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Strength Design of Marine Diesel Engine Crankshaft*

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The author describes empirical formulae of bending and torsional stress concentration factors for a crankshaft of marine engine and analytical methods for calculation of bending and torsional moment as well. He also describes a method of dynamical stress measurement and cites some examples of the results of analysing the stresses on crank fillet and additional stresses due to vibrations in crankshaft.

Furthermore, bending and torsional fatigue tests of model crankshafts and defective materials making use of large specimens are summarized.

The author concludes that the stresses of crankshaft can be approximately calculated by his investigations and strength design of crankshaft may become more precise when the additional stresses due to vibration are exactly calculated.

1. Introduction

In the designing of marine engine, the sizes and dimensions of crankshaft have been generally chosen to comply with the requirements of the classification societies and those of the engine performance, because both stresses and strengths have not been precisely estimated. Nevertheless, in the above design practice established by past experiences, the rate of failure of the marine engine crankshaft has been very low.

In recent years, however, the rate of crankshaft failure is becoming higher gradually and this fact may be caused by the conventional design practice becoming in appropriate due to the remarkable increase of engine power. Therefore, investigations on stressing and strengths of marine engine crankshaft have recently been made by many investigators, intending to establish a new design practice.

The recent investigations may be broadly divided into the following items:

- (1) Distribution of bending and torsional moment in multithrow crankshaft^{(1) (2)}.
- (2) Bending and torsional stress concentration factors on crank fillet^{(3) ~ (5)}.
- (3) Torsional and axial vibration in crankshaft^{(6) (7)}.
- (4) Rotary bending and twisting fatigue tests on model crankshafts^{(8) ~ (10)}.
- (5) Fatigue tests on defective material^{(11) (12)}.

(6) Dynamical stress measurements on crank fillet in service conditions⁽¹³⁾.

The author and his colleagues have studied the above items for about ten years and deduced a proposal for strength design of marine engine crankshaft.

2. Analytical calculation of stresses on crank fillet

2.1 Stress concentration factors on crank fillet

Statical stress measurements have been carried out by many investigators^{(3) ~ (5) (14)} and the stress concentration factors defined by respective methods have been reported.

Bending stress concentration factor of a crankshaft under a concentrated load shown in Fig. 1, may be defined as follows:

Fig. 1

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(1) Maximum stress on fillet divided by a nominal stress on crank pin center; $\sigma_{pf}/(WL/4Z)^{(14)}$.

(2) Maximum stress on fillet divided by a nominal stress on crankweb; $\sigma_{pf}/(M_w/Z_w)^{(3)(4)}$.

(3) Maximum stress on fillet divided by a nominal stress M_w/Z ; $\sigma_{pf}/(M_w/Z)^{(5)}$.

Where, W , L and M_w are shown in Fig. 1. Z : section modulus of crank pin or journal against pin fillet or journal fillet, respectively. Z_w : section modulus of crankweb.

One of the author's colleague has statically measured the fillet stress on 151 crankshafts by using wire strain gauges of 2~5mm in gauge length, and derived the following empirical formulae⁽⁵⁾ for a solid crankshaft of marine diesel engine. The nomenclatures in these formulae, are shown in Fig. 2.

The stresses on crank fillet under twisting moment consist of a simple shear stress and two dimensional normal stresses as shown in Fig. 3 except that on crank plane.

$$\alpha_{kb} = 4.84 f_1 f_2 f_3 f_4 f_5 \dots (1)$$

$$f_1 = 0.420 + 0.160 \sqrt{d/r - 6.864}$$

$$f_2 = 1 + 81 \{ 0.769 - (0.407 - s/d)^2 \} (\delta/r)(r/d)^2$$

$$f_3 = 0.285(2.2 - b/d)^2 + 0.785$$

$$f_4 = 0.444(d/t)^{1.4}$$

$$f_5 = 1 - (s/d + 0.1)^2 / (4t/d - 0.7)$$

where, $8 \leq d/r \leq 27$, $0 \leq \delta/r \leq 1$, $-0.3 \leq s/d \leq 0.3$

$$1.33 \leq b/d \leq 2.1, 0.36 \leq t/d \leq 0.56$$

It is difficult, therefore, to give a general

Fig. 2

Fig. 3 Principal stresses on fillet

empirical formula to calculate the stresses due to twisting moment on any portion of crank fillet.

