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Heat Transfer between Metallic Surfaces in Contact*

(3rd Report, The Effect of Oxidation of Contacting Surfaces)

By Konomo SANOKAWA**

The effect of the oxidation of surfaces on thermal contact resistance is investigated. If the surface of a metal is oxidized, the thermal contact resistance becomes greater and its change with the increase of contact pressure is slower than in pure metal contact. From the theoretical analysis these phenomena are explained only by the fact that the oxide of a metal has poor thermal conductivity compared with the pure metal.

1. Introduction

When the surfaces of metals in contact are oxidized or eroded, the surfaces are covered with oxide film or rust. And the thermal contact resistance across the oxide film is supposed to be different from the pure metal contact. Since the thermal conductivity of this oxide film or rust is generally smaller than that of metal and the surface roughness is very irregular, these seem to play important roles in the cause of larger thermal resistance.

The experiments about the effects of oxidation of surfaces by the other investigators^{(1) (2)} have been available and they have understood this as a function of thickness of oxide film on the surface. These results show that oxidation of surfaces causes larger thermal resistance compared with the pure metal contact; however, they were not likely to get a quantitatively decisive relation between thermal contact resistance and the thickness of an oxide film.

In this report, we will discuss this point.

Fig. 1 Al

Fig. 2 Fe

Fig. 3 Fe

Fig. 4 Fe

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** Research Fellow, Japan Atomic Energy Research Institute, Tokai-mura, Naka-gun, Ibaraki-ken.

2. Effect of oxidation of surfaces in contact

Now we assume that the surfaces in contact are covered with thin films the substance of which becomes different from the metal by oxidation or erosion and so on. Supposing that the thickness of the layer is d_0 , and the thermal conductivities of metal and the oxide layer are k and k' respectively, then the thermal contact resistance including this oxide layer is supposed to be approximately the sum of the thermal contact resistance without oxide layer and the equivalent length $d_0 k/k'$ caused by the oxide layer.

The growth of oxide layer depends upon the substance of metal; however, in most cases thickness of oxide films grown when left in air at normal temperature or in hot gas atmosphere was found to be 1μ or less. When the surface is eaten by iron rust and becomes irregularly rough, the maximum height of roughness is within the range of several microns to 30μ , and in case of erosion by chemicals as sulfuric acid, chloric acid, nitric acid and so on, we observed the maximum height of roughness was sometimes up to 50μ . Figs. 1~4 show the oxidized or eroded surfaces of test pieces, half of which were abraded until the pure metal plane appeared. Figs. 5~8 show the recorded profiles of oxidized surface roughness, and a rather flat portion of surface profile means pure metal plane, therefore the difference between the maximum height and this flat part of profile is considered to be the approximate thickness of the layer whose substance is different from pure metal. And the results of these measurements have led us to the conclusion as already mentioned. Namely the thickness of oxide layer in most cases is supposed to be at most 50μ .

$\times 1000(\text{V-axis}) \times 15(\text{H-axis})$
Fig. 5 Al

$\times 1000(\text{V-axis}) \times 50(\text{H-axis})$
Fig. 6 Br

$\times 1000(\text{V-axis}) \times 50(\text{H-axis})$
Fig. 7 Cu

$\times 500(\text{V-axis}) \times 50(\text{H-axis})$
Fig. 8 Fe

We do not have the slightest idea about thermal conductivity of oxide layer, but it is certainly worse than that of pure metal. Supposing that it is $1/10$ of pure metal, the value $d_0 k/k'$ will be 500μ in equivalent length. Taking into account two oxide layers of both surfaces in contact, the equivalent length of oxide layers becomes of the order of 1 mm. The equivalent length of the resistance of constricting and spreading heat flows is of the order of several centimeters, as was seen in the 1st and 2nd reports. Therefore we might say that the effect of oxide films on thermal contact resistance is not so remarkable as far as the thickness is concerned.

