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Bending Fatigue Strength of Gears under Various Loading Conditions*

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In this paper, the authors investigated the bending fatigue properties and the behaviour of residual stress generated on the root fillet under the loading conditions corresponding to repeatedly reversed gears and idle gears, by using the bending fatigue testing machine of new type, which was developed by the authors.

The main results obtained from this experiment are summarised as follows.

(1) The bending fatigue limit of repeatedly reversed gears increases with the number of load cycles in one loading stage n .

And there exists a linear relationship between σ_{up}'' and $\log_{10}n$.

(2) The bending fatigue limit of idle gears decreases by about 40 % compared with that in one side loading conditions.

(3) In repeatedly reversed gears, compressive residual stress is generated on the root fillet of the load application side and tensile residual stress on that of the opposite side alternately, with the periodical change in the direction of load application.

And in idle gears, compressive residual stresses are generated by alternating load on both sides of the root fillet.

1. Introduction

In the previous papers⁽¹⁾, some points as mentioned below were made clear by investigating the mechanism of bending fatigue breakage of gear tooth from the standpoints of the changes in microstructure and in residual stresses at the root fillet in the fatigue process, and moreover by examining the bending fatigue strength of gears under reverse loading conditions.

(1) When the applied load exceeds the fatigue limit, both sliplines and cracks appear first at the compressive side, then later at the tensile side. When the applied load corresponds to the fatigue limit, a crack appears only at the compressive side and grows to some extent, then after a certain number of load cycles it ceases its growth, which is called a non-propagating crack. And when the applied load is below and closely approaching the fatigue limit, no sign of sliplines can be observed at the tensile side by the micro-

scope of magnification 400, but at the compressive side sliplines can be observed distinctly. But no crack initiates on both sides.

(2) When the pulsating load is applied on one side of the flank of the gear tooth after a certain number of load cycles on the other side [Fig. 1 (c)], tooth breakage occurs earlier and the fatigue limit decreases by about 20 % compared

Fig. 1 Various loading conditions for gears

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with that in one side loading conditions [Fig. 1 (a)].

(3) Tensile residual stress is generated at the compressive side and compressive residual stress at the tensile side by the load repetitions, and their magnitude changes with the number of load cycles. When the pulsating load is applied on one side of the flank of the gear tooth after a certain number of load cycles on the other side, compressive residual stress is still generated at the tensile side (after reverse) and tensile residual stress at the compressive side (after reverse).

From these results it can be conjectured that the bending fatigue limits of the repeatedly reversed gears [Fig. 1 (d)], in which the direction of load application changes periodically with a constant interval of load repetitions, and the idle gears [Fig. 1 (b)], in which the direction of load application changes alternately, decrease considerably.

In this paper, the authors investigated the bending fatigue properties and the behaviour of residual stresses generated on the root fillet under the loading conditions corresponding to repeatedly reversed gears and idle gears by using the bending fatigue testing machine of new type.

2. Experimental procedure

2.1 Test gears and bending fatigue testing machine

Test gears are of S15C carbon steel and cut with a hob and their dimensions are as follows: module $m=4$, pressure angle $\alpha=20^\circ$, number of teeth $z=18$, face width $b=8.0$ mm, stress concentration factor $\alpha_f=1.57$ (tensile side).

The nominal proportional limit load and the nominal yielding point load of the test gears,

① Injection pump ④ Test gear
② Pressure cylinder ⑤ Micro-switch
③ Pressure controller ⑥ Pressure detector

Fig. 2 Bending fatigue testing machine

which were determined by the static bending test, were 400 kg and 450 kg respectively.

The bending fatigue testing machine used in this experiment, which was developed by the authors, consisted of three main parts, namely, two fuel injection pumps for Diesel-motor①, two pressure cylinders②, and two pressure controllers③, as shown in Fig. 2.

Oil pressure of two cylinders, which are connected by pipes with fuel injection pumps and pressure controllers, is set up independently, even in motion, by operating the pressure controllers and this fatigue testing machine is applicable to the bending fatigue test of gears, which are used under various loading conditions, such as repeatedly reversed gears, idle gears etc.

