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Grinding Methods of Gears with Circular-Arc Tooth-Curve*

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Gears with circular-arc tooth-curve can easily be ground with the cup-shaped grinding wheel and a desirable crowning can readily be given to the tooth surface. The gear grinding machine can be of the simplest construction. Further, it can grind parallel-depth toothed spiral bevel- and hypoid-gears theoretically correctly with great ease. Therefore, high precision, good tooth-bearing gears can be made at the first setting without any trial cutting. The parallel-depth tooth system should be adopted especially when small numbers of various gears are to be produced.

However, since the cup-shaped grinding wheel and the tooth surface being ground contact as if to envelop each other, the arc of contact of the abrasive grain is long and the cooling and flushing effects of the grinding fluid cannot be achieved sufficiently. Consequently, there is a fear of burning of the tooth surface.

By the addition of a small revolving motion ($r=0.5$ mm, $n=60$ rpm) to the shaft of the grinding wheel, intermittent grinding is effected, where the grinding fluid flushes and cools the grinding region sufficiently, and the grinding horsepower decreases remarkably because of the decrease of grinding friction. Thus, the grinding efficiency is markedly increased. Even the plunge grinding, or the so-called Formate Grinding, can be performed successfully.

1. Introduction

Ideal gearing lies in the smooth transmission of large horsepower in a small, compact space. The demand for quiet operating high-speed gears has recently become more and more pronounced.

Even the Gleason Works, renowned for its outstanding workmanship in precision machining and its series of equipment to limit heat treatment distortion to a very minimum, recommends that the tooth surfaces of spiral bevel gears and hypoid gears, for use with more than 40 m/sec of pitch line velocity, should be finished by grinding. Thus the field where the hardened gears must be ground precisely will become larger and larger. And the problem lies in the practicability of precision grinding with high efficiency.

Circular-arc-toothed gears can easily be ground with cup-shaped grinding wheels, and the spiral

bevel gears and hypoid gears by the Gleason Works system boast their excellence with circular-arc tooth-curves. Recently the same Works is going to adopt the circular-arc tooth-curve for cylindrical gears (Zerate gears), too. Desirable crowning can easily be given to the circular-arc-toothed gears and the grinding machine can be of the simplest construction. Thus, circular-arc-toothed gear grinding machines deserve much study and development.

The authors have been engaged a long time in the study of the grinding methods for the circular-arc-toothed gears, and the experimental results on its simplification and efficient grinding are reported in this paper.

2. The parallel-depth tooth system is recommended for bevel gears with circular-arc tooth-curve

A circular-arc-toothed bevel gear is generated by the principle shown in Fig. 1. A tooth surface of the generating crown gear is represented by a face-mill gear-cutter or a cup-shaped wheel rotating around shaft O_c , and the tooth surface is revolved around the shaft of the crown gear O to

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produce the rotating circular-arc-toothed crown gear. Now, if the pinion workpiece B is mounted to mesh ideally with this crown gear and rolled, a tooth surface of the spiral bevel pinion is generated. If the cutter axis is held perpendicular to the pitch plane, the axes of the cutters generating the pinion and the gear can be held parallel with each other, and, in special cases, they can be set to coincide. The gear mating with the pinion B is cut in a similar way by setting the gear workpiece similar to that of the pinion workpiece. In Fig. 1, the tooth of the gear generated is shown in the state of meshing with the tooth of the pinion.

The external conical cutting or grinding surface (radius r_a) generating the concave tooth surface of the pinion can be brought into close contact with the internal conical cutting or grinding surface (radius r_i), and both the surfaces generated by these cutting or grinding surfaces would be in close mesh with each other*. In actual practice, r_a is made slightly larger than r_i so that the tooth surfaces may come into close contact at the center of face-width P , and the concave tooth surface is relieved slightly towards both ends, or crowning

Fig. 1 Generating principle of spiral bevel gears

* See Appendix (1)

is given. Thus, theoretically correct gear cutting is performed readily with high accuracy. Good tooth bearing can be obtained at the first setting without any trial cutting.