Now we calculate the thermal resistance of a unit cell shown in Fig. 9 as a model of metals, the surfaces of which are covered with different substances. We assume that the thermal conductivity of the substance within $0 \leq z \leq z_2$ is k' , and denote the temperatures of four regions of Fig. 9 by θ_I , θ_{II} , θ_{III} and θ_{IV} , then the equations of heat conduction are

$$\nabla^2 \theta_I = 0 \quad (z_2 \leq z, 0 \leq r \leq r_2) \quad (1)$$

$$\nabla^2 \theta_{II} = 0 \quad (z_1 \leq z \leq z_2, 0 \leq r \leq r_2) \quad (2)$$

$$\nabla^2 \theta_{III} = 0 \quad (0 \leq z \leq z_1, 0 \leq r \leq r_1) \quad (3)$$

$$\nabla^2 \theta_{IV} = 0 \quad (0 \leq z \leq z_1, r_1 \leq r \leq r_2) \quad (4)$$

Boundary conditions which should be satisfied are

$$\text{at } z_2 \leq z, \quad Q = 2\pi k \int_0^{r_2} \frac{\partial \theta_I}{\partial z} r dr \quad (5)$$

$$\text{at } z_1 \leq z \leq z_2, \quad Q = 2\pi k' \int_0^{r_2} \frac{\partial \theta_{II}}{\partial z} r dr \quad (6)$$

$$\text{at } z=0, \quad \theta_{III} = 0 \quad (7)$$

$$\theta_{IV} = 0 \quad (8)$$

$$\text{at } z=z_2, \quad \theta_I = \theta_{II} \quad (9)$$

$$k \frac{\partial \theta_I}{\partial z} = k' \frac{\partial \theta_{II}}{\partial z} \quad (10)$$

$$\text{at } z=z_1, \quad \theta_{II} = \theta_{III} \quad (0 \leq r \leq r_1) \quad (11)$$

$$k' \frac{\partial \theta_{II}}{\partial z} = k' \frac{\partial \theta_{III}}{\partial z} \quad (0 \leq r \leq r_1) \quad (12)$$

$$\theta_{II} = \theta_{IV} \quad (r_1 \leq r \leq r_2) \quad (13)$$

$$k' \frac{\partial \theta_{II}}{\partial z} = \lambda \frac{\partial \theta_{IV}}{\partial z} \quad (r_1 \leq r \leq r_2) \quad (14)$$

Fig. 9

$$\text{at } r=r_1, \quad \theta_{\text{II}}=\theta_{\text{IV}} \dots\dots\dots (15)$$

$$k' \frac{\partial \theta_{\text{II}}}{\partial r} = k \frac{\partial \theta_{\text{IV}}}{\partial r} \dots\dots\dots (16)$$

$$\text{at } r=r_2, \quad \frac{\partial \theta_{\text{I}}}{\partial r} = 0 \dots\dots\dots (17)$$

$$\frac{\partial \theta_{\text{II}}}{\partial r} = 0 \dots\dots\dots (18)$$

$$\frac{\partial \theta_{\text{IV}}}{\partial r} = 0 \dots\dots\dots (19)$$

From Eqs. (5) and (17), the solution θ_{I} of Eq. (1) is expressed as

$$\theta_{\text{I}} = T_1 + \frac{Q}{k\pi r_2^2} z + \sum_{n=1}^{\infty} A_n e^{-(\gamma_n/r_2)(z-z_2)} J_0\left(\frac{\gamma_n}{r_2} r\right) \dots\dots\dots (20)$$

where T_1 and A_n are arbitrary constants and γ_n is a root of $J_1(\gamma_n)=0$.

From Eqs. (6) and (18), the solution θ_{II} of Eq. (2) is expressed as

$$\theta_{\text{II}} = T_2 + \frac{Q}{k'\pi r_2^2} z + \sum_{n=1}^{\infty} (B_n e^{(\gamma_n/r_2)(z_2-z)} + C_n e^{-(\gamma_n/r_2)(z_2-z)}) J_0\left(\frac{\gamma_n}{r_2} r\right) \dots\dots\dots (21)$$

where T_2 , B_n and C_n are arbitrary constants.

