clearance between the cylinders is small and the cylinders are rotating in the same direction with approximately equal velocity. Following this analysis, we can get approximately the disturbance when only the inner cylinder is rotating and the outer one is at rest.

Making the value of $u_1(y)$ with one at $y=1/2$ and comparing the results of Eq. (14) with those deduced from the analysis of Taylor, the simple form of Eq. (14) is found to agree with the analysis with good approximation as shown in Fig. 17.

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Fig. 17 Shape of the disturbance u'

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An Analysis of Combustion Knock in a Diesel Engine*

By Fujio NAGAO**, Makoto IKEGAMI***, and Koji OSHIMA****

The present work theorizes the phenomenon of rapid combustion in a diesel engine by introducing a concept that the ignition delay of the injected fuel element varies from time to time according to the changes in temperature and pressure of the charge. In this process the rate of heat release is multiplied by the rate of shortening of the ignition delay, thus causing an explosive combustion. In this paper, a theoretical analysis was conducted to establish the course of the changing delay and thereby the physical and chemical features of the process were described by simultaneous differential equations. Discussions were made on the influences of the following factors on the maximum rate of heat release; namely, ignition delay, injection rate, pilot injection and vaporization velocity of fuel. The results show that the present theory can successfully describe the qualitative nature of the combustion knock in a diesel engine.

1. Introduction

The so-called diesel knock, which originates

from the rapid pressure rise occurring in the cylinder, has a close connection with the quantity of combustible mixture that concerns itself with the early stages of inflammation, namely the auto-ignition. Such a quantity of the combustible mixture, however, cannot be measured directly and there is almost no theory by which we can predict the relationship between the ignition delay and

* Received 20th October, 1965.

** Professor, Faculty of Engineering, Kyoto University.

*** Assistant Professor, Faculty of Engineering, Kyoto University, Sakyo-ku, Kyoto.

**** Student, Postgraduate School, Kyoto University.

the rate of pressure rise or between the rate of fuel injection and the rate of pressure rise, excluding an empirical one presented by Austen and Lyn⁽¹⁾. The strongest obstacle to theorize them is the fact that the physical processes, such as atomization, distribution, mixing and vaporization of the fuel injected, are intricately interwoven with the chemical processes involving the complex phenomena such as preflame reaction.

In order to reveal the essential nature of the combustion knock, it will be of value to study each of the stated processes. At the same time, it seems equally important to find out the interrelation between the injection rate and the rate of heat release. In the present paper, without paying any detailed attention to the above-stated processes, an approach to the latter direction is planned by introducing a model of the combustion system, the essential of which lies in the concept that the ignition delay of every fuel element injected at different time grows shorter according to the changing state parameters of the cylinder charge. The analysis led to the conclusion that the qualitative nature of the combustion knock could be described somewhat successfully by introducing such a model.

2. Ignition delay and its time-variation

2.1 Increase in heat-release rate ensured from the time-variation of the ignition delay

Assume that fuel droplets distributed in a spray form over the space of the combustion chamber behave thermally independently of one another. If the combustion chamber were an infinite space, then the rate of heat release would be equal to the injection rate since the ignition delay of all droplets would be the same. When the volume of the combustion chamber is finite as is in the actual case, the ignition delay of successive fuel elements becomes shorter as the pressure and the temperature rise in the cylinder after the combustion starts, even if each fuel element is thermally insulated from another. The degree of the shortening of the delay will be to such extent that, in the stages of controlled combustion, fuel can burn immediately after it comes into the combustion chamber; namely, it burns almost without delay. In short, the fuel introduced earlier has a longer delay whereas the one introduced later has a shorter one. It is anticipated that fuel burns crowdedly sometime in the middle of the combustion stages when the ignition delay changes rapidly with time, leading to a higher rate of heat release than the injection rate. In Fig. 1 is shown schematically the relationship between the course

of the injection rate $B(t)$ and the development of heat-release rate $\dot{Q}(t)$ in which t denotes time. Each of the strips drawn in the figure indicates the same fuel element. Since the less the duration assigned to burning of a fuel element the higher becomes the rate of heat release, the density of the strips is proportional to the ratio of $\dot{Q}(t)$ to the corresponding rate of injection. In particular, when these strips coalesce in a point, the combustion would occur sometime all at once, thus resulting in an infinitely high rate of heat release. If such a process constitutes the origin of rapid combustion, we shall be able to obtain some fundamental knowledges on the diesel knock. In order to analyze the combustion process from this point of view, we ought to describe the influence of time-variation of the ignition delay on the heat-release rate and thereafter how the variation of the delay develops under the changing pressure and temperature.

