n} + \left(\frac{G}{\alpha}\right)^2 \frac{2}{\kappa} \left(\frac{1}{\zeta_m}\right) \left(\frac{1}{1 - \nu^2}\right) \frac{1}{F_1 F_n} \log_e \frac{p_1}{p_n} = \frac{g}{p_0 v_0} \zeta_m \left(\frac{1}{1 - \nu^2}\right) p_1^2 \left[1 - \left(\frac{p_n}{p_1}\right)^2\right] \dots\dots\dots (14)$$

Hence the final equation for the leakage for a straight through type radial labyrinth gland is

$$G = \alpha \frac{1}{\sqrt{1 - \nu^2}} \sqrt{F_1 F_n} \sqrt{\frac{p_1^2}{p_0 v_0} g \zeta_m} \sqrt{\frac{1 - (p_n/p_1)^2}{(n-1) + \frac{2}{\kappa} \frac{1}{\zeta_m} \frac{1}{1 - \nu^2} \log_e (p_1/p_n)}} \dots\dots\dots (15)$$

in which if we put the mean area of the throttling opening $\sqrt{F_1 F_n} = F$ constant, Eq. (15) is shown as the leakage formula for a straight through type axial labyrinth gland and in case when the value of $\nu = 0$, Eq. (15) is identical with the Egli's formula⁽⁴⁾.

State points at each end of successive expansions in a radial flow type labyrinth gland have nearly equal temperature to each other, so that the relation between the state points at the end of the i th throttling and one of the $(i-1)$ th throttling may be given as $p_i v_i = p_{i-1} v_{i-1}$ where v_i and v_{i-1} are the specific volumes of fluid at the ends of adiabatic changes at the i th and the $(i-1)$ th throttlings respectively.

On the other hand, combining the equation of continuity and the above relation, we obtain

$$\frac{v_i}{v_{i-1}} = \frac{c_i F_i}{c_{i-1} F_{i-1}} = \frac{p_{i-1}}{p_i}$$

We now write $\nu_c = \bar{c}_{i-1} / c_{i-1}$ (carry-over factor), and next equation is obtained.

$$\nu = \frac{\bar{c}_{i-1}}{c_i} = \frac{\bar{c}_{i-1}}{c_{i-1}} \frac{c_{i-1}}{c_i} = \nu_c \frac{F_i}{F_{i-1}} \frac{p_i}{p_{i-1}} \dots\dots\dots (16)$$

in which for the mean values of F_i / F_{i-1} and p_i / p_{i-1} of the labyrinth gland, $(\sqrt{r_1 r_n} + \Delta r) / \sqrt{r_1 r_n}$ and $(p_i / p_{i-1})_m$ are given respectively, then the carry-over factor of the labyrinth gland is calculated from the experimental results.

The value of ζ_m in Eq. (15) is nearly equal to unity in general so that the approximate value of ν_c is obtained from Eqs. (15) and (16) by using the value of $\zeta_m = 1$ in Eq. (15). But in the case when the value of ν_c or Δp_i is so large that $-\nu^2 \Delta p_i^* / p_{i-1}$, that is $\zeta_m - 1$, is not negligible, more accurate value of ν_c has to be calculated from Eqs. (15) and (16) once again by using the assumed value of

$$\zeta_m = (1 - \nu^2 \Delta p_i^* / p_{i-1})_m = [1 - \{\nu^2 / (1 - \nu^2)\} (\Delta p_i / p_{i-1})]_m$$

4. Experimental results

An experimental result of pressure distribution in the labyrinth gland is shown in Fig. 4 where the pressure drops at the first throttling are particularly larger than the others and increase with an increase of the clearance of the fin. The others do not show such differences from each other at the same clearance of the fin in contrast to the first.

As there is no carry-over to the first throttling, the carry-over has to be considered to the other throttling except the first. In the case when the pressure of the first expansion chamber is given, the carry-over factor of the labyrinth gland — expressed as simply “the carry-over factor” hereafter — is obtained from Eqs. (15) and (16). The pressure of the first expansion chamber p_1 is obtained from the next ordinary equation which is derived on the assumption that the change of state is adiabatic.

$$G = \alpha F_1 \sqrt{\frac{2g\kappa}{\kappa-1} \frac{p_0}{v_0} \left(\frac{p_1}{v_0}\right)^{2/\kappa} \left[1 - \left(\frac{p_1}{p_0}\right)^{\frac{\kappa-1}{\kappa}}\right]} \quad \dots(17)$$

where this paper deals with the case when the value of the pressure ratio of p_1 to p_0 is larger than the critical pressure ratio, that is, the velocity from the first throttling is below acoustic. In the range that the velocity is larger than the acoustic one, the leakage through labyrinth gland is constant and not affected by the number of fins. Therefore the leakage formula for this case will be explained by other form.