The solution θ_{III} of Eq. (3) is expressed as

$$\theta_{\text{III}} = \bar{A}_3 z + \sum_{n=1}^{\infty} D_n \sinh \frac{\alpha_n}{r_1} z J_0\left(\frac{\alpha_n}{r_1} r\right) + \sum_{n=1}^{\infty} E_n \sin \frac{n\pi}{z_1} z I_0\left(\frac{n\pi}{z_1} r\right) \dots\dots\dots (22)$$

where $\bar{A}_3$, D_n and E_n are arbitrary constants and α_n is a root of $J_0(\alpha_n)=0$.

The solution θ_{IV} of Eq. (4) is expressed as

$$\theta_{\text{IV}} = \bar{A}_4 z + \sum_{n=1}^{\infty} F_n \sinh \frac{\beta_n}{r_1} z C_0\left(\frac{\beta_n}{r_1} r\right) + \sum_{n=1}^{\infty} G_n \sin \frac{n\pi}{z_1} z D_0\left(\frac{n\pi}{z_1} r\right) \dots\dots\dots (23)$$

where $\bar{A}_4$, F_n and G_n are arbitrary constants and $C_0\left(\frac{\beta_n}{r_1} r\right)$ and $D_0\left(\frac{n\pi}{z_1} r\right)$ are functions defined as follows:

$$C_0\left(\frac{\beta_n}{r_1} r\right) = J_0\left(\frac{\beta_n}{r_1} r\right) - \frac{J_1\left(\frac{\beta_n}{r_1} r_2\right)}{Y_1\left(\frac{\beta_n}{r_1} r_2\right)} Y_0\left(\frac{\beta_n}{r_1} r\right) \dots\dots\dots (24)$$

$$D_0\left(\frac{n\pi}{z_1} r\right) = I_0\left(\frac{n\pi}{z_1} r\right) + \frac{I_1\left(\frac{n\pi}{z_1} r_2\right)}{K_1\left(\frac{n\pi}{z_1} r_2\right)} K_0\left(\frac{n\pi}{z_1} r\right) \dots\dots\dots (25)$$

And β_n is a root of $C_0(\beta_n)=0$ (3).

From Eqs. (9), (10), (11), (13) and (15), we get

$$T_2 = T_1 + \frac{Q}{k\pi r_2^2} z_2 - \frac{Q}{k'\pi r_2^2} z_2 \dots\dots\dots (26)$$

$$\bar{A}_3 = \bar{A}_4 = \frac{T_1}{z_1} + \frac{Q}{k\pi r_2^2} \frac{z_2}{z_1} - \frac{Q}{k'\pi r_2^2} \frac{z_2 - z_1}{z_1} \dots\dots\dots (27)$$

$$A_n = B_n \left(1 + \frac{\frac{k'}{k} - 1}{\frac{k'}{k} + 1}\right) \dots\dots\dots (28)$$

$$C_n = B_n \frac{\frac{k'}{k} - 1}{\frac{k'}{k} + 1} \dots\dots\dots (29)$$

$$G_n = E_n \frac{I_0\left(\frac{n\pi}{z_1} r_1\right)}{D_0\left(\frac{n\pi}{z_1} r_1\right)} \dots\dots\dots (30)$$

$$D_n = \sum_{m=1}^{\infty} 2B_m \left(e^{(\gamma_m/r_2)(z_2-z_1)} + \frac{\frac{k'}{k} - 1}{\frac{k'}{k} + 1} e^{-(\gamma_m/r_2)(z_2-z_1)} \right) \frac{1}{\sinh \frac{\alpha_n}{r_1} z_1} \frac{\alpha_n}{\left(\frac{\gamma_m}{r_2} r_1\right)^2 - \alpha_n^2} \frac{J_0\left(\frac{\gamma_m}{r_2} r_1\right)}{J_1(\alpha_n)} \dots\dots\dots (31)$$