Contrary to this, in the case of the tapered-depth system with the root line of the tooth running towards the apex of the pitch cone, the cutter axis must be held perpendicular to the root line of the tooth. As shown by O_{cP} and O_{cG} in Fig. 2, the cutter axes for the pinion and for the gear are slanted in the opposite directions by the amount of root angle of each, and it is impossible to make the cones represented by inner and outer cutting edges coincide. There would be no other means than to make both tooth surfaces conjugate some how at the center of the tooth width and manage to finish the concave and the convex tooth surfaces. Here lies great difficulty.

The adoption of the parallel-depth tooth system is a great first step towards simplification.

Some apprehensions that might be felt about the adoption of the parallel-depth tooth system may be removed in the following manner.

a) The parallel-depth tooth has extra depth at the toe and is stubby at the heel. It is recommended that the addendum and the dedendum of this gear be equal to those at two-thirds of the width inside the heel of the Gleason tapered-depth tooth system. With this tooth proportion, the tooth would not be under-cut at the toe, nor would it be too stubby at the heel. As the tooth surfaces

Fig. 2 Cutting of tapered-depth spiral tooth

are crowned and relieved slightly at the tip, the tooth bearing area is not any smaller with the parallel-depth tooth than with tapered-depth tooth.

b) Pinions with very few teeth might have pointed tips of teeth at the toe, in which case the tips of the teeth can safely be cut off at the toe.

c) In the cutting method of finishing both surfaces simultaneously with a cutter of spread blades, the thickness of the tooth would become greater towards the heel, because the width of the tooth space is constant. In such cases, the situation can be readily coped with by the use of a cutter of a smaller radius so that the spiral angle may become greater and the circular tooth space greater towards the heel (see Fig. 3).

Thus, all the difficulties conceivable with the parallel-depth tooth system can be removed by proper design considerations.

The parallel-depth tooth system has also a big advantage in the case of mass production of gears by the Formate gear cutting method. Gears of

3:1 ratio and higher, or $\tan \delta_2 \geq 3$ (Fig. 4), $\delta_2 = 71^\circ 30'$, are close to crown gears, and both tooth surfaces can be finished simultaneously by simple plunge cutting with a cutter of spread blades. The tooth profile thus finished is straight, and the mating pinion is finished as follows: One tooth surface of the Formate gear is represented by a cutter to mate closely to the cutter for the gear; and one tooth surface of the pinion is produced by the generating motion to mesh ideally with the Formate gear. In the case of the parallel-depth tooth, this generation can be made ideally without the slightest approximation. The opposite tooth surface of the pinion can be finished by the same method. As gears of a large number of teeth can be cut readily and efficiently, the Formate gear cutting method is employed widely in mass production.

3. Hypoid gear grinder made experimentally at Kyushu University

Fig. 5 is a photograph of the hypoid gear grinder made experimentally at Kyushu University. The cutter driving unit of the hypoid gear generator, which had been made previously at Kyushu University, was replaced with the grinding unit, and the dressing and truing unit for the grinding wheel was added.

The principle of generation is as described in Section 2. The gear being finished and the generating crown gear are rotated always in one direction, and indexing is made continually for each turn of the crown gear. Therefore, the rotating

A: middle cone distance
R: cutter radius

Fig. 3 Relationship between cutter radius and tooth-thickness in spread-blade method

Fig. 4 Formate gear cutting method

Max. $P_d=4$, max. dia. of gear=12" (1:10), vertical movement of work spindle $=\pm 1\frac{3}{4}$ in., main motor 2 HP, motor for rapid feed=3 HP, motor for grinding spindle ≈ 2 HP

Fig. 5 Spiral bevel- and hypoid-gear grinder

body corresponding to the cradle of the Gleason generator is a rotor with the authors' generator.