Fig. 4 Pressure distribution in labyrinth gland

Fig. 5 Effect of R_e on ν_c

For calculation of the carry-over factor ν_c , the value of discharge coefficient α which is determined experimentally with a single radial fin whose shape is the same as one of the labyrinth gland is used. (see Supplement)

4.1 Effect of Reynolds number R_e on the carry-over factor ν_c

Value of Reynolds number for unit area of the throttling opening at the mean radius of the labyrinth gland is given from the next equation.

$$R_e = \frac{\delta c \rho}{\mu} = \frac{\delta c \rho g}{\mu g} = \frac{\delta}{\mu g} \frac{G}{2\pi \sqrt{r_1 r_n} \delta} = \frac{1}{2\pi g} \frac{G}{\sqrt{r_1 r_n} \mu} \quad \dots(18)$$

where μ is the fluid viscosity, ρ is the fluid density, c is the flow velocity and δ is the clearance of the fin tip.

Fig. 5 shows an effect of Reynolds number on the carry-over factor, where ν_c decreases with a decrease of the clearance of the fin tip δ and is almost independent of R_e . On the basis of this experimental fact, it seems that the flow pattern in the expansion chamber may be almost unchanged by the quantity of the flow. From observed results by means of aluminium powder sprinkled on the water and of smoke in the air⁽⁵⁾⁻⁽⁷⁾, it has been reported that eddy patterns in the chamber are determined by geometrical dimension ratio of the chamber.

4.2 Effect of the ratio of the width of the expansion chamber s to the tip clearance of the fin δ on the carry-over factor ν_c

An effect of the ratio s/δ on the factor ν_c in the range of the height of the fin h from 1.5 mm to 15 mm is shown in Fig. 6 where the factor ν_c decreases rather rapidly as the ratio s/δ increases until 20~30 and continues to decrease rather slowly for a larger value of the ratio s/δ . In the case when the ratio is a lower value, the factor ν_c approaches unity. This experimental fact means that the large part of the kinetic energy of the fluid from the throttling is carried over and its characteristic may be similar to one of the rough surface.

Fig. 6 Effect of s/δ on ν_c

On the contrary, in the case when the ratio s/δ is a higher value, the space of the expansion chamber becomes large proportionally in the radial direction, then the greater part of the kinetic energy from the throttling may be converted to heat and the value of the factor ν_c becomes lower. On the basis of these results, we find that a very large value of the ratio is required to take a very small value of the factor ν_c . The characteristic of the experimental results that the values of the carry-over factor are distributed in some range at the constant ratio s/δ as shown in Fig. 6 is caused by the variation of the height of the fin. Therefore the factor ν_c may be also affected by the ratio of the fin height to the width of the chamber.

4.3 Effect of the ratio of the fin height h to width of the expansion chamber s on the carry-over factor ν_c

Fig. 7 shows the effect of the ratio h/s on the factor ν_c in the range of the clearance of the fin tip δ from 0.2 mm to 1.0 mm. The factor ν_c decreases with a decrease of the clearance δ in all range and has its minimum value at the ratio h/s about 0.25. Similar result was also obtained for the axial labyrinth gland⁽⁸⁾. When the ratio h/s is a very low value, the factor ν_c rises rapidly and approaches unity. This means that the carry-over effect is large, in other words, the form of the expansion chamber is so flat that its performance is not sufficient. On the other hand, the factor ν_c takes its lower value in the range of the ratio h/s between 0.8 and 1.0.

In case when the ratio h/s takes its value nearly 0.25, it is explained that many unstable eddies occur particularly in the expansion chamber so that the kinetic energy of the fluid from the throttling is in the greater part converted to heat, then the residual energy becomes smaller owing to the flow resistances caused by eddies themselves and by the fluid surrounding the wall. Therefore it is concluded that the chamber having this ratio has an effective performance. The ratio of the energy which turns into heat to the kinetic energy of the fluid from the throttling is presented by $(1-\nu_c^2)$.

Fig. 7 Effect of h/s on ν_c

As the carry-over factor ν_c is almost independent of Reynolds number and affected by the geometrical dimensions of the chamber s , h and δ as before mentioned, the factor ν_c will be represented as follows.

$$\nu_c = \nu_c(s/\delta, h/s, \delta/h) \dots\dots\dots (19)$$

Next, some consideration is presented to make clear this function on flow in the expansion chamber.