$$F_n = - \sum_{m=1}^{\infty} 2B_m \left(e^{(\gamma_m/r_2)(z_2-z_1)} + \frac{\frac{k'}{k}-1}{\frac{k'}{k}+1} e^{-(\gamma_m/r_2)(z_2-z_1)} \right) \times \frac{1}{\sinh \frac{\beta_n}{r_1} z_1} \frac{\beta_n}{\left(\frac{\gamma_m}{r_2} r_1 \right)^2 - \beta_n^2} \frac{1}{\left(\frac{r_2}{r_1} \right)^2 \left\{ C_0^2 \left(\frac{\beta_n}{r_1} r_2 \right) / C_1^2(\beta_n) \right\} - 1} \frac{J_0 \left(\frac{\gamma_m}{r_2} r_1 \right)}{C_1(\beta_n)} \dots (32)$$

Substituting these relations into Eqs. (20)~(23), we get

$$\theta_I = T_1 + \frac{Q}{k\pi r_2^2} z + \sum_{n=1}^{\infty} B_n \left(1 + \frac{\frac{k'}{k}-1}{\frac{k'}{k}+1} \right) e^{-(\gamma_n/r_2)(z-z_2)} J_0 \left(\frac{\gamma_n}{r_2} r_1 \right) \dots (33)$$

$$\theta_{II} = T_1 + \frac{Q}{k\pi r_2^2} z - \frac{Q}{k'\pi r_2^2} (z_2 - z) + \sum_{n=1}^{\infty} B_n \left(e^{(\gamma_n/r_2)(z_2-z)} + \frac{\frac{k'}{k}-1}{\frac{k'}{k}+1} e^{-(\gamma_n/r_2)(z_2-z)} \right) J_0 \left(\frac{\gamma_n}{r_2} r_1 \right) \dots (34)$$

$$\theta_{III} = \left(\frac{T_1}{z_1} + \frac{Q}{k\pi r_2^2} \frac{z_2}{z_1} - \frac{Q}{k'\pi r_2^2} \frac{z_2 - z_1}{z_1} \right) z + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} 2B_m \left(e^{(\gamma_m/r_2)(z_2-z_1)} + \frac{\frac{k'}{k}-1}{\frac{k'}{k}+1} e^{-(\gamma_m/r_2)(z_2-z_1)} \right) \times \frac{\sinh \frac{\alpha_n}{r_1} z}{\sinh \frac{\alpha_n}{r_1} z_1} \frac{\alpha_n}{\left(\frac{\gamma_m}{r_2} r_1 \right)^2 - \alpha_n^2} J_0 \left(\frac{\gamma_m}{r_2} r_1 \right) \frac{J_0 \left(\frac{\alpha_n}{r_1} r_1 \right)}{J_1(\alpha_n)} + \sum_{n=1}^{\infty} E_n \sin \frac{n\pi}{z_1} z I_0 \left(\frac{n\pi}{z_1} r_1 \right) \dots (35)$$

$$\theta_{IV} = \left(\frac{T_1}{z_1} + \frac{Q}{k\pi r_2^2} \frac{z_2}{z_1} - \frac{Q}{k'\pi r_2^2} \frac{z_2 - z_1}{z_1} \right) z - \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} 2B_m \left(e^{(\gamma_m/r_2)(z_2-z_1)} + \frac{\frac{k'}{k}-1}{\frac{k'}{k}+1} e^{-(\gamma_m/r_2)(z_2-z_1)} \right) \times \frac{\sinh \frac{\beta_n}{r_1} z}{\sinh \frac{\beta_n}{r_1} z_1} \frac{\beta_n}{\left(\frac{\gamma_m}{r_2} r_1 \right)^2 - \beta_n^2} \frac{1}{\left(\frac{r_2}{r_1} \right)^2 \left\{ C_0^2 \left(\frac{\beta_n}{r_1} r_2 \right) / C_1^2(\beta_n) \right\} - 1} J_0 \left(\frac{\gamma_m}{r_2} r_1 \right) \frac{C_0 \left(\frac{\beta_n}{r_1} r_2 \right)}{C_1(\beta_n)} + \sum_{n=1}^{\infty} E_n \sin \frac{n\pi}{z_1} z \frac{I_0 \left(\frac{n\pi}{z_1} r_1 \right)}{D_0 \left(\frac{n\pi}{z_1} r_1 \right)} D_0 \left(\frac{n\pi}{z_1} r_1 \right) \dots (36)$$