Let z be the number of teeth of the generated bevel gear and δ be the pitch cone angle. Then the number of teeth z_c of the generating crown gear is expressed by

$$z_c = z / \sin \delta \quad (\text{see Fig. 1})$$

z_c is in general not an integer. Now z_c' is chosen as follows,

$$z_c' = az \pm b$$

where a and b are integers and b is a prime number to z . For each turn of the crown gear or the rotor, the bevel gear rotates a turns and b pitches. Namely the grinder will grind a tooth space at b pitches spacing. Since z is prime to b , a different tooth is finished per each turn of the rotor and all the teeth are finished in z turns of the rotor.

The rotor should be rotated at the rate of z/z_c per each turn of the workpiece, but actually it is rotated at the rate of z/z_c' . Therefore, the rotor is given a compensating rotation by $(z/z_c - z/z_c')$ by the use of a differential gear system. However, this compensating rotation is removed completely in the idle-running period after the generating period. Namely, the differential gear system is given a reverse rotation by the amount of compensating rotation given to it in the generating period, so that the compensating rotation may be cancelled and the correct indexing accomplished. In the idle-running period, the rotor is turned rapidly by a rapid feed motor, and the sliding base carrying the workhead retreats so that the bevel gear may be free from the grinding wheel. Thus, the hypoid gear grinder is equipped with the necessary mechanisms, but the simplest possible construction with sufficient rigidity was adopted for maximum ac-

curacy.

Fig. 6 shows the gear train of this grinder and Fig. 7 the plan of the rotor part. The distance from the center of the rotor to the shaft of the grinding wheel is set exactly by the rotation of the double eccentric cylinder D.

All the important parts were finished very carefully. The maximum accumulative pitch error of the master worm wheel which rotates the work spindle was limited within 12 seconds and the cyclic error motion of master worm wheel caused per each turn of the master worm within 8 seconds. In the case of a large grinder manufactured recently, these errors were limited to within 6 and 2 seconds, respectively. The rotor was balanced and the backlash of the gears was adjusted to a very minimum. The work spindle was braked with a constant load, so that the accuracy during working might be maintained constant.

4. Grinding method

4.1 Features of grinding method with cup-shaped grinding wheel

The cup-shaped grinding wheel has greater rigidity than the dish-shaped grinding wheel, and is more suited to precision grinding. However, as the surface of the grinding wheel and the tooth surface being ground contact as it were enveloping each other, the arc of contact is long and the cooling and flushing effects of the grinding fluid cannot be achieved sufficiently. Consequently,

Fig. 6 Gear train of the grinder shown in Fig. 5

Fig. 7 Rotor of the grinder

there is a fear of burning of the tooth surface.

When a spiral bevel gear is generated by grinding, however, the lines of contact of the grinding wheel surface representing one tooth surface of the crown gear and the tooth surface being ground are as shown in Fig. 8. As the grinding is performed on these contact lines, it begins near line 1 and progresses through lines 2, 3 and finishes around line 6. The abrasive grain runs parallel with the root land of the tooth and cuts these contact lines obliquely. Thus the grinding fluid can be drawn in more easily and the cutting length of an abrasive grain is not too long. Thus, grinding is practicable, but the grinding should be performed skilfully with possible burning of the tooth surface in mind.

4.2 Grinding method adopted

Sharply dressed grinding wheels are used in the rough grinding, with ample grinding fluid poured, and the finish grinding is done lightly with smoothly trued grinding wheels, so that the wear of the grinding wheels may be nil. By this method, the authors succeeded in finishing the tooth surfaces on the whole circumference with accurate pitch and profile, without truing the grinding wheel halfway. A vitrified wheel of 60~80-J-m made of single crystal Alundum grain was used. As for the grinding fluid, Noritake-Cut was squirted at a rate of 10 l/min in the direction of wheel rotation so that the fluid may be drawn between the grinding wheel surface and the tooth surface. After truing, it is important that the nearly loosened grains be washed away completely by brushing the grinding wheel surface with the grinding fluid poured amply.

Contrary to conventional methods, the grinding wheel is set to the bottom of the tooth space from the very beginning, and the bevel gear is rotated minutely to bring the tooth surface into light contact with the grinding wheel face and then the grinding by the generating motion is started. If there is not accumulation of error by eccentricity or other causes, any heat-treatment distortion can be ground off safely with a grinding

Fig. 8 Line of contact of spiral bevel gear tooth-surface with the conical grinding wheel surface

wheel of good sharpness.