For the sake of convenience we now introduce time variable z for scaling the injection time, by which we are able to distinguish it from the time variable t for measuring the heat release-rate $\dot{Q}(t)$. Let us take the commencement of the injection to be the origin of both time variables and assume the ignition delay of the fuel element introduced in the beginning to be a given magnitude τ_0 . If we write the ignition delay of fuel element introduced at an arbitrary time as τ , the following relation has to hold for the fuel element under consideration.

$$z = t - \tau \quad \dots\dots\dots (1)$$

Fig. 2 shows a comprehensive diagram of the changing ignition delay mechanism. As can be recognized from the figure, a fuel element introduced at z during an infinitesimal period Δz , which amounts to $B(z)\Delta z$, releases heat at a rate $\dot{Q}(t)$ during an infinitesimal period Δt . It will be

Fig. 1 Relation between injection rate $B(t)$ and heat-release rate $\dot{Q}(t)$

reasonably assumed that the heat introduced $B(z)dz$ is equal to the heat released $\dot{Q}(t)dt$. This may be described as follows:

$$\dot{Q}(t) = \frac{B(z)}{\frac{dt}{dz}} = \frac{B(z)}{1 + \frac{d\tau(z)}{dz}} \quad \dots\dots\dots(2)$$

The initial ignition delay τ_0 depends on the pressure and temperature of the quiescent gas before the combustion; for example, it obeys the Wolfer's formula in the following⁽²⁾.

$$\tau_0 = c p_0^{-n} e^{E/RT_0} \text{ msec} \quad \dots\dots\dots(3)$$

where p_0 denotes the initial pressure, T_0 the initial temperature, and c , n and E/R constants. Wolfer gave $c=0.44$, $n=1.19$ and $E/R=4650^\circ\text{K}$ when p_0 was expressed in ata. After the combustion starts, the ignition delay grows shorter with the rises in both pressure and temperature in the combustion chamber. The ignition delay of successive fuel element cannot be determined by any empirical formula since the pressure and temperature change from time to time during the delay period of a considering fuel element.

2.2 Ignition delay under changing pressure and temperature

Although many research works have been conducted experimentally to determine the ignition delay in the bomb or in the engine, these ignition delay data have been gained under the condition in which the process is assumed to be in a fixed state. Therefore, it is necessary for the present purpose to estimate the ignition delay of a changing environment during the induction period. There is almost no established rule of predicting it from the delay data gained at a fixed state, except one presented by Livengood and Wu⁽³⁾. Their method will be summarized briefly in the following: Let us put $p(t)$ and $T(t)$ as the changing pressure and temperature respectively. The ignition delay at a fixed environment is assumed to

be given as a function of instantaneous values of p and T . We denote this delay as $\sigma = \sigma(p, T)$. The reaction velocity during the delay period is not constant when p and T are changing. We assume that there is always a fixed relationship between the concentration of active center of reaction x and the relative time t/σ . Thus we can write the equation of reaction as follows:

$$\frac{d}{dt} \left(\frac{x}{x_c} \right) = \phi \left(\frac{t}{\sigma} \right) \quad \dots\dots\dots(4)$$

where ϕ denotes the function of reaction rate and x_c the critical concentration (constant). When $x/x_c=1$, fuel ignites. Integrating this equation, we get

$$\frac{x}{x_c} = \int_0^\tau \phi \left(\frac{t}{\sigma} \right) dt = 1 \quad \dots\dots\dots(5)$$

where τ is the unknown ignition delay. If we assume that for a fixed state process the reaction rate does not change with time (zero-th order reaction), the following relation has to hold:

$$\phi(t/\sigma) = 1/\sigma \quad \dots\dots\dots(6)$$

Therefore Eq.(5) can be written as the following form.

$$\int_0^\tau \frac{dt}{\sigma} = 1 \quad \dots\dots\dots(7)$$

This equation may be solved with the aid of ignition delay data and a knowledge of the state-time history of the process. By using this equation, Livengood and Wu tried to predict the condition of autoignition in a gasoline engine from the delay data gained by Rapid Compression Machine and the state history of gas measured by the velocity of sound technique. The predicted conditions of autoignition showed a fairly good agreement with the experimental ones. Eq. (7) seems applicable as well for predicting the ignition delay in a diesel engine which is now considered. As the ignition delay data, we use Wolfer's formula, that is,

$$\sigma(p, T) = c p^{-n} e^{E/RT} \quad \dots\dots\dots(8)$$

Since σ is a function depending on state parameters and does not indicate the ignition delay itself under the changing environment, σ may be designated as the ignition delay function. Now our problem is reduced to solving τ out of Eq.(7) by the aid of Eq.(8).