4.4 Consideration of the flow state in the expansion chamber

A flow in the i th expansion chamber between the i th throttling fin and the $(i+1)$ th for the radial labyrinth gland may be considered as shown in Fig. 8 which is similar to the Groddeck's consideration⁽⁹⁾ for the axial labyrinth gland. The flow passing through the clearance δ broadens out proportionally in the radial distance as shown by dotted line, being decelerated by the resistances and arrives at the $(i+1)$ th throttling fin with the residual velocity $\bar{c}_i$.

Consider a small part of the flow in the expansion chamber 1256341 in parallel with the fin. It is assumed that the pressures on the surfaces 125 and 436 are constant and at the interface 23 between the almost static part 25632 and the moving part 12341 the flow receives the resistance force dW_1 caused by the fluid at the interface, in the same way at the boundary surface 14 the flow is resisted by the force dW_2 caused by the fluid on the wall surface and also it is assumed that the moving part changes its velocity from $\bar{c}$ to $\bar{c} + d\bar{c}$, where the secondary flow in the peripheral direction is negligible.

Equation for the balance of forces at the small part 12341 is

$$\frac{G}{g} d\bar{c} + \pi D \Delta z dp + dW_1 + dW_2 = 0 \dots\dots\dots (20)$$

where $G = \gamma \pi D \Delta z \bar{c}$ is the quantity of the flow, γ is the specific volume of the fluid and g is the acceleration of gravity. The resistance forces dW_1 and dW_2 are represented as follows.

Fig. 8 Flow in expansion chamber

$$dW_1 = f_1 \frac{\gamma}{2g} \bar{c}^2 \pi D dl, \quad dW_2 = f_2 \frac{\gamma}{2g} \bar{c}^2 \pi D dl$$

where f_1 and f_2 are the coefficients of friction on the wall and at the interface respectively.

On the other hand, the equation of continuity is

$$D \Delta z \bar{c} = \text{constant}, \quad \frac{dD}{D} + \frac{d(\Delta z)}{\Delta z} + \frac{d\bar{c}}{\bar{c}} = 0 \quad \dots\dots(21)$$

Since the pressure rise of the flow through the chamber is so small as negligible, Eq. (20) is

$$G \frac{d\bar{c}}{\bar{c}^2} = -\frac{f_1 + f_2}{2} \gamma \pi D dl \quad \dots\dots(22)$$

Putting $f_1 + f_2 = f$ (coefficient of resistance in the expansion chamber), Eq. (22) is

$$\Delta z \frac{d\bar{c}}{\bar{c}} = -\frac{f}{2} dl \quad \dots\dots(23)$$

From Eq. (21)

$$\Delta z \frac{d\bar{c}}{\bar{c}} = -\Delta z \frac{dD}{D} - d(\Delta z) \quad \dots\dots(24)$$

Combining Eq. (23), Eq. (24) and $dD = 2dl$, we get

$$\left. \begin{aligned} \Delta z dD + D d(\Delta z) &= f \frac{D dD}{4} \\ \therefore d(D \Delta z) &= \frac{f}{8} d(D^2) \end{aligned} \right\} \quad \dots\dots(25)$$

From the boundary condition $D = D_i$ and $\Delta z = \delta$, Eq. (25) is

$$D \Delta z - D_i \delta = \frac{f}{8} (D^2 - D_i^2) \quad \dots\dots(26)$$

Putting the relation $D = D_i + 2l$ into Eq. (26)

$$\left(1 + \frac{2l}{D_i}\right) \Delta z - \delta = \frac{f}{2} \left(1 + \frac{l}{D_i}\right) l \quad \dots\dots(27)$$

in which the term l/D_i is so small as negligible, then Eq. (27) is

$$\Delta z = \frac{f}{2} l + \delta \quad \therefore d(\Delta z) = \frac{f}{2} dl \quad \dots\dots(28)$$

Substituting Eq. (28) into Eq. (23), we get

$$\frac{d\bar{c}}{\bar{c}} = -\frac{d(\Delta z)}{\Delta z} \quad \therefore \bar{c} = \bar{c}_i \frac{\delta}{\Delta z} \quad \dots\dots(29)$$

Using Eq. (28), Eq. (29) is

$$\bar{c} = \frac{c_i}{(f/2)(l/\delta) + 1} \quad \dots\dots(30)$$

in which putting $l = s$, the residual velocity in the i th expansion chamber is obtained by

Fig. 9 Effect of Re on f

$$\bar{c}_i = \frac{c_i}{(f/2)(s/\delta) + 1} \quad \dots\dots(31)$$

Consequently the carry-over factor ν_c is

$$\nu_c = \frac{\bar{c}_i}{c_i} = \frac{1}{(f/2)(s/\delta) + 1} \quad \dots\dots(32)$$

which is represented as a function of the ratio s/δ and the coefficient of resistance in the expansion chamber f .