A mirror finish of 0.8μ can readily be achieved by one round of finish grinding after finish truing, following 2 or 3 rounds of roughing with 0.05~0.08 mm of depth of cut (see Table. 1).

When especially high precision is required, semi-finishing is performed before finish grinding, and with 2 or 3 times of sparking out after finish grinding, a mirror finish of 0.4μ is obtained as shown in Fig. 9. It is believed that this is one of the highest precision bevel gears ever ground.

5. Key to increase grinding efficiency

A slight error in relative mounting positions of a pair of bevel gears causes a great change in the bearing of teeth. The gear cutting and finish grinding should be performed with the apex of the pitch cone exactly at the machine center (O, in Fig. 7). On this grinder, the machine center is accurately indicated by the ball gauge, and the gear workpiece can easily be set at the correct mounting distance. When a set of pinion and gear is assembled, the apexes of both pitch cones can be made to coincide by setting each at the correct mounting distance from the intersection of the center lines of both shafts.

The workpiece should, of course, be so mount-

Table 1 Truing and grinding data

	Truing by diamond dresser		Grinding	
	Depth of cut mm	Feed mm/min	Depth of cut mm	Pitch circle velocity m/min
Rough grinding	0.03	1000	0.05~0.08	100~120
Semi-finishing	0.02	300	0.005~0.01	50~80
Finish grinding	(0.03-0.01-0)	30	0.005~0.01	50~80

$P_d=4$, $\alpha=20^\circ$, $\beta=0^\circ$, $z=16$, Cr-M₀ steel hardened, $H_s \approx 80$
Fig. 9 Zerol bevel gear ground on the authors' machine

ed that there may be no runout or wobble of the gear. To describe this in more detail, reference surfaces ① and ② are prepared near the tooth, and the cutting is made with zero runout of reference surface ① and zero wobble of the surface ②.

After hardening, the hole ③ is finished by grinding with reference to the surfaces ① and ②, and then the work piece is mounted on the accurate mandrel by the hole, and the surface ① and the mounting surface ④ or the surface ② are finished by grinding. Next, the workpiece is mounted on the grinder so that the runout and wobble of the surfaces may be nil. The mounting distance must be determined correctly as stated before. Any heat-treatment distortion should, of course, be minimized.

6. Profile modification method

It is of special importance with the hardened gears to give tip relief and root relief to the tooth profile. In the case of the circular-arc-toothed gears, an unmodified tooth profile can be obtained by truing with the diamond dresser moved in the direction of the cone surface element of the grinding wheel. If the diamond dresser is moved on a straight line contacting this cone surface at the pitch point P and cutting the cone surface element obliquely, the surface of the grinding wheel swells out gradually towards the tip and the root of the tooth and the modifying grinding wheel can easily be obtained.

7. Accuracy and performance of the spiral bevel gear finished by the authors' grinder

Fig. 11 shows the results of tests, on a Wakuri's single-flank bevel gear testing machine, on the parallel-depth toothed spiral bevel gears with

H_1 =mounting distance in grinding

H =mounting distance in cutting

Fig. 10 Mounting distance and reference-surfaces of gear

number of teeth $z_1=16$, $z_2=32$, $P_d=4$ and $\beta=35^\circ$, ground on the authors' grinder. The recording pen of the testing machine describes a true circle when there is no error. The protrusion on the top of the circle represents the advance of the driven gear caused by the insertion of a film of 0.12 mm in thickness between the mating gear teeth, and shows the magnification of this testing machine and also the gearing motion of a pair of the teeth. There is not the slightest error in either the profile or the pitch. The tooth surface is crowned and the tooth bearing is satisfactory.