The author's colleague has carried out statical stress measurements on 22 model cranks by using wire strain gauges of 2mm in gauge length, and established the following empirical formulae⁽²⁾ to calculate the torsional stress concentration factor on the crank plane, on which the factor is simply defined as follows:

$$\alpha_{kt} = \tau_{pf} / (T/Z_p)$$

where τ_{pf} : maximum shear stress on fillet on crank plane.

T : twisting moment.

Z_p : polar modulus of crank pin or journal.

$$\alpha_{kt} = 1.75 q_1 q_2 q_3 \dots (2)$$

$$q_1 = 31.6(0.152 - r/d)^2 + 0.67$$

$$q_2 = 1.04 + 0.317 s/d$$

$$q_3 = 1.31 - 0.233 b/d$$

where, $0.022 \leq r/d \leq 0.143$, $-0.286 \leq s/d \leq 0.222$

$$1.14 \leq b/d \leq 2.00, 0.30 \leq t/d \leq 0.588,$$

$$0.489 \leq l_p/d \leq 0.857$$

2.2 Bending and twisting moment

The bending and torsional stresses are calculated by multiplying the stress concentrating factors to the nominal stresses which are given by the quotient of the moments in the crankweb divided by section modulus of crank pin.

Bending moment M_w in crankweb can be computed by presuming the following;

(1) Multi-throw crankshaft is convertible to

Fig. 4 Calculation of bending moment

a continuous beam shown in Fig. 4.

(2) The beam is supported at the center of journal bearing having k_n in spring constant and subjected to piston load at the center of each supporting span.

(3) The complicated deflection of crankweb due to twisting moment is neglected.

Because the spring constant is usually unknown, in numerical calculation⁽²⁾ conducted with the digital computer the specific rigidity A defined by $\{EI_n/(k_n l_n^3)\}$ is put at a certain value, where E : Young's modulus of crankshaft material, I_n : moment of inertia of area of crank pin, l_n : distance between supporting points. The presumption of A is examined by comparison between the calculated stress and the measured one^{(2) (13)}.

On the other hand, the bending moment in crankweb can be approximately estimated by the following method. That is, the continuous beam calculation is made with three throws, neglecting the effects of piston loads acting on the throws distant from the objective one, as shown in Fig. 4.

The bending moments M_{w1} and M_{w2} subjected to the crankweb of the center throw are calculated by Eq. (3)

$$\left. \begin{aligned} M_{w1} &= (a_1 l + a_2 l_w) F_{r1} + (a_3 l + 0.5 l_w) F_{r2} \\ &\quad - (a_4 l + a_2 l_w) F_{r3} \\ M_{w2} &= -(a_4 l + a_2 l_w) F_{r1} + (a_3 l + 0.5 l_w) F_{r2} \\ &\quad + (a_1 l + a_2 l_w) F_{r3} \end{aligned} \right\} \dots\dots (3)$$

where, a_1 , a_2 , a_3 and a_4 are given by Table 1, against the respective values of A . The values of A related with fabrication of engine bed and presumed by the dynamical stress measurements are also tabulated for reference in this table. Concentric components of piston force F_{r1} , F_{r2} and F_{r3} are given in by appendix A.

Twisting moment in crank journal is calculated by Eq. (4), and tangential component of piston force F_t is given in Appendix A.