2.3 Ignition delay of fuel injected at a given time

For the convenience sake, we adopt the following notation:

$$f(h) = \int_0^h \frac{dt}{\sigma(t)} \quad \dots\dots\dots(9)$$

where h denotes a variable. If p and T are uniform over the total gas inside the combustion chamber, then the function $f(h)$ of the considered

Fig. 2 Comprehensive diagram of changing ignition delay

fuel element depends only on the time progress of p and T . Although a fuel element injected at different time has a different value of $f(h)$, $f(h)$ is the integral of a common function of $1/\sigma$. From the fact that the difference in $f(h)$ is merely the range of the integration, we can determine the ignition delay τ of the fuel injected at a given time z .

In the case of $z < \tau_0$: Let $t' = t - z$ be the time elapsed after injection of the considering fuel element. When t' is between zero and $\tau_0 - z$, no fuel has ignited yet in the combustion chamber, and when $t' = \tau_0 - z$, fuel injected starts to burn. Separating the left side of Eq. (7) into two parts at $t' = \tau_0 - z$, we obtain

$$1 = \int_0^{\tau_0 - z} \frac{dt}{\sigma(t)} + \int_{\tau_0 - z}^{t'} \frac{dt}{\sigma(t)} \quad (10)$$

$\sigma(t)$ in the first integral being equal to τ_0 and the second integral being able to alter the range of integration, Eq. (10) is reduced to

$$1 = \frac{\tau_0 - z}{\tau_0} + f(t' - \tau_0 + z) \quad (11)$$

As the ignition delay $\tau(z)$ is such t' as to fulfil this equation, we can reach the final expression

$$\tau(z) = f^{-1}(z/\tau_0) + \tau_0 - z \quad [z < \tau_0] \quad (12)$$

where f^{-1} denotes the inverse function of $f(h)$.

In the case of $z \geq \tau_0$: The considered injection time is later than the earliest inflammation by $z - \tau_0$. If we trace back to the start of burning in measuring time in Eq. (7), we have

$$1 = \int_{\tau_0 - z}^{t'} \frac{dt}{\sigma(t)} - \int_{\tau_0 - z}^0 \frac{dt}{\sigma(t)} \quad (13)$$

or

$$1 = f(t' - \tau_0 + z) - f(z - \tau_0) \quad (14)$$

Putting $\tau(z)$ on t' in this equation and solving it, ignition delay finally becomes

$$\tau(z) = f^{-1}\{f(z - \tau_0) + 1\} + \tau_0 - z \quad [z \geq \tau_0] \quad (15)$$

2.4 Relationships between $z(t)$, $f(h)$ and $\sigma(h)$

As the time variable z is related with the time variable t by Eq. (1), we get the following relationship for $z < \tau_0$.

$$t = f^{-1}(z/\tau_0) + \tau_0 \quad (16)$$

or

$$z = \tau_0 f(t - \tau_0) \quad (17)$$

The latter expression implies that the z versus t curve is quite similar to $\tau_0 f(h)$ for $z < \tau_0$. The gradient of the z versus t curve is, from Eqs. (9) and (16),

$$\frac{dt}{dz} = \frac{\sigma(t)}{\tau_0} \quad (18)$$

In the case of $z \geq \tau_0$, we get a similar relationship from Eq. (15), that is,

$$t = f^{-1}\{f(z - \tau_0) + 1\} + \tau_0 \quad (19)$$

Transforming Eq. (19) by using the next function

$$F(h) \equiv \tau_0(h - \tau_0) \quad (20)$$

then we get

$$F(t) = F(z) + \tau_0 \quad (21)$$

Through a second transformation

$$G(h) \equiv F(h) + \tau_0 \quad (22)$$

we obtain a simple expression as follows:

$$F(t) = G(z) \quad (23)$$

We can now find out the following geometrical relations when the coordinate (F, h) is superimposed on the coordinate (z, t) , as shown in Fig. 3. Drawing a curve ABC as $F(h)$, it is clear from Eqs. (17) and (20) that for $z < \tau_0$ the z versus t curve coincides with $F(h)$ while, for $z \geq \tau_0$, i.e. in the region below point D, the curve $z(t)$ departs from $F(h)$. Since $G(h)$ is a parallel displacement of $F(h)$ by τ_0 , we may draw $G(h)$ as the curve A'B'C'. A fuel element that burns at time t , as represented by line OH, has been injected at time z which is represented as line OK=OL. Therefore, $G(z)$ can be represented by line B'L. At the same time, $G(z)$ is equal to $F(t)$ from Eq. (23), namely, equal to CH. In other words, through the four apices of a rectangle JB'CP pass the following four curves; $z=t$, $G(h)$, $F(h)$ and $z(t)$ respectively. Thus $z(t)$ becomes uniquely determinable.