Test gears④ is fixed to the holder with a shaft and two teeth of the internal segment gear.

When a loaded gear tooth breaks, the driving electric motor stops by the working of micro switch⑤.

And two pressure detectors⑥ using electric resistance wire strain gauges are provided to detect the pressure.

The position and direction of the application of load on the test gear are shown in Fig. 3.

Notations	Dimensions
l	7.69 mm
s	7.67 "
ρ	2.34 "
y	1.77 "
θ	60°
d	1.00 mm

Fig. 3 Positions of load application and X-ray incidence

2.2 Measurement of residual stress

The behaviour of residual stresses generated on the root fillet of gear tooth under the loading conditions corresponding to repeatedly reversed gears and idle gears was investigated by X-ray method which may be said to be a unique method of nondestructive measurement of stresses existing on the local parts of specimens.

The position of X-ray incidence was selected on the point determined by Hofer's method as shown in Fig. 3.

This is because the position determined in this way shows a good agreement with the point where the tooth breakage actually occurs.

3. Experimental results and discussions

3.1 Experimental results of bending fatigue test

Fig. 4 shows the results of the bending fatigue test which was carried out under various loading conditions. In this figure, the case $n=1$ corresponds to the loading condition of idle gears [Fig. 1 (b)] and $n=\infty$ to the one side loading conditions [Fig. 1 (a)].

The true stress at the root fillet of the load application side (tensile side) σ_t , and that at the opposite side (compressive side) σ_c in this figure were calculated by the following equations⁽²⁾ (S. Fig. 3).

$$\sigma_t = \left(1 + 0.08 \frac{s}{\rho}\right) \left\{ 0.66 \sigma_{tNb1} + 0.40 \sqrt{\sigma_{tNb1}^2 + 36 \tau_N^2} + 1.15 \sigma_{tNc} \right\} \quad \dots\dots\dots (1)$$

$$\sigma_c = \left(1 + 0.08 \frac{s}{\rho}\right) \left\{ 0.66 \sigma_{cNb1} - 0.40 \sqrt{\sigma_{cNb1}^2 + 36 \tau_N^2} + 1.40 \sigma_{cNc1} + \sigma_{cNb2} \right\} \quad \dots\dots\dots (2)$$

$$\left. \begin{aligned} \sigma_{tNb1} &= -\sigma_{cNb1} = \frac{6P \sin \theta l}{s^2 b} \\ \sigma_{tNc} &= \sigma_{tNc1} + \sigma_{tNb2} = -\left(1 + \frac{6y}{s}\right) \frac{P \cos \theta}{sb} \\ \sigma_{cNc1} &= -\frac{P \cos \theta}{sb}, \quad \sigma_{cNb2} = \frac{6P \cos \theta y}{s^2 b} \\ \tau_N &= \frac{P \sin \theta}{sb} \end{aligned} \right\} \quad \dots\dots\dots (3)$$

where

σ_{tNb1} , σ_{cNb1} : nominal stresses on the critical section based on bending component of force, at the tensile side and at the compressive side respectively

σ_{tNc1} , σ_{tNb2} , σ_{cNc1} , σ_{cNb2} : nominal stresses on the critical section based on compressive component of force

τ_N : nominal stress on the critical section based

on shearing force, which is assumed to be distributed uniformly all over the section

The stress concentration factor α_f at the root fillet is obtained by the following equation.

$$\alpha_f = \frac{\sigma_t}{\sigma_{tNc} + \sigma_{tNb1}} \quad \dots\dots\dots (4)$$

Fig. 4 shows that no $S-N$ curve crosses each other and both the cyclic life and the fatigue limit increase with an increase in the number of load cycles n .

$\sigma_{up}''-n$ curve becomes nearly a straight line when plotted on semilog paper with n on the log scale and σ_{up}'' on the uniform scale as shown in Fig. 5 (σ_{up}'' : fatigue limit in the loading conditions corresponding to those of repeatedly reversed gears).