where $C_1(\beta_n)$ is a function defined by

$$C_1(\beta_n) \equiv J_1(\beta_n) - \frac{J_1 \left(\frac{\beta_n}{r_1} r_2 \right)}{Y_1 \left(\frac{\beta_n}{r_1} r_2 \right)} Y_1(\beta_n) \dots (37)$$

From Eqs. (12), (14) and (16), we get

$$-T_1 \frac{1}{2} \frac{r_1}{z_1} + \sum_{n=1}^{\infty} B_n \left\{ \left(-e^{-(\gamma_n/r_2)(z_2-z_1)} + \frac{\frac{k'}{k}-1}{\frac{k'}{k}+1} e^{-(\gamma_n/r_2)(z_2-z_1)} \right) J_1 \left(\frac{\gamma_n}{r_2} r_1 \right) - \sum_{m=1}^{\infty} 2 \left(e^{(\gamma_n/r_2)(z_2-z_1)} + \frac{\frac{k'}{k}-1}{\frac{k'}{k}+1} e^{-(\gamma_n/r_2)(z_2-z_1)} \right) \frac{1}{\tanh \frac{\alpha_m}{r_1} z_1} \frac{\alpha_m}{\left(\frac{\gamma_n}{r_2} r_1 \right)^2 - \alpha_m^2} J_0 \left(\frac{\gamma_n}{r_2} r_1 \right) \right\} - \sum_{n=1}^{\infty} E_n (-1)^n I_1 \left(\frac{n\pi}{z_1} r_1 \right) = \frac{Q}{k\pi r_2} \frac{1}{2} \frac{r_1}{r_2} \left(1 - \frac{k}{k'} \right) \dots (38)$$

$$\begin{aligned}
& T_1 \frac{\lambda}{k} \frac{1}{\beta_n} \frac{r_1}{z_1} + \sum_{m=1}^{\infty} B_m \frac{\beta_n}{\left(\frac{\gamma_m}{r_2} r_1\right)^2 - \beta_n^2} J_0\left(\frac{\gamma_m}{r_2} r_1\right) \left[\frac{k'}{k} \frac{r_1}{r_2} \gamma_m \left(-e^{(\gamma_m/r_2)(z_2-z_1)} + \frac{k'/k-1}{k'/k+1} e^{-(\gamma_m/r_2)(z_2-z_1)} \right) \right. \\
& \left. + \frac{\lambda}{k} \beta_n \left(e^{(\gamma_m/r_2)(z_2-z_1)} + \frac{k'/k-1}{k'/k+1} e^{-(\gamma_m/r_2)(z_2-z_1)} \right) \frac{1}{\tanh \frac{\beta_n}{r_1} z_1} \frac{1}{\left(\frac{r_2}{r_1}\right)^2 \left\{ C_0^2 \left(\frac{\beta_n}{r_1} r_2 \right) / C_1^2(\beta_n) \right\} - 1} \right] \\
& + \frac{\lambda}{k} \sum_{m=1}^{\infty} E_m (-1)^m \frac{\frac{m\pi}{z_1} r_1 \beta_n}{\left(\frac{m\pi}{z_1} r_1\right)^2 + \beta_n^2} I_0\left(\frac{m\pi}{z_1} r_1\right) = \frac{Q}{k\pi r_2} \frac{1}{\beta_n} \frac{r_1}{r_2} \left\{ \frac{k}{k'} \left(\frac{\lambda}{k} \frac{z_2-z_1}{z_1} + \frac{k'}{k} \right) - \frac{\lambda}{k} \frac{z_2}{z_1} \right\} \\
& (n=1, 2, \dots) \dots\dots\dots (39)
\end{aligned}$$