As a result of the dynamical stress measurements, it is generally presumed that the twisting moment in driving side fillet of crank pin is nearly equal to that in crank journal.

$$T_n = r \sum_{i=1}^n F_{ti} \dots\dots\dots (4)$$

Table 1 Coefficients a_1 , a_2 , a_3 and a_4

A	a_1	a_2	a_3	a_4	Fabrication of engine bed
0	-0.100 0	0.125 0	-0.075 0	-0.025 0	Neglecting deflection of supporting point
0.015	-0.072 7	0.073 9	-0.059 7	-0.000 3	Cast iron bed $p_{me} \leq 10 \text{ kg/cm}^2$
0.630	-0.055 3	0.040 6	-0.053 2	0.014 7	Cast iron bed $10 \leq p_{me} < 16 \text{ kg/cm}^2$
0.045	-0.042 8	0.018 4	-0.043 3	0.024 6	Weld construction $p_{me} \leq 12 \text{ kg/cm}^2$
0.060	-0.033 9	0.002 3	-0.031 9	0.031 7	Weld construction $p_{me} > 12 \text{ kg/cm}^2$

Note, p_{me} : Indicated mean effective pressure kg/cm^2

3. Dynamical stress measurement on crankshaft and analysis of the measured values

3.1 Method of dynamical measurement

Dynamical stress measurement of crankshaft has been hitherto carried out with the use of slip-rings, but it is almost impossible to take out the lead wire from strain gauge to the slip-ring in service condition of marine engine.

The author and his colleagues invented an FM strain telemeter applicable to the crankshaft in even the service condition by the aid of Primo Co. Ltd. Transmitter of the strain telemeter containing three measuring elements is 1 kg in weight, $50 \times 60 \times 100 \text{ mm}$ in outer dimensions and is applicable to the crankshaft of a high speed engine.

Fig. 5 shows the connection of the strain gauges on crank fillet in crank plane. The bending and torsional stresses are calculated by Eq. (5), as the circumferential strain is negligible and a simple shear stress exists in the fillet in crank plane.

$$\left. \begin{aligned} \sigma_x &= E \varepsilon_b / (1 - \nu^2), \quad \sigma_y = \nu E \varepsilon_b / (1 - \nu^2) \\ \tau &= E \varepsilon_t / 2(1 + \nu) \end{aligned} \right\} \dots\dots (5)$$

where, σ_x : bending stress in axial direction.

σ_y : bending stress in circumferential direction.

τ : torsional stress.

Fig. 5

ε_b : measured bending strain.
 ε_t : measured torsional strain.
 ν : poisson's ratio.

3.2 Results of measurements

The author and his colleagues have hitherto measured the dynamical stresses on crank fillets of 17 marine diesel engines, 11 two cycle low speed engines and 6 four cycle medium speed ones, and obtained enormous records of measurements.

In this paper, the author illustrates two examples. One is that of four cycle engine, for the sake of comparison between the analytical calculation and the measured one. The other is that of two cycle engine and it explains a stress analysis method relating to fatigue failure of crankshaft and shows the effects of additional stresses caused by torsional and axial vibrations.

Fig. 6 illustrates the measured stress patterns and calculated ones on pin fillet of free end side of No. 2 crankthrow of a four cycle engine (6 cylinders, cylinder bore 270 mm, stroke 420 mm, engine output 900 PS at 390 rpm). In this figure, the calculated pattern of bending stress at 390 rpm coincides generally with the measured one, but compressive stress of the measured one is larger than that of the calculated one which is computed by assuming A as 0.015. It may be presumed that better coincidence will be obtained when A is assumed as 0.03 or a little more. Torsional stress on crank pin fillet of free end side is smaller than that of the calculated one at the journal between No. 2 pin and No. 3 pin.

Fig. 7 (a), (b) illustrate the stress patterns of a two cycle engine (9 cylinders, cylinder bore 860 mm, stroke 1600 mm, engine output 19600 PS at 115 rpm).

Fig. 7 (a) shows the stresses on pin fillet of

Fig. 6 No. 2 throw of 4-cycle 6-cylinder engine at 390 rpm

free end side of No. 5 crank throw at 98 rpm, and Fig. 7 (b) on pin fillet of free end side of No. 8 crank throw at 110.7 rpm.