The ratios of fatigue limits for various loading conditions to that in one side loading conditions ($\sigma_{up}''/\sigma_{up}$) are shown in Table 1.

It is seen from Table 1 that the ratio $\sigma_{up}''/\sigma_{up}$ becomes smaller with a decrease in n and that the bending fatigue limit in the case of $n=1$ is about 60% of that in the case of $n=\infty$.

From Fig. 5, the relation between n and σ_{up}'' can be expressed as Eq. (5), by applying the least square method.

$$\sigma_{up}'' = 21.1 + 1.56 \log_{10} n \quad (1 < n < 10^8) \quad \dots\dots\dots (5)$$

Further, assuming that the extension of the straight line passes through the point A ($n=10^8$,

Fig. 4 Results of repeatedly reversed bending fatigue test

Table 1 Number of load cycles n and bending fatigue limit ratio $\sigma_{up}''/\sigma_{up}$

n	$\sigma_{up}''/\sigma_{up}$
∞	1
3×10^6	0.92
1×10^6	0.90
4.8×10^4	0.84
1.6×10^4	0.82
1	0.63

$\sigma_{up}'' = \sigma_{up}$), Eq. (5) can be written as Eq. (6).

$$\sigma_{up}'' = \sigma_{wp} + \frac{\sigma_{up} - \sigma_{wp}}{8} \log_{10} n \quad (1 < n < 10^8) \dots (6)$$

σ_{up} : bending fatigue limit in one side loading conditions

σ_{wp} : bending fatigue limit in alternating loading conditions

The fact that both cyclic life and fatigue limit decrease with a decrease in n may be attributed to "fatigue Baushinger's effect", to the change in residual stresses generated on the root fillet by load repetitions and to the stress amplitude becoming relatively larger with smaller values of n (loading conditions approach the alternating loading conditions with a decrease in n).

The form of the fracture of gear teeth is concave in the tip direction and the position of crack initiation nearly coincides with the intersecting point of the critical section by Hofer's method with the fillet curve.

And it is observed that the concavity of the fracture of tooth tends to be smaller, as the applied load approaches the fatigue limit.

3.2 Change in residual stress

Fig. 6 shows the changes in the residual stresses, generated on the root fillet by the load repetitions under various loading conditions in the

Fig. 5 Relation between n and σ_{up}''

Fig. 6 Change in residual stresses by load repetition ($P=0.92 P_{up}$)

case of $P=300$ kg (corresponding to $0.92 \sigma_{up}$).

The changes in residual stresses under one side loading conditions were already described in detail in the previous paper⁽¹⁾.

Before the first reverse in the repeatedly reversed bending fatigue test, residual stresses on the root fillet show, naturally, quite the same behaviour as in one side loading conditions, but after a certain number of load cycles after reverse compressive residual stress is generated at the tensile side (after reverse) and tensile residual stress at the compressive side (after reverse) and the residual stress values approach those in one side loading conditions with the number of load cycles, depending on the applied load levels and the number of load cycles before reverse N_p . In other words, under the loading conditions corresponding to repeatedly reversed gears, compressive residual stress is generated on the root fillet of the load application side and tensile residual stress on that of the opposite side alternately, with the periodical change in the loading direction.

Under the loading conditions corresponding to idle gears ($n=1$), compressive residual stress is generated on the root fillets of both sides. In this case, the absolute value of residual stress is smaller in comparison with that in one side loading conditions, and has a maximum value at about $N=4 \times 10^4$, then decreases with the number of load cycles, and at about $N=10^5$ residual stress is almost relieved.

According to the experimental results obtained in the previous paper⁽¹⁾, it may safely be considered that in the case of $n=1$, microcracks appear at $N=4 \times 10^4$, and at $N=10^5$ they grow up considerably and these microcracks relieve the residual stress on the root fillet.

The present experimental results show the same tendency as the hitherto available data⁽³⁾ on the change in the residual stress on the surface of unnotched or notched specimens under alternating loading conditions.

As compressive residual stress is generated at the tensile side and tensile residual stress at the compressive side, the loading conditions at the root fillet become partly alternating conditions.