$$\begin{aligned}
& \sum_{m=1}^{\infty} \sum_{m'=1}^{\infty} 2B_m (-1)^n J_0\left(\frac{\gamma_m}{r_2} r_1\right) \left(e^{(\gamma_m/r_2)(z_2-z_1)} + \frac{\frac{k'}{k}-1}{\frac{k'}{k}+1} e^{-(\gamma_m/r_2)(z_2-z_1)} \right) \\
& \times \left[\frac{k'}{k} \frac{\frac{n\pi}{z_1} r_1 \alpha_{m'}}{\left(\frac{n\pi}{z_1} r_1\right)^2 + \alpha_{m'}^2} \frac{\alpha_{m'}}{\left(\frac{\gamma_m}{r_2} r_1\right)^2 - \alpha_{m'}^2} + \frac{\lambda}{k} \frac{\frac{n\pi}{z_1} r_1 \beta_{m'}}{\left(\frac{n\pi}{z_1} r_1\right)^2 + \beta_{m'}^2} \frac{\beta_{m'}}{\left(\frac{\gamma_m}{r_2} r_1\right)^2 - \beta_{m'}^2} \right. \\
& \left. \times \frac{1}{\left(\frac{r_2}{r_1}\right)^2 \left\{ C_0^2 \left(\frac{\beta_{m'}}{r_1} r_2 \right) / C_1^2(\beta_{m'}) \right\} - 1} \right] + E_n \frac{n\pi}{2} \left\{ \frac{k'}{k} I_1\left(\frac{n\pi}{z_1} r_1\right) - \frac{\lambda}{k} \frac{I_0\left(\frac{n\pi}{z_1} r_1\right)}{D_0\left(\frac{n\pi}{z_1} r_1\right)} D_1\left(\frac{n\pi}{z_1} r_1\right) \right\} = 0 \\
& (n=1, 2, \dots) \dots\dots\dots (40)
\end{aligned}$$

where $D_1\left(\frac{n\pi}{z_1} r_1\right)$ is a function defined by

$$D_1\left(\frac{n\pi}{z_1} r_1\right) \equiv I_1\left(\frac{n\pi}{z_1} r_1\right) - \frac{I_1\left(\frac{n\pi}{z_1} r_2\right)}{K_1\left(\frac{n\pi}{z_1} r_2\right)} K_1\left(\frac{n\pi}{z_1} r_1\right) \dots\dots\dots (41)$$

Assuming that

$${}_5a_{1,1} = -\frac{1}{2} \frac{r_1}{z_1} \dots\dots\dots (42)$$

$$\begin{aligned}
{}_5a_{1,2m} = & \left(-e^{(\gamma_m/r_2)(z_2-z_1)} + \frac{k'/k-1}{k'/k+1} e^{-(\gamma_m/r_2)(z_2-z_1)} \right) J_1\left(\frac{\gamma_m}{r_2} r_1\right) \\
& - \sum_{m'=1}^n 2 \left(e^{(\gamma_m/r_2)(z_2-z_1)} + \frac{k'/k-1}{k'/k+1} e^{-(\gamma_m/r_2)(z_2-z_1)} \right) \frac{1}{\tanh \frac{\alpha_{m'}}{r_1} z_1} \frac{\alpha_{m'}}{\left(\frac{\gamma_m}{r_2} r_1\right)^2 - \alpha_{m'}^2} J_0\left(\frac{\gamma_m}{r_2} r_1\right) \dots\dots\dots (43)
\end{aligned}$$

$${}_5a_{1,2m+1} = -(-1)^m I_1\left(\frac{n\pi}{z_1} r_1\right) \dots\dots\dots (44)$$