This crankshaft has three vibration modes as shown in Appendix B, C, that is, one node torsional vibration having a natural frequency of 406 c/min, two nodes torsional vibration having a natural frequency of 1056 c/min and one node axial vibration having a natural frequency of 680 c/min.

In Fig. 7 (b), sixth order resonance of the axial vibration is remarkably observed in bending stress pattern.

3.3 Additional stresses due to vibrations

The records of measurements such as Fig. 7 (a), (b) are examined by harmonic analysis in order to reveal the additional stresses of vibrations.

Fig. 8 (a), (b) illustrate the harmonic components of bending and torsional stress on the pin fillet of free end side of No. 5 crank throw and Fig. 9 (a), (b) those of No. 8 crank throw of the two cycle engine.

Table 2 shows the harmonic components of stresses at the respective resonance points, and from this table the following facts are generally observed as characteristics of crankshaft vibration of a large two cycle engine.

(1) Additional bending stresses are induced by both of the one node and the two nodes torsional vibrations, and the amplitudes of the additional stresses become large in the vicinity of the nodule

(a) No. 5 throw of 2-cycle 9-cylinder engine at 98 rpm

(b) No. 8 throw of 2-cycle 9-cylinder engine at 110.7 rpm
 Fig. 7

point.

(2) Axial vibration does not induce any additional torsional stress.

(3) Bending stress has many harmonic components due to the axial vibration and torsion induced, but torsional stress has comparatively a few components.

3.4 Stress analysis on fillet

As the measured stresses in Figs. 6 and 7 show stress components, the principal stresses are to be computed from Eq. (6) in order to compare with fatigue strength of crankshaft.

Fig. 8 Harmonic components of No. 5 throw

$$\left. \begin{aligned} \sigma_1 &= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} \\ \sigma_2 &= \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} \\ \theta &= \frac{1}{2} \tan^{-1} \frac{2\tau}{\sigma_x - \sigma_y} \end{aligned} \right\} \dots\dots\dots (6)$$

where, σ_1 , σ_2 : maximum and minimum principal stress, respectively

θ : inclination of σ_1 to crank pin axis.

The above computation is made with an interval

Fig. 9

Table 2 Additional stresses due to resonances

Position of fillet		No. 5 throw						No. 8 throw					
		Torsional				Axial		Torsional				Axial	
Vibration mode													
Additional stress		Induced bend		Torsion		Bend		Induced bend		Torsion		Bend	
		Amp.	rpm	Amp.	rpm	Amp.	rpm	Amp.	rpm	Amp.	rpm	Amp.	rpm
4th order	one node	—	—	0.6	102	—	—	0.9	104	0.85	102	—	—
5th order	one node	—	—	—	—	—	—	1.0	82	0.5	82	—	—
6th order	one node	—	—	0.6	68	2.1	111	1.4	69	0.9	68	3.3	111
8th order	one node	—	—	—	—	0.4	84	0.2	51	—	—	0.6	85
9th order	one node	0.4	45	0.9	45	1.25	76	1.6	45	1.1	45	1.7	77
12th order	two node	0.9	88	0.9	88	—	—	0.5	87	0.35	88	—	—
15th order	two node	0.4	72	0.25	72	—	—	—	—	—	—	—	—

Notes, Amp.: Harmonic component of additional stress in kg/mm²
rpm: Resonance rpm of engine revolution

of 10 degrees of rotating angle on the measured shown in Fig. 7 (a), (b).

Fig. 10 (a), (b) illustrate the principal stresses σ_1 , σ_2 and θ on a coordinate, on which positive values of σ_1 are plotted on the first or the second quadrants and positive values of σ_2 are plotted on the first or the fourth quadrants.

In these figures the points A, A' ... and B, B' ... correspond to those shown in Fig. 7 (a), (b).