Further, as these residual stresses change with the number of load cycles, it is difficult at the present stage to determine the influence of residual stress on the crack initiation and the fatigue strength of gears.

4. Conclusion

From the results obtained in this investigation, the following should be taken into consideration

in the practical design of gears.

(1) The bending fatigue limit of repeatedly reversed gears σ_{up}'' increases with the number of load cycles in one loading stage n , and there is the following relationship between them

$$\sigma_{up}'' = \sigma_{wp} + \frac{\sigma_{up} - \sigma_{wp}}{8} \log_{10} n \quad (1 < n < 10^8)$$

This formula may be applicable to tempered gears in general use.

(2) The bending fatigue limit of idle gears σ_{wp} is about 60 % of that in one side loading conditions σ_{up} .

Hence, special attention should be paid to the fact that in idle gears the ratios of bending fatigue strength to surface durability become smaller in comparison with the case of general gearing.

(3) When the number of load cycles n in one side and that in the other side of the gear

tooth differ from each other, it would be safe to take a smaller n for the determination of the bending fatigue limit σ_{up}'' .

(4) In repeatedly reversed gears, compressive residual stress is generated on the root fillet of the load application side and tensile residual stress on that of the opposite side alternately, with the periodical change in the direction of load application.

(5) In idle gears, compressive residual stress is generated on the root fillets of both sides by the load repetitions.

References

- (1) T. Aida and S. Oda: *Bulletin of JSME*, Vol. 9, No. 36 (1966), p. 793.
- (2) T. Aida and Y. Terauchi: *Trans. Japan Soc. Mech. Engrs.*, Vol. 27, No. 178 (1961), p. 853.
- (3) S. Taira and K. Honda: *Proc. 5th Japan Congr. Test. Mat.*, (1962), p. 8.

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Tragfähigkeit und Verlustleistung von Schraubenradgetrieben*

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Der vorliegende Aufsatz enthält die Untersuchungen an Schraubenradgetrieben. Aus Versuchen wurden Gesetzmäßigkeiten und Berechnungsgleichungen für die Tragfähigkeit der Schraubenradgetriebe abgeleitet.

Spezifischer Verschleiß, Grenzlast, Verlustleistung, Reibzahl und Rauheit der Zahnflanken wurden für die verschiedenen Drehzahlen, Öle und Werkstoffpaarungen ermittelt.

Die Grenzlast steigt mit der Drehzahl bei der Werkstoffpaarung Stahl/Bronze, sie fällt mit der Drehzahl bei Stahl/Stahl und sie ändert sich nur wenig mit der Drehzahl bei Stahl/Grauguß. Die für Schraubenradgetriebe ermittelte Grenzlast steht in Beziehung zu der im FZG-Stirnradtest für die gleiche Werkstoff- und Ölpaarung ermittelten Grenzlast. Für die Vergleichsrechnung sind die Hertz'sche Pressung und die Gleitgeschwindigkeit am Zahnkopf maßgebend. Zum Berechnen bei der beiden Größen sind neue Gleichungen angegeben.

1. Einleitung

Bekanntlich sind Schraubenradgetriebe wegen ihrer ungünstigen Punktberührung gegenüber Schneckenge trieben und Hypoidgetrieben weniger tragfähig. Da Schraubenradgetriebe deshalb bisher

nur für Sonderfälle verwendet wurden, liegen sehr wenige diesbezügliche Versuchsergebnisse vor.

Aber für die Theorie der Zahnradpaarung ist sehr beachtlich, daß sich alle Zahnradpaarungen (Stirnradgetriebe, Kegelradgetriebe, Hypoidgetriebe und Schneckengetriebe) hinsichtlich der Berechnung der hierbei auftretenden Kräfte, Bewegungen und Verlustleistungen auf Schraubenradgetriebe zurückführen lassen. Die Zielsetzung der Untersuchungen ist also folgende:

(1) wie liegt die Grenzlast (Freßlast) bzw. Tragfähigkeit bei Schraubenradgetrieben im Ver-

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