Next, we derive the differential quotient of $z(t)$ for $z \geq \tau_0$. Differentiating Eq. (23), we have the following relation.

$$\left(\frac{dt}{dz}\right)_P = \frac{(dG/dh)_{h=z}}{(dF/dh)_{h=t}} = \frac{(dG/dh)_{B'}}{(dF/dh)_C} \quad (24)$$

This relation indicates that the gradient of the curve $z(t)$ at point P is given by the ratio of the slope at B' to that at C. Using Eqs. (20), (22) and (9), we obtain

Fig. 3 Geometrical relationships between $z(t)$, $F(h)$ and $G(h)$

$$\frac{dt}{dz} = \frac{d\{f(z)\}}{d\{f(t)\}} = \frac{\sigma(t)}{\sigma(z)} \quad \dots\dots\dots(25)$$

This final expression can be easily extended to the case of $z < \tau_0$: Since the state parameters are invariable during the initial delay period, $\sigma(z)$ is clearly equal to τ_0 for $t < \tau_0$, therefore Eq. (25) is reduced to Eq. (18). This matter is quite equivalent to drawing a straight line OA' as the continuation of $G(h)$ in the range $h < \tau_0$. Finally, Eq. (25) will serve to calculate the relation between the injection rate $B(z)$ and heat-release rate $\dot{Q}(t)$. Putting Eq. (25) into Eq. (2), we have

$$\dot{Q}(t) = B(z) \frac{\sigma(z)}{\sigma(t)} \quad \dots\dots\dots(26)$$

It is noted that the ignition delay is implicit in Eqs. (25) and (26).

3. Combustion knock without considering fuel vaporization

3.1 Governing equations

Changing ignition delay and its effect on the heat-release rate were described in the last chapter. In order to estimate the time developments of heat-release, pressure and temperature we make the following assumptions in the calculation:

(1) Each of fuel elements does not release any heat during the delay period and as soon as the fuel element ignites, it is converted into final products releasing the total energy of fuel.

(2) Fuel droplets are fine and uniformly distributed over the combustion chamber. The pressure and temperature are macroscopically uniform, but there is no microscopic interaction between fuel particles.

(3) Both the total weight of gas and the volume of the combustion chamber do not change throughout the combustion and the gas obeys the ideal air law.

(4) Vaporization time of a fuel droplet is negligibly short.

Denoting gas weight and volume of the chamber as G and V respectively, we have the following relations:

$$\dot{Q} = c_v G \dot{T} \quad (\text{First law of thermodynamics}) \dots\dots\dots(27)$$

$$pV = GRT \quad (\text{Ideal gas law}) \dots\dots\dots(28)$$

where c_v denotes specific heat at constant volume ($=0.171 \text{ kcal/kg}^\circ\text{K}$), and R gas constant ($=29.3 \text{ m}^\circ\text{K}$). Unknown variables are $\dot{Q}$, p , T , σ and z . For these five unknowns, Eqs. (8), (25), (26), (27) and (28) are given as the simultaneous differential equations. Among variables, B , $\dot{Q}$ and G are intensive quantities so that we can consider them as the quantities per unit weight of gas. Therefore G is unity in the present case. Because

of nonlinearity of the governing equations, all solutions were obtained by numerical method.

3.2 Influence of ignition delay

At first, the calculations were made for the following conditions; initial pressure $p_0 = 40 \text{ kg/cm}^2$ and injection rate $B = 0.2 \text{ kcal/g}$ of air/msec (of course $B = 0$ when $z < 0$). In order to evaluate the effect of the initial ignition delay, the following four cases were taken; $\tau_0 = 0.5, 0.75, 1$ and 1.25 msec, in which the initial gas temperature T_0 was properly selected so as to fit the stated conditions. The solutions are inclusively drawn in Fig. 4.

As seen from the figure, $z(t)$ starts with a gradient of unity ($dz/dt = 1$) and accelerates toward the maximum gradient at $z = \tau_0$ where $\dot{Q}$ reaches its maximum. Thereafter the curve $z(t)$ quickly approaches the asymptotic line $z = t$ and $\dot{Q}(t)$ does $B(t)$. Since the pressure rise is proportional to $\int \dot{Q}(t) dt$, the pressure-time curve is S-shaped at first and asymptotically approaches a certain line. It can be seen that both the maximum rate of heat-release and the maximum rate of pressure rise increase rapidly with the initial

Fig. 4 Influence of initial ignition delay under a step-functional injection rate (numerical values written in each peak position of $\dot{Q}$ indicate the maximum rate of pressure rise)

ignition delay. This is in accordance with the trend in practical engines.

3.3 Influence of injection rate

Under the initial conditions of $p_0=40 \text{ kg/cm}^2$ and $\tau_0=1 \text{ msec}$ ($T_0=892^\circ\text{K}$), the influence of injection rate B on the heat-release rate was investigated. The results of numerical calculation are presented in Fig. 5. As can be seen from this, the injection rate has a large influence on the maximum rate of heat release and its dependence can be roughly expressed by the following form in the range where the calculations were made:

$$\dot{Q}_{\max} \propto B^{2.7} \dots \dots \dots (29)$$

This result seems to be supported by the current understanding that reduction in the injection rate is effective for reducing the combustion knock. In the later stages of combustion when the maximum heat-release rate is over, the ignition delay of the fuel element tends to zero so that a larger injection rate is allowable in such stages. Consequently, such an injection rate diagram may be ideal that the rate is low during the delay period but is high after the delay period is finished.