$${}_5b_1 = \frac{1}{2} \frac{r_1}{r_2} \left(1 - \frac{k}{k'} \right) \dots\dots\dots (45)$$

$${}_5a_{2m,1} = \frac{\lambda}{k} \frac{1}{\beta_n} \frac{r_1}{z_1} \dots\dots\dots (46)$$

$$\begin{aligned}
{}_5a_{2n,2m} = & \frac{\beta_n}{\left(\frac{\gamma_m}{r_2} r_1\right)^2 - \beta_n^2} J_0\left(\frac{\gamma_m}{r_2} r_1\right) \left[\frac{k'}{k} \frac{r_1}{r_2} \gamma_m \left(-e^{(\gamma_m/r_2)(z_2-z_1)} + \frac{\frac{k'}{k}-1}{\frac{k'}{k}+1} e^{-(\gamma_m/r_2)(z_2-z_1)} \right) \right. \\
& \left. + \frac{\lambda}{k} \beta_n \left(e^{(\gamma_m/r_2)(z_2-z_1)} + \frac{\frac{k'}{k}-1}{\frac{k'}{k}+1} e^{-(\gamma_m/r_2)(z_2-z_1)} \right) \frac{1}{\tanh \frac{\beta_n}{r_1} z_1} \frac{1}{\left(\frac{r_2}{r_1}\right)^2 \left\{ C_0^2 \left(\frac{\beta_n}{r_1} r_2 \right) / C_1^2(\beta_n) \right\} - 1} \right] \dots\dots\dots (47)
\end{aligned}$$

$${}_5a_{2n,2m+1} = \frac{\lambda}{k} (-1)^m \frac{\frac{m\pi}{z_1} r_1 \beta_n}{\left(\frac{m\pi}{z_1} r_1\right)^2 + \beta_n^2} I_0\left(\frac{m\pi}{z_1} r_1\right) \dots (48)$$

$${}_5b_{2n} = \frac{1}{\beta_n} \frac{r_1}{r_2} \left\{ \frac{k}{k'} \left(\frac{\lambda}{k} \frac{z_2 - z_1}{z_1} + \frac{k'}{k} \right) - \frac{\lambda}{k} \frac{z_2}{z_1} \right\} \dots (49)$$

$${}_5a_{2n+1,2m} = \sum_{m'=1}^n 2(-1)^n J_0\left(\frac{\gamma_m}{r_2} r_1\right) \left(e^{(\gamma_m/r_2)(z_2-z_1)} + \frac{\frac{k'}{k}-1}{\frac{k'}{k}+1} e^{-(\gamma_m/r_2)(z_2-z_1)} \right) \\ \times \left[\frac{\frac{k'}{k} \frac{n\pi}{z_1} r_1 \alpha_{m'}}{\left(\frac{n\pi}{z_1} r_1\right)^2 + \alpha_{m'}^2} \frac{\alpha_{m'}}{\left(\frac{\gamma_m}{r_2} r_1\right)^2 - \alpha_{m'}^2} + \frac{\lambda}{k} \frac{\frac{n\pi}{z_1} r_1 \beta_{m'}}{\left(\frac{n\pi}{z_1} r_1\right)^2 + \beta_{m'}^2} \right. \\ \left. \times \frac{\beta_{m'}}{\left(\frac{\gamma_m}{r_2} r_1\right)^2 - \beta_{m'}^2} \frac{1}{\left(\frac{r_2}{r_1}\right)^2 \left\{ C_0^2 \left(\frac{\beta_{m'}}{r_1} r_2 \right) / C_1^2(\beta_{m'}) \right\} - 1} \right] \dots (50)$$

$${}_5a_{2n+1,2n+1} = \frac{n\pi}{2} \left\{ \frac{k'}{k} I_1\left(\frac{n\pi}{z_1} r_1\right) - \frac{\lambda}{k} \frac{I_0\left(\frac{n\pi}{z_1} r_1\right)}{D_0\left(\frac{n\pi}{z_1} r_1\right)} D_1\left(\frac{n\pi}{z_1} r_1\right) \right\} \dots (51)$$