The parallel-depth toothed spiral gears thus finished on this grinder were mounted on M-company's precision jig-boring machine 6AJB, and a noise test was conducted. Gg. D. T. E. Light Oil was used as the lubricant. The noise was much smaller than the conventional lapped bevel gears. The ground bevel gears made regular and pleasant sound. Three sets of bevel gears are used on this jig-boring machine, and they run at about twice the number of revolutions of the drilling spindle. The overall noise level of the machine was measured with a microphone set at the usual working location of the operator and facing in the direction of the drilling spindle. The noise level was 78 phons at 500 rpm of the drilling spindle. The boring machine operated as quietly as SIP6H (77 phons), the spindle of which was directly driven by belt from a motor, and far more quietly than SIP4G (84 phons). This shows clearly the excellence of the ground spiral bevel gears and the careful assembly of the gears.

$P_d=4$, $\beta=35^\circ$, $z_1=16$, $z_2=32$

Amount of crowning=0.11 mm

Fig. 11 The accuracy curve of the ground bevel gears obtained by Wakuri's single-flank gear-testing machine

8. Intermittent grinding method*

In grinding the circular-arc-toothed gears, as mentioned above, there is a fear of burning the tooth surface. The authors have well conducted the rough grinding of the gears, using sharp dressed and properly selected grinding wheels and pouring an ample amount of grinding fluid. But this is in the case of generating gear grinding. In the Formate gear grinding, as the gear surface comes in contact with that of the wheel all over, it is no wonder that the surfaces are doomed to burn. After many investigations the authors invented a new grinding method, or the "intermittent gear grinding" method. The principle of this method is as follows: The axis of a cup grinding wheel is revolved at medium or fairly low speed in a small circle while the wheel is rotated around its own axis at high speed. Fig. 12 shows a schematic diagram of the grinder on which the circular-arc-toothed cylindrical gear is being ground. The axis is revolved around the center O_A . Fig. 13 shows a concave tooth surface being ground by the outer conical surface of the

wheel. When the axis comes on the point a , the point a_1 laid on the line extrapolated from $O_A a$ by the distance r_a measured from the point a , is ground most deeply. And also, when the axis comes to the points b and c , b_1 and c_1 are ground most deeply, respectively. Therefore, as the grinding points move from a_1 to b_1 c_1 on the tooth surface, the concave-tooth surfaces are finished to a radius of $(r_a + e)$, where e is the radius of revolution. When the axis come on the point b' , the gap between the wheel and the tooth surface reaches an amount of $2e$. If the grinding fluid is squirted in the direction of arrow in Fig. 13, the fluid will be drawn between the wheel and the surface, and while the gap is made between them, flushing and cooling of the fluid are fully accomplished. Fig. 14 shows a convex tooth surface being ground with the inner conical surface of the wheel. The convex tooth surface is finished to a circular-arc having a radius of $(r_i - e)$.

In the intermittent grinding, the normal width of the tooth space must be larger than the sum of the thickness of the wheel $(r_a - r_i)$ and the double eccentricity $2e$, or the opposite tooth surface of the adjacent tooth will be ground. By adjusting e so as to satisfy the equation $w = (r_a - r_i) + 2e$, both concave and convex tooth surfaces can be ground in one operation. By adjusting e , the

Fig. 12 Schematic diagram of intermittent gear-grinding machine

Fig. 13 Schematic diagram of a concave tooth being finished by the outer conical surface of wheel

Fig. 14 Schematic diagram of a convex tooth being finished by the inner conical surface of wheel

Fig. 15 Experimental gear-grinding-machine having ability of intermittent grinding.

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amount of crowning can be varied to some extent too.

9. Gear grinder having ability of "intermittent grinding"

Fig. 15 shows the circular-arc-toothed gear grinder made at Kyushu University by the authors. The generating grinding is done by giving a rolling motion to the gear being ground so as to mesh ideally with the circular-arc-toothed rack which is represented by the cup wheel. Fig. 16 shows a schematic diagram of the intermittent grinding unit. The hole for the adjusting cylinder B is bored in the revolving cylinder A with an eccentricity of e_0 , and also the hole for the grinding wheel axis is bored in B with the same eccentricity. By turning the cylinder B, the wheel axis can be set to a position so as to have the desired radius of revolution, which can be varied in the range from zero to $2e$. In the experiment reported herein, the cylinder A was revolved by gears and V belts at a speed of 60 rpm and the radius e was set at 0.5 mm. The grinding spindle was directly coupled to the motor shaft running at a speed of 1700 rpm.