3.4 Effect of pilot injection

Next, the injection rate versus time relation was estimated for a given pressure development. From the viewpoint of knock suppression, the maximum rate of pressure rise is recommended to be less than $4 \text{ kg/cm}^2/\text{sec}$ in crank angle in the practical engine. Therefore we can assume the ideal

Fig. 5 Influence of injection rate

pressure development to be a ramp function at a fixed rate of $\dot{p}=36 \text{ kg/cm}^2/\text{msec}$ at an engine speed of 1500 rpm. In Fig. 6 are shown the calculated injection rate time relations, together with the given pressure developments of $\dot{p}=36 \text{ kg/cm}^2/\text{msec}$ and $24 \text{ kg/cm}^2/\text{msec}$. It can be seen from the figure that a quite low rate of pressure rise can be realized by adopting the pilot injection characteristics; namely, considerably high injection rate at first which is followed by a gradual decrease until the ignition and a rapid growth to a constant injection rate. If the injection rate is not high enough in the earliest stage, then the pressure rise will be less sharp at its very beginning since the initial high injection rate concerns with the curvature of the pressure rise.

4. Combustion knock with fuel vaporization considered

4.1 Relation between injection rate and velocity of vaporization

Because all of the physical properties of spray, such as distribution and vaporization of fuel, were neglected in the foregoing treatment, the numerical values of heat-release rate obtained from the foregoing calculations gave impractically large ones. For example, let us consider an engine operating at a speed $n=1500 \text{ rpm}$, excess air ratio $\lambda=1.5$ and duration of injection $\theta=20^\circ$. If the stoichiometric air L_0 and real calorific value H_u are 14.5 kg/kg and 10^4 kcal/kg respectively, then the average injection rate B will be

$$B = \frac{6nH_u}{(1+\lambda L_0)\theta} = 0.198 \text{ kcal/g of air/msec} \dots (30)$$

The injection rate $0.2 \text{ kcal/g of air/msec}$, the

Fig. 6 Effect of pilot injection

value of which was presented in the previous chapter, was a proper estimate. Further, if the initial ignition delay is 1 msec, that is, 9° in crank angle, which seems to be an entirely reasonable value for a practical engine, then the theoretical maximum rate of pressure rise, from Fig. 4, amounts to $2990 \text{ kg/cm}^2/\text{msec}$ or $332 \text{ kg/cm}^2/^\circ$. This is extraordinarily high for the value of usual operating condition, considered from our experience. One of the causes of its overestimate seems to be the fact that the vaporizing velocity of the fuel injected was assumed to be infinite in the calculations.

Hitherto, several investigations were carried out about the vaporization process of finely atomized fuel droplets. Among them, the simplest theory will be sufficient for evaluating the effect of velocity of fuel vaporization on combustion knock, because no quantitative argument is necessary for our present purpose. Hence, assuming an iso-thermal quasi-stationary process of vaporization of an aggregate of the same-sized fuel particles and neglecting the heat-up time, we determine the relation between injection rate and velocity of vaporization. Thus the droplet diameter d at time z is

$$d^2 = d_0^2 - Kz \quad (31)$$

where d_0 is the initial diameter and K the coefficient of velocity of vaporization. K is constant in the iso-thermal quasi-stationary process. Change in weight of a particle w per unit time is

$$-\frac{dw}{dz} = -\frac{d}{dz} \left\{ \frac{\pi}{6} \bar{\gamma}_b d^3 \right\} = \frac{\pi}{4} \bar{\gamma}_b K \sqrt{d_0^2 - Kz} \quad (32)$$

where $\bar{\gamma}_b$ denotes the specific weight of the fuel in liquid phase. As the initial weight of a particle is equal to $(\pi/6)\bar{\gamma}_b d_0^3$, vaporization velocity $\psi(z)$, when fuel of unit weight is instantaneously introduced, is

$$\psi(z) = \begin{cases} \frac{3}{2z_0} \sqrt{1 - \frac{z}{z_0}} & [0 \leq z/z_0 < 1] \\ 0 & [1 \leq z/z_0] \end{cases} \quad (33)$$

where z_0 is vaporization time: $z_0 = d_0^2/K$(34)

When the injection rate $B(z)$ is given, the rate of vaporization $C(z)$ can be obtained by superposition using $\psi(z)$ as the weighting function, namely,

$$C(z) = \int_0^z B(\theta) \psi(z-\theta) d\theta \quad (35)$$

If $B(z)$ is a step function, $C(z)$ is

$$C(z) = \begin{cases} B \left\{ 1 - \left(1 - \frac{z}{z_0} \right)^{3/2} \right\} & [0 \leq z/z_0 < 1] \\ B & [1 \leq z/z_0] \end{cases} \quad (36)$$