The dotted lines shown in Fig. 6 represent the calculated values from Eq. (32) for various values of coefficient f . They are rather in good agreement with the characteristics of the experimental results. Considering from Eq. (19), the coefficient of resistance in the chamber f seems to be a function of the ratios h/s and δ/h . Putting the carry-over factor ν_c obtained from the experimental results and the ratio s/δ into Eq. (32), the coefficient f is given.

Fig. 9 shows an effect of Reynolds number on the coefficient f , where the coefficient f is independent of Re . This fact is natural because the factor ν_c is also independent of Re as shown in Fig. 5. Fig. 10 shows an effect of the ratio δ/h on the coefficient f , where f increases in proportion to the cubic root of the ratio δ/h . Fig. 11 shows the correlation of the ratio h/s and $f/(\delta/h)^{1/3}$,

where the ratio $f/(\delta/h)^{1/3}$ takes rather large value at the ratio h/s about 0.25 and 0.8~1.0, and it is found that this characteristic varies inversely in contrast to the factor ν_c shown in Fig. 7.

Fig. 10 Effect of δ/h on f

Fig. 11 Correlation of h/s and $f/(\delta/h)^{1/3}$

It may be concluded that the coefficient of resistance in the expansion chamber f is a function of the ratios δ/h and h/s , and the carry-over factor ν_c is determined from Eq. (32) which shows the correlation of the ratios δ/h , h/s and s/δ .

5. Conclusion

(1) A theoretical formula Eq. (15) for calculating the leakage through the straight through type radial labyrinth gland is derived taking the carry-over of kinetic energy of the fluid from the preceding throttling into consideration. In the case when the mean value of the throttling opening in Eq. (15) is a given constant value F , Eq. (15) is shown as the leakage formula for the straight through type axial labyrinth gland.

(2) The carry-over factor in the formula is evaluated from the experimental results which are obtained from the radial labyrinth gland having a typical shaped fin of flat tip. The factor ν_c is almost independent of Reynolds number and is affected by the ratios of dimensions of the expansion chamber s , h and δ where s is the width of the chamber, h is the height of the fin and δ is the tip clearance. The factor ν_c decreases rather rapidly as the value of the ratio s/δ increases until 20~30 and has its minimum value at the ratio h/s about 0.25.

(3) A consideration for the carry-over is presented assuming the flow issuing from the

throttling fin in the expansion chamber as shown in Fig. 8, and it is shown in Eq. (32) that the carry-over factor is a function of the ratio s/δ and the coefficient of resistance in the expansion chamber which depends on the ratios h/s and δ/h . The coefficient f increases approximately in proportion to the cubic root of the ratio s/h , and the ratio $f/(\frac{\delta}{h})^{\frac{1}{3}}$ has its rather large value at the ratio h/s about 0.25 and 0.8~1.0.

6. Supplement : On the discharge coefficient

The experiment is performed to determine the discharge coefficient of a single radial fin for three distinct types of fin — A, B and C types — as shown in Fig. 12. The instrumental equipment is the same as one of the labyrinth gland in Fig. 1 and this experiment is also carried out on five kinds of clearance of the fin tip from 1.0 mm to 0.2 mm for each type of the fin in the same way for the labyrinth gland. The inlet pressures on both left and right-hand sides near the fin are measured by the mercury manometers and for measurement of atmospheric pressure the barometer is used.

6.1 Experimental results

The formula for the calculation of the flow coefficient α is given by the next equation which is derived on the assumption that the velocity from the fin is below acoustic.

$$\alpha = \frac{G}{F \left\{ 2g \frac{\kappa}{\kappa-1} \left(\frac{p_0}{p_n} \right) \left(\frac{p_n}{p_0} \right)^{\frac{2}{\kappa}} \left[1 - \left(\frac{p_n}{p_0} \right)^{\frac{\kappa-1}{\kappa}} \right] \right\}^{\frac{1}{2}}} \quad \dots\dots\dots(33)$$

where G is the actual flow measured, p_0 and p_n are pressures at the inlet and outlet respectively and F is the area of the throttling opening.