10. Experiment on intermittent gear grinding by generating process

The experiment was conducted, using the circular-arc-toothed cylindrical gears (module=4.85, $z=32$, $\alpha=20^\circ$, $b=40$ mm, $\beta=0^\circ$, Ni-Cr-Mo steel, hardness $H_s=80$) and a cup wheel of single crystal Alundum grain 60~80-J-m-Vit. The grinding surface of the wheel was dressed sharp for rough grinding and was trued smooth for finish grinding as follows:

a) Truing A diamond dresser is passed on the wheel surface at a speed of 30 mm/min.

Fig. 16 Schematic diagram of intermittent grinding unit

Depths of cuts of each pass are 0.05, 0.03, 0.01 mm, and zero.

b) Dressing At first, the wheel surface is trued smooth as mentioned above, and then the diamond dresser is passed once on the surface with a cut of 0.03 mm in depth at a speed of 1000 mm/min. Screw-shaped ridges of 0.03 mm in height and 0.6 mm in pitch are left on the wheel surface. With this wheel, free and cool grinding is performed and the ground surfaces of fairly good roughness are obtained on account of generating grinding.

To know the approximate temperature rise in the gear tooth during grinding a thermocouple was inserted to a depth of about 0.6~0.8 mm under the tooth surface near the pitch point. Noritake-Cut was used as the grinding fluid, and an amount of 3.2 l/min was poured in the direction of wheel rotation. Flow-out speed of the fluid is about 100 m/min.

If the grinding wheel contacts with the tooth surface M times to generate its profile, the profile of tooth ground is constructed with straight lines having a number of M . In this experiment, M is 24 and the maximum polygonal profile error of the tooth caused by the intermittent grinding is about $1.2\mu^*$.

Fig. 17 shows the record of the temperature variation close to the tooth surface in rough grinding. The line of contact on which the grinding is taking place advances from the tip to the root of the

Fig. 17 Recorded temperature curves in rough grinding by generating process

tooth, and as the grinding line moves to the measuring point or the pitch point, the temperature rises gradually and reaches maximum at the pitch point, then decreases gradually.

In the case of the intermittent grinding, the recorded temperature curves are waving with the period of intermittent grinding, while the temperature rise is in a range of from one-third to one-fourth that of the conventional grinding. Effects of sharp dressing on the temperature rise are remarkable too. Therefore, it is highly recommended that the wheel be dressed sharply in rough

* See Appendix (2)

grinding.

Fig. 18 shows a case of the finish grinding with a cut of 0.014 mm in depth. The temperature rise was only 6 °C in the intermittent grinding, while it was about 30 °C in the conventional grinding.

Thus, in the intermittent grinding, we can perform efficiently cool grinding with a depth of cut so deep that it can not be expected in the conventional grinding. But, it is likely that the grinding accuracy may be decreased owing to the revolving member added to the grinding unit. To the contrary, however, the accuracy of the gear finished by the intermittent grinding is better than that of the gears by the conventional grinding. Fig. 19 shows the error in action of the gear measured with a Wakuri's gear testing machine. The tested gears are ground by the intermittent grinding. In this figure, the mountain-shaped line projecting out of the circle in the upper part of it is drawn purposely by inserting a film of 0.12 mm in thickness between the mating teeth. The magnification of the machine and the gearing motion

Fig. 18 Recorded temperature curves in finish grinding by generating process

$m=4.85$, $\alpha=20^\circ$, $\beta=0^\circ$, $b=40$,
 $z_1=32$, $z_2=32$, $N_1\text{-C}_r\text{-M}_0$ steel, $H_S \cong 85$
Fig. 19 Accuracy curve of the cylindrical gears, obtained by Wakuri's gear testing machine

of a pair of the teeth are clearly shown by this test. From this accuracy curve, we know that the gears tested have a high degree of accuracy in pitch and profile, and will transmit an ideal smooth motion.