The coefficient of vaporization is estimated on the basis of Sitkei's description⁽⁴⁾. Owing to quasi-stationary process, the heat transferred from the surrounding air to the surface of droplet, q , is equal to the heat carried away by mass transfer. Thus we have

$$q/\pi d^2 = \alpha(T - T_L) = \beta(p_L - p_K)l \quad (37)$$

where α and β denote transfer coefficients of heat and matter respectively, T and T_L temperatures in the surrounding and at the surface of the droplet, p_K and p_L partial pressures of fuel vapor in the surrounding and at the surface respectively, and l latent heat plus superheat. Assuming that the droplet diameter is so small that the fuel vapor is transferred outward by molecular diffusion, namely, the Nusselt number is two, we get the following relationship.

$$\alpha d/\lambda = \beta d/D_p = 2 \quad (38)$$

where λ denotes the thermal conductivity and D_p the diffusion coefficient. If we neglect any interaction between droplets, p_K is naught in Eq. (37), thus leading to

$$\lambda(T - T_L) = D_p l p_L \quad (39)$$

p_L and D_p are given by the following formulae:

$$D_p = D_{p0} \frac{T}{T_n} \frac{p_n}{p} \quad (40)$$

$$p_L = a e^{-b/T_L} \quad (41)$$

where p denotes the total pressure, p_n and T_n reference pressure and temperature (273°K and 1 kg/cm^2 respectively), and a, b and D_{p0} constants. Solving simultaneously Eqs. (39) through (41), we obtain p_L , T_L and D_p . The velocity of vaporization of a droplet is

$$-\frac{dw}{dz} = \pi d^2 \beta p_L \quad (42)$$

so that K and z_0 become, by comparing Eq. (42) with Eq. (32),

$$K = \frac{8D_p p_L}{\bar{\gamma}_b} \quad (43)$$

and

$$z_0 = \frac{d_0^2 \bar{\gamma}_b}{8D_p p_L} \quad (44)$$

4.2 Results of calculation

Given $p_0 = 40 \text{ kg/cm}^2$ and $T_0 = 892^\circ\text{K}$ (see Fig. 5), K and z_0 were calculated by using Sitkei's data⁽⁴⁾ ($D_{p0} = 0.121 \times 10^{-4} \text{ m/hr}$, $a = 6.0 \times 10^7 \text{ kg/m}^2$, $b = 4.15 \times 10^3$ °K). We get $T_L = 602^\circ\text{K}$ and $K = 1.30 \times 10^2 \mu^2/\text{msec}$ at $l = 280 \text{ kcal/kg}$, so that for two kinds of initial droplet diameters $d_0 = 10 \mu$ and 20μ

Fig. 7 Vaporization velocity of a step-functional injection rate

the vaporization time z_0 is 0.77 msec and 3.10 msec respectively. Fig. 7 shows the vaporization rate versus time relation $C(z)$ of these droplets at $B = 0.2$ kcal/g of air/msec. Using the vaporization rates thus obtained, the heat-release rate $\dot{Q}(t)$ as well as the pressure developments was computed. The results are summarized in Fig. 8, in which vaporization rates are assumed not to be influenced by combustion throughout the process. It can be seen that the heat-release rate and the rate of pressure rise are remarkably lower than those when the influence of the vaporization is neglected ($d_0 = 0 \mu$ in the figure). The maximum rate of heat release at 20μ of initial diameter is only about one tenth that at 0μ . By the way, another finding is that the larger the ignition delay the later comes the moment of maximum rate of heat release.

Assuming the droplet diameter to be 20μ and neglecting the temperature dependence on the vaporization velocity, the heat-release rate was calculated for various ignition delays under the same condition of Fig. 4. Fig. 9 shows the results. As seen from the figure, $\dot{Q}_{\max}$ rapidly decreases as the initial ignition delay τ_0 is reduced. The maximum rates of pressure rise become nearer to the practical value as compared with the case of Fig. 4; if we compute them in terms of crank angle basis at an engine-speed of 1500 rpm, they are 80, 33, 17 and 6.3 kg/cm²/deg for $\tau_0 = 1.25, 1.0, 0.75$ and 0.5 msec respectively.

In the above-stated theory, the vaporizing

velocity of fuel was calculated on the assumption of molecular diffusion process. This assumption is correct only when the droplets are very fine or quiescent. However, there are large particles and a large relative-velocity between air and a droplet owing to injection, turbulent diffusion and air swirl. It can be considered that the calculation based on the molecular diffusion gives the lower limit of the theoretical rate of heat release whereas the calculation of the infinite vaporizing velocity describes its higher limit.