When the variations of magnitudes and direction of the principal stresses are taken into account, it is difficult to find out any fatigue test that can be conducted in such complicated stress condition. The author therefore, proposes to adopt an equivalent stress amplitude to fatigue failure.

The equivalent stress amplitude is given by a summation of components of the principal stresses σ_1 and σ_2 on a certain axis in which the amplitude becomes maximum. When the maximums of abso-

lute values of σ_1 and σ_2 are nearly equal as shown in Fig. 10 (a), (b), the axis of the equivalent stress can be conventionally taken as a bisection line OP of angle δ which is the complementary angle between the maximum of σ_1 and the minimum of σ_2 . Fig. 11 (a), (b) illustrate the double amplitude of equivalent stress obtained by the above calculation comparing with the peak to peak values of bending and torsional stresses on the crank fillets of the two-cycle diesel engine.

It is obviously known that the equivalent stress amplitude is a little larger than the bending stress through the range of revolution except the resonance points of the torsional vibration.

4. Fatigue strength of crankshaft

Fatigue tests^{(9)~(11)} of model crankshaft having a pin diameter of 70 ~ 150 mm have been recently conducted in Japan. Fig. 12 (a), (b) illustrate the summarized results of these fatigue tests. The bending fatigue strengths of the crank fillet, (nominal fatigue strength of crank pin) $\times$ (stress concentration factor), are plotted against those of the smaller specimen taken from the crankweb perpendicularly to the metal flow in Fig. 12 (a), and torsional fatigue strengths of crank fillets are plotted against

Fig. 11 Peak to peak values in one cycle of 2-cycle 9-cylinder engine

those of small specimen taken from the journal longitudinally to the metal flow in Fig. 12 (b).

It will be known from these figures that the bending fatigue strengths of the crank fillet are generally equal to those of the small specimens and the torsional fatigue strengths of the crank fillet is equal to 0.7~1.0 times those of the small specimens. In the above comparison, the size effect of fatigue strength is roughly considered as an equivalency of stress gradient between the crank fillet and the small specimen.

The fatigue strength of crank fillet, therefore, will be estimated from the test result on the fatigue strength of the small specimen which has an equal stress gradient in subsurface to that of the crank fillet⁽¹⁵⁾.

Although the fatigue strength of crankshaft made of sound material can be estimated as the above, the fatigue strength of large crankshaft of marine diesel engine may be decreased by harmful

(a) σ_{wpf} (bending fatigue limits of crank fillets) and σ_{wt10} (bending fatigue limits of 10φ specimens taken from crank web tangentially)

(b) τ_{wpf} (torsional fatigue limits of crank fillets) and τ_{wt7} (torsional fatigue limits of 7φ specimen taken from journal)

Fig. 12 Fatigue strengths of crank specimens

defects such as segregation cracks or sand marks.

Bending and torsional fatigue tests^{(11) (12)} have been hitherto conducted with large specimens of 50~100 mm in diameter. As the result of these tests the fatigue strength of the large sound specimens is given by Eq. (7) and the effect of material defects on the fatigue strength is shown in Fig. 13.

$$\sigma_{w0} = 18 \left\{ 1 + \frac{2}{3} \left(\frac{\sigma_B}{42} - 1 \right) \right\}, \quad \tau_{w0} = 11 \left\{ 1 + \frac{2}{3} \left(\frac{\sigma_B}{42} - 1 \right) \right\} \quad (7)$$

Where, σ_B : tensile strength of material, kg/mm²

σ_{w0} : bending fatigue limit, kg/mm²

τ_{w0} : torsional fatigue limit, kg/mm²

It is known from Fig. 13 that the fatigue strength is reduced by the defects in proportion to the logarithmic value of the defect size, and the coefficients A_d of the fatigue strength reduction are tabulated in Table 3 relating to the kinds of the defects.