Fig. 20 shows a chart of roughness of the tooth surfaces ground by the intermittent grinding. There are some scratches on the surface ground by the conventional method, but no scratch on the surface ground by the intermittent grinding. This results from the effective flushing of the fluid, with which grinding chips and loose abrasive grains trapped in the pockets of the wheel surface may be swept away.

There is a simple effective grinding method, in which the wheel surface is grooved at several places with a pencil grinder, such as shown in Fig. 21. The grinding action of this wheel is the same as that of the conventional one, but the tooth sur-

Fig. 20. Roughness of tooth surface before and after intermittent grinding

Fig. 21 Cup-shaped grinding wheel grooved with a pencil grinder

face being ground is washed by the fluid in the grooves. When a large amount of fluid is poured, its cooling effect will be fairly good. Fig. 22 shows the temperature curve during the rough grinding with a cut of 0.112 mm in depth, in which the amount of fluid poured was 11 l/min. The maximum temperature rise was 122°C with the conventional grinding wheel, while it was only 29°C with the wheel having eight grooves. Grinding with the grooved wheel may be one way to prevent grinding burns, but its cooling and flushing effects are far inferior to those of the intermittent grinding, whose temperature rise was only 14°C.

11. Experiment on intermittent gear grinding by Formate process

In the Formate gear grinding, the intermittent grinding method displays its amazing ability to the full extent. In the conventional Formate grinding, an abrasive grain in the wheel contacts with the whole length of a tooth curve, while in the intermittent grinding, the grain contacts from b_1 to p in Fig. 23, where f is the feed of the wheel per one revolution of the grinding shaft*. In the experimental gears, θ_a is 0.07 rad., while the full contact angle θ is 0.344 rad. ($r_a=117.45$ mm, $e=0.5$ mm, $f=1.17$ μ /rev). Moreover, in the intermittent grinding, a gap is made during the non-contact

Fig. 22 Recorded temperature curves in grinding cylindrical gears by three different methods

Fig. 23 Schematic diagram for calculation of the arc of contact in intermittent grinding by Formate process

period as mentioned before.

The experiment was conducted by applying the wheel to the tooth surface, in the direction of the pitch line of the grinding rack. The gears were cut by the Formate method and then they were hardened by liquid-carburizing to a hardness of $H_s=80$. The dimensions of the gears and the grinding wheel used are as described in the section of generating grinding. Fig. 24 shows the recorded temperature curves in this Formate rough grinding. The temperature rise in grinding with a sharply dressed wheel is 80 °C in the conventional and is only 15 °C in the intermittent grinding.

In the Formate finish grinding, to know the time when the wheel must be re-trued to recover its cutting ability, the following procedure was taken: At first, the temperature curves are recorded, using a fresh-trued wheel. An example of these curves is shown in Fig. 25, in which the total depth being ground is 0.056 mm, the time needed is 48 sec., and the feed per revolution $f=1.2$ μ . And then, the grinding is continued without truing until one hundred teeth are ground. Fig. 26 shows the recorded tempera-

Fig. 24 Recorded temperature curves in Formate rough grinding

Fig. 25 Recorded temperature curves soon after truing

* See Appendix (3)

ture curves when each last tooth is ground. In the conventional Formate grinding, the fluctuations in temperature are more than 40 °C. They are indications of abnormal grinding or dullness of the wheel, accompanied with abnormal grinding noises. When the wheel becomes too dull, the workpiece cannot be ground on the first cut, but by the continuous feeding of the work, the theoretical depths of cut are accumulated, and the elastic deformation of the machine parts is increased. When the contact force to penetrate the abrasive grains into the workpiece reaches a maximum value, a cutting with a large depth of cut is suddenly done and a large amount of heat is generated.