5. Consideration on the theory

Although the qualitative nature of the relation between injection rate $B(z)$ and heat-release rate $\dot{Q}(t)$ was fairly well established, there are several kinds of assumptions put in this theory which hinder its application to the practical case. In order to improve the theory we must consider the following matters in the first place.

(1) Ignition delay: Strictly speaking, Wolf-er's formula here adopted is valid for a relatively long ignition delay. In the present calculation an extrapolation was made to too short delay. Therefore a more proper selection of the expression for the ignition delay will be required when quantitative results are expected. According to the present knowledge, the ignition delay consists of two parts, namely, the physical delay and the chemical delay. Since the latter is very short at a high temperature range, the physical

Fig. 8 Influence of particle size

Fig. 9 Influence of ignition delay under consideration of fuel vaporization (notice the scale of $\dot{Q}$)

delay occupies an important part of the total induction period. In such a condition, the approximation of the zero-th order reaction is not correct and a more adequate treatment becomes necessary. However, it does not seem difficult to make up for the defect of our theory by applying the many detailed theories and their results concerning the physical delay⁽⁵⁾.

(2) Heat evolution of an element of fuel: In the theory it was assumed that each of the fuel elements did not produce any appreciable energy release during the delay period whereupon it was suddenly converted into final products of reaction. Fig. 10 shows schematically the discontinuous evolution of heat of an element as drawn in the solid line. This discontinuous process is the first approximation which was already adopted by Crocco and Cheng in the analysis of combustion instability of a rocket motor⁽⁶⁾. The actually encountered heat evolution, however, will follow an imperfect step-function like the broken line drawn in the same figure, which originates from several effects; namely, gradual oxidation during the delay period, decrease in the concentration of reactants, their poor mixing and so forth. The precise descriptions of them would require detailed knowledges of all the intermediate processes. This is impossible at present. In order to estimate the effect of deviation from the pure discontinuity, let us consider the reaction time defined as the half-value width of the rate diagram of heat-release evolution. Since the reaction time may be considered to be the same order of the rapidness of autoignition of homogeneous mixture in a gasoline engine, we can expect that the magnitude of the reaction time will be small enough to neglect in the calculation of pressure and temperature-developments when the velocities of injection and vaporization are relatively low.

(3) Influence of flame propagation: The considered combustion system has a positive feedback loop through the changing delay time sensitive to the previous courses of temperature and

Fig. 10 Schematic combustion progress of an element of fuel

pressure. Owing to this, a rapid burning can occur even if microscopic structure of dispersed fuel particles is essentially adiabatic without occurrence of any flame propagation. On the other hand, many experimental observations made by high-speed photography of the diesel combustion chamber show that an ignitable mixture burns very rapidly once ignition occurs from one or several nuclei, the flame spreading over the spray jet and covering the entire combustion space within a very short duration, just like the flame propagation. Austen and Lyn, who inquired into the combustion process of a direct-injection engine, stated that the process occurring in early stages of combustion lies in the border region of simultaneous autoignition and flame propagation⁽¹⁾. Since the flame propagation can get ahead of successive autoignition when the latter process is slow, it seems probable that simultaneously with successive autoignition the flame propagates both apparently and substantially. But, the speed of propagation itself cannot be so fast as considered from the normal flame speed in a gasoline engine. Alcock and Scott saw no correlation between the diesel knock and the flame speed and found that the spreading velocity of apparent flame front was far less than the velocity of detonation or shock wave⁽⁷⁾. These matters probably support the fact that the flame propagation alone cannot produce an explosive nature of combustion in a diesel engine.

6. Conclusion

From the abovementioned analysis, we reach the following conclusion: The nature of the combustion knock in a diesel engine can be well described by introducing the concept that the ignition delay of the injected fuel element varies from time to time according to the changes in temperature and pressure of the charge. In this process, the rate of heat release is multiplied by the shortening velocity of the ignition delay thus resulting in an explosive nature of combustion. By using this theory, we can evaluate the effects of various factors on the rate of pressure rise in the cylinder.

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Discussion

M. OGASAWARA: I would pay my respects to your skillful approach to the very complex phenomenon by introducing a new method. Although some assumptions made in the paper might have been purposely adopted for removing the complexity in the treatment of the phenomenon, I want to ask you about them in the following points:

(1) In the first part of your paper, you put an implied assumption of simultaneous injection and evaporation as well as formation of a homogeneous mixture. Indeed too high rate of pressure rise could be modified toward a reasonable one by taking account of fuel vaporization in the second part of your paper, but the second part depended on the same assumption that the ignition of the fuel occurred after its perfect evaporation. In the actual case, however, the greater part of the fuel will fly in the state of droplet after the injection and atomization and will ignite on its path. Therefore, the lifetime of its burning must be taken into consideration in the theoretical treatment. So, I think the assumption (1) used in Chapter 3 does not hold. Further, it may be expected that the rate of pressure rise will be better approximated to the actual value by taking account of the lifetime of burning. What do you think about it?