Consequently, the fatigue strength of large crankshaft is given by Eq. (8).

$$\sigma_{wd} = \sigma_{w0} - A_d (1 + \log_{10} \lambda) \quad (8)$$

where, σ_{wd} : fatigue limit of defective material

λ : length or diameter of defect in mm.

5. Conclusions

As a summary of this paper the author should like to add the following:

(1) Stress concentration factors of bending and twisting stress for marine diesel engines crankshaft have been established.

(2) Calculating method of bending moment and twisting moment on multi-throw crankshaft

Fig. 13 Effect of material defects on fatigue strength

Table 3 Coefficients A_d

Material	Defect	Stress	A_d kg/mm ²
Cast steel (SC 46)	Transverse hole	Bending	5.5
	Blow hole	Bending	5.0
Forged steel (SF 50~55)	Crack	Bending	7.5
	Transverse hole	Torsion	3.5

have been approximately completed by the aids of dynamical stress measurements of crankshafts.

(3) Fatigue strengths of marine engine crankshafts can be estimated from the results of fatigue tests, when the crank material is sound or defective.

(4) Additional stresses due to the vibrations in crankshaft can be studied by analysing the results of dynamical stress measurement of crankshaft.

(5) Despite the above studies, it is at present impossible to synthesize the actual stress pattern from the mechanically calculated ones and the vibratory ones, because the phase difference between them can not be analysed.

(6) Therefore, the author presumes that the safety factor, ratio of the fatigue strength to the exactly calculated stress amplitude, cannot be reduced to a value smaller than 1.5~2.0.

Appendix A

$$F_{ri} = \left\{ \cos \theta_n - \frac{\sin \theta_n \sin (\theta_n + \delta_i)}{\sqrt{\lambda^2 - \sin^2 (\theta_n + \delta_i)}} \right\} \times \left[\frac{\pi D^2}{4} P_{(\theta_n + \delta_i)} - \frac{W_{ri}}{g} r \omega^2 \right] \times \left\{ \cos (\theta_n + \delta_i) + \frac{\cos 2(\theta_n + \delta_i)}{\lambda} \right\} \dots\dots\dots (a)$$

$$F_{tn} = \left(\sin \theta_n + \frac{\sin \theta_n \cos \theta_n}{\sqrt{\lambda^2 - \sin^2 \theta_n}} \right) \times \left[\frac{\pi D^2}{4} P_{(\theta_n)} - \frac{W_{rn}}{g} r \omega^2 \left(\cos \theta_n + \frac{\cos 2\theta_n}{\lambda} \right) \right] \dots\dots\dots (b)$$

where, F_{ri} : external force component from i -th throw pin to the n -th pin on which bending stress is calculated.

F_{tn} : tangential force of n -th throw.

$P_{(\theta_n)}$: cylinder pressure of n -th throw.

W_r/g : mass of reciprocating part.

D : cylinder bore.

r : rotating radius of crankshaft.

δ_i : phase angle between the n -th throw and the i -th throw.

θ_n : rotating angle of the n -th throw.

λ : ratio of connect-

ing-rod length to rotating radius.

ω : angular velocity of crankshaft.

Append-Fig. (a) External force acting on the crankshaft

Appendix B

$J_1 \sim J_9$: moment of inertia of each crank throw

$J_{K1} \sim J_{K2}$: moment of inertia of coupling

J_t, J_f : moment of inertia of thrust collar and fly wheel

J_p : moment of inertia of propeller (6 blades)

$l_1 \sim l_p$: equivalent length (dimensions are modified in this figure)

Append-Fig. (b) Torsional vibration mode of 2-cycle 9-cylinder engine

Appendix C

$M_1 \sim M_9$: mass of each crank throw

M_i : mass of intermediate shaft

$K_1 \sim K_9$: spring constant of crank throw

M_{tf} : mass of thrust collar and fly wheel

M_p : mass of propeller and propeller shaft

K_t : spring constant of thrust bearing block

Append-Fig. (c) Axial vibration mode of 2-cycle 9-cylinder engine

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