Thus a sudden temperature rise is caused. The following temperature drop may be caused by the decrease in the depth of cut. So the fluctuation of temperature is caused by repeating a cycle of cutting and rubbing the tooth surface. A slight symptom of this phenomenon can be seen at the end of the temperature curve of the first tooth grinding.

The total wear of the wheel was about 20 μ . So the conventional Formate gear grinding can never be used.

In the case of the intermittent grinding, a normal good state of grinding was continued from the beginning to the end of the operation, accompanied with a pleasant sound. The wear of the wheel was less than 10 μ . The wheel can well stand the grinding of one hundred teeth, having a stock removal of 0.056 mm in each tooth (the precision finishing stock is usually about 0.01 mm), while the grinding time per tooth is only 48 sec. The wheel surface is kept in a good condition for a long grinding time.

Fig. 26 Recorded temperature curves after 5.6 mm grinding

The horsepower required in the intermittent grinding is very small; which is the most amazing fact in grinding. Fig. 27 shows the recorded input of the electric motor which is coupled directly to the wheel spindle. The approximate effective electric power needed for the grinding is expressed by

$$W = \eta(W_i - W_i),$$

where W_i , W_i , and η are the input during grinding, input idle running, and the efficiency of the motor, respectively. The effective power needed for the intermittent grinding with a feed of 0.21 mm/min, and 0.07 mm/min are one-eighth and one-sixth of that of the conventional grinding, respectively. As the stock removals of these grinding methods are equal, the maximum removal rate in the actual short grinding time of the intermittent grinding becomes greater. So the decrease of input may be ascribed partially to the thicker chips. But, the main reason for this will be the decrease in the frictional force between the wheel and the tooth being ground, which results from the effective action of the fluid. It can also be considered that the lubricating quality of the fluid is important in reducing the grinding friction.

These experiments show clearly that the intermittent grinding is effective not only in cooling and flushing but also in reducing the generation of heat. As the grinding time is immensely shortened and the surface finish is much improved by the intermittent grinding, this method will also be applied profitably to some other grinding fields.

Work piece: Ni-Cr steel ($H_s=75$)

Fig. 27 Recorded input of the motor of grinding shaft

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Appendix

(1) The pressure angles of both the right and the left cutting edges are α_n ; therefore all spiral bevel gears can be cut with the Gleason type spiral bevel gear cutter but only of No. Zero.

(2) The tooth profile is generated by the rack which moves as long as L along the pitch line. And in the intermittent grinding, the number of contacts of the wheel with the tooth surface to generate its profile is $M = \frac{NL}{V}$, where V is the velocity of the rack and N is the revolutions of the cylinder A per unit time. The polygonal profile error ϵ can be calculated by the following equation: $2\rho\epsilon = \left(\frac{\rho\theta}{2}\right)^2$, where θ is the angle of rotation of

the gear during one cycle of intermittence of the wheel, and ρ is the maximum radius of curvature of the tooth form.

(3) In Fig. 23, b is the present position of wheel axis, and p is the point where the abrasive grain begins to part from the tooth curve which has been finished just before one cycle of intermittence of the wheel. O_{Af} is the position of the center of this curve, and f is the feed of work-piece per one cycle, which is obtained by multiplying the axial feed of the wheel per one cycle by $\tan\alpha_n$. Seeing $\triangle pbO_{Af}$ the following equations are obtained;

$$(r_a + e)^2 = r_a^2 + (e + f)^2 - 2r_a(e + f)\cos\theta_a$$

$$\theta_a = \cos^{-1} \left[\frac{(r_a + e)^2 - r_a^2 - (e + f)^2}{2r_a(e + f)} \right]$$

The equation concerning a convex tooth is expressed by

$$\theta_i = \cos^{-1} \left[\frac{r_i^2 - (r_i - e)^2 + (e + f)^2}{2r_i(e + f)} \right]$$

The angle of contact for the whole face width $\theta = \sin^{-1} \left(\frac{b}{2r \cos \beta} \right)$ where β is a helix angle at the center of the face, θ_a and θ_i are very small as compared to θ , excluding the gears having a very narrow face-width.