(2) Regarding the ignition delay, E/R in Eqs. (3) and (8) is, in my opinion, not so well-founded from a theoretical viewpoint, therefore an empirical value of 4650 °K must be used in those formulae instead of E/R . I think that putting E as activation energy will exaggerate the chemical cause of the ignition delay. As the physical delay is more predominant than the chemical delay in a hot atmosphere as exists in the diesel engine, it will be more reasonable to suppose that the above expression of the delay includes both delays, won't it? In short, I will recommend you to use not E/R but pure numerical value in the expression of the delay.

Y. IGUCHI: I would like to express my appreciation for your efforts with which you have developed a theory of the combustion knock in a diesel engine, the nature of which was very difficult to be approached. But, I am sorry for lack of any experimental confirmation of the theory.

(3) In your paper, it is assumed that the

ignition delay becomes shorter from time to time after the combustion starts. However, the delay does not always become shorter depending on a decrease in oxygen content due to combustion and changes in the pressure and temperature caused by crank motion. What do you think about it?

(4) The present theory depends on a concept that the fuel introduced burns in order. But, in the actual case, it seems possible that the fuel introduced later burns earlier than that introduced earlier, doesn't it?

(5) Effect of pilot injection will differ largely with the timing of fuel injection. How about this?

Authors' closure

(1) Neglecting the ignition and burning of the individual droplets en route, authors put an assumption that the vaporized fuel, which was the total of the individual droplets, came into the reaction of autoignition. It is true that burning in the state of droplet is one of the most important factors in investigating the later stages of combustion, that is, after-burning period, but the total evaporation seems to play a more important role than the burning of droplets in the stages of knocking combustion since it comes very soon after the ignition. For this reason, the burning of the individual droplets was neglected in the present treatment. When the flame front formed around each of droplets is taken into consideration, assumptions (1) and (2) in Chapter 3 are not true and the analysis will be very difficult.

(2) Authors agree with Dr. Ogasawara on the concept of the ignition delay, as has been described in Chapter 5. Instead of E/R , another notation should have been adopted in the delay formula. Because the authors' analysis started with the theory of chain reaction, on the basis of which both Wolfer's formula and Livengood-Wu's theory were derived, the authors carelessly used the stated notation thus leading to an impression of the activation energy.

(3) Indeed the authors are sorry that the theory has not been yet undergone experimental confirmation, but the theoretical results obtained seem to agree with our experience in tendency. The answer for the first question is as follows: As has been pointed out, it does not always follow

that the ignition delay of successive fuel elements becomes short, if we take into consideration the retardation of burning due to decrease in the oxygen content as well as the change in crank angle. But, their influences on the combustion knock do not seem very large in themselves. The influence of crank angle, which does not seem difficult to analyze, will be examined in the near future.

(4) Authors cannot fully understand the meaning of the question. But, in the earliest model described in Chapter 2, it is illogical that the fuel introduced later burns earlier than that introduced earlier; otherwise the ignition delay of a given history of pressure and temperature cannot be reduced to a certain value, thus preventing any progress in arguments. In the modified model presented in Chapter 4, the stated matter is true since the sequence of evaporation of the fuel does not necessarily accord with that of injection, where no sooner has any fuel portion evaporated

than it comes into the process of induction. In the actual combustion chamber, the sequence of ignition will not be uniform owing to several causes. If their rules are once known, it will be possible to improve the present theory.

(5) The injection timing at pilot injection is considered from two viewpoints: The one is the interval between the auxiliary injection and the main injection, and the other is voluminal change in combustion chamber caused by a piston movement. The latter effect may be easily estimated by appending to Eqs. (27) and (28) the term of voluminal change. But, this is not carried out yet. As regards the interval of the injections, it can be expected from Fig. 6 that an insufficient interval will result in a high rate of pressure rise, missing the effectiveness of the pilot injection, whereas an excessive interval will yield a low rate of pressure rise, to be sure, but a long injection period.

Errata (Vol. 10, No. 38, April, 1967)

Author	Page	Column	Line	For	Read
S. KITAGAWA et al.	245	left	1 from bottom	desinated	designated
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S. MATSUSHIMA	260	Ref. (1)	5 7	Vol. 5, No. 12 (1956), Vol. 8, No. 742;	Vol. 5, No. 12 (1954), Vol. 8, No. 74 (1959), p. 844, 937; Vol. 9, No. 87 (1960), p. 742;
		Ref. (13)	26 from bottom	for Metallurgy, (1957),.....	for Metallurgy), (1957),.....
		Ref. (20)	6 from bottom	and R. P. Feler,	and P.R.Felgar,.....