On the other hand, the formula for the calculation of Reynolds number R_e for unit area of the throttling opening is given by the next equation.

$$R_e = \frac{\delta c \rho}{\mu} = \frac{\delta c \rho g}{\mu g} = \frac{\delta G}{\mu g 2 \pi r \delta} = \frac{G}{2 \pi g r \mu} \quad \dots\dots\dots(34)$$

where c is the discharge velocity from the throttling, ρ is the density, μ is the fluid viscosity and r is the radius from the center of the shaft to the fin. Fig. 13 shows an effect of Reynolds number R_e on the discharge coefficient α of A-2 type whose shape is the same as one of the labyrinth gland. The discharge coefficient increases with an increase of the clearance of the fin tip and it becomes a nearly constant value for a larger value of Reynolds number than 3000~4000. These characteristics

Type	No.	1	2	3	4	5	6	7
A	t mm	3.0	2.0	1.0	0.5	0.2		
	h mm		15 ~3					
	D φ mm	90	40 90	90	90	90		
B	t mm	2.0	1.5	1.0	0.5	0.25		
	D φ mm = 90							
	θ° = 20, 45, 70							
C	t mm	2.0	1.5	1.0	0.5	1.0	1.0	1.0
	H mm	0.5	0.75	1.0	1.25	1.3	1.6	1.9
	D φ mm = 90							

Fig. 12 Shape and dimension of fin

Fig. 13 Effect of R_e on α

have similar tendency to other types of fin. Fig. 14 shows that the discharge coefficient is independent of the height of the fin.

Effects of the ratio δ/t on the discharge coefficient at the value of Re 4000—the discharge coefficient is almost independent of Reynolds number of larger value than this one—are shown in Figs. 15~19.

In Fig. 15, the curves of the coefficient α approach a constant value with an increase of the ratio δ/t and become larger at lower value of the ratio δ/t . In Fig. 16 (B type, $\theta=20^\circ$), the curves of the discharge coefficient α have their maximum value at the range of the ratio δ/t between 0.4 and 1.0, and some curves take a larger value than unity. For this characteristic it is said that the flow rate is increased by the diffuser effect at the tapered part of the fin tip. In Fig. 18(B type, $\theta=70^\circ$), its characteristic is similar to A type and seems to be almost independent of the taper at the fin tip. In Fig. 17 (B type, $\theta=45^\circ$), the curves of the discharge coefficient α seems to have mean characteristic between the types of $\theta=20^\circ$ and of $\theta=70^\circ$.

C type is designed for the reason that contraction of the flow may be promoted by the eddies occurring in the semi-circular pocket at the fin tip. Fig. 19 shows the correlation of the coefficient α and the ratio δ/t on types C-1~C-4, where the curves α have good characteristic at the low values of the thickness t , of clearance δ and at the large pocket. But in case of a deep pocket and a thick fin as in types C-5~C-7, the performance is not so good.

From a few studies⁽¹⁰⁾ on annular orifice of A type, it has been shown that the discharge coefficient α is affected by Reynolds number and the ratio δ/t , and increases in general with an increase of

Fig. 14 Effect of fin height

Fig. 15 Value of α for δ/t (A type)

Fig. 16 Value of α for δ/t (B type, $\theta=20^\circ$)

Fig. 17 Value of α for δ/t (B type, $\theta=45^\circ$)

Fig. 18 Value of α for δ/t (B type, $\theta=70^\circ$)

Fig. 19 Value of α for δ/t (C type)

Fig. 20 Diagrammatical sketch of flow pattern

R_e , although at higher value of R_e it decreases slightly, and that it approaches a constant value at higher value of the ratio δ/t . Those characteristics are comparatively similar to our experimental results of A type.

6.2 Observation of the flow pattern

The flow patterns at the fin tip are observed by means of aluminium powder sprinkled on the water using the two-dimensional water channel model. Fig. 20 shows diagrammatical sketches of the flow patterns on B and C types. On B type, an eddy occurs at the tapered part of the fin tip and at the wall side respectively. Denoting these eddies as (a) and (b) respectively, (a) does not change its position and occurs always in the contracted flow and transfers its position afterward with an increase of Reynolds number, its shape being elongated. As the value of contraction is almost independent of the thickness δ , it may be said that the smaller the value of thickness, the better will be the performance of the discharge coefficient. These characteristics are similar to A type.

On the C type, one or a few eddies occur in the pocket according to the shape of the pocket and another occurs also at the wall side. Denoting again the eddies as (a') and (b) respectively, (a') be-

$$R_e = 2.9 \times 10^3, t/\delta = 0.65$$

Fig. 21 A flow pattern of C type

comes large in proportion to the space of the pocket and its shape is elongated in case of small clearance of fin tip. Fig. 21 shows an example of photograph where the thickness of the fin tip is rather thin and the pocket is large and the flow is contracted by the eddy in the pocket to the wall side.

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