Eqs. (38), (39) and (40) are the same as the following equations, since $n \rightarrow \infty$.

$$T_1 {}_5a_{1,1} + \sum_{m=1}^n B_m {}_5a_{1,2m} + \sum_{m=1}^n E_m {}_5a_{1,2m+1} = \frac{Q}{k\pi r_2} {}_5b_1 \dots (52)$$

$$T_1 {}_5a_{2n,1} + \sum_{m=1}^n B_m {}_5a_{2n,2m} + \sum_{m=1}^n E_m {}_5a_{2n,2m+1} = \frac{Q}{k\pi r_2} {}_5b_{2n+1} \dots (53)$$

$$\sum_{m=1}^n B_m {}_5a_{2n+1,2m} + E_n {}_5a_{2n+1,2n+1} = 0 \dots (54)$$

If we denote ${}_5D_0$ and ${}_5D_1$ by

$${}_5D_0 = \begin{array}{c|cc} & m=1, 2, \dots & \\ \hline & {}_5a_{1,2m} & {}_5a_{1,2m+1} \\ \hline n=1, 2, \dots & {}_5a_{2n,2m} & {}_5a_{2n,2m+1} \\ & {}_5a_{2n+1,2m} & {}_5a_{2n+1,2m+1} \\ \hline & 0 & \end{array} \dots (55)$$

$${}_5D_1 = \begin{array}{c|cc} & m=1, 2, \dots & \\ \hline & {}_5a_{1,2m} & {}_5a_{1,2m+1} \\ \hline n=1, 2, \dots & {}_5a_{2n,2m} & {}_5a_{2n,2m+1} \\ & {}_5a_{2n+1,2m} & {}_5a_{2n+1,2m+1} \\ \hline & 0 & \end{array} \dots (56)$$

T_1 is expressed as

$$T_1 = \frac{Q}{k\pi r_2} \frac{{}_5D_1}{{}_5D_0} \dots (57)$$

If we write the temperature drop ΔT in the region $0 \leq z \leq z_2$ as

$$\Delta T = \lim_{z \rightarrow \infty} \left\{ \theta_1 - \frac{Q}{k\pi r_2^2} (z - z_2) \right\} = T_1 + \frac{Q}{k\pi r_2^2} z_2 \\ = \frac{Q}{k\pi r_2} \left(\frac{{}_5D_1}{{}_5D_0} + \frac{z_2}{r_2} \right) \dots (58)$$

Then the thermal resistance per unit area between $-z_2 \leq z \leq z_2$ is calculated as

$$R_5 = \frac{2\Delta T}{\left(\frac{Q}{\pi r_2^2}\right)} = \frac{2r_2}{k} \left(\frac{{}_5D_1}{{}_5D_0} + \frac{z_2}{r_2} \right) \dots (59)$$

The equivalent length l_5 for k is

$$\frac{l_5}{r_2} = 2 \left(\frac{{}_5D_1}{{}_5D_0} + \frac{z_2}{r_2} \right) \dots (60)$$

Tables 1 and 2 show some examples of calculated value l_5 as a function of $z_2 - z_1$. In these examples we assumed $k/k' = 10$ just for a trial, and under this assumption the value of $z_2 - z_1$ is more likely to give little influence on l_5 . These results quite agree with the before-mentioned rough consideration.

Figs. 10~13 show the profiles of oxidized or eroded surface roughness, which are very irregular and rough. The state of roughness is well under-

stood under magnification by means of a microscope. Fig. 17 shows the surface which was exposed to hot air and was covered with a thin oxide film.

Fig. 14 Al

×45

Fig. 15 Fe

×45

Table 1

Equivalent length l_s for Al-Al in air with oxide films on the surfaces
 $k=178 \text{ kcal/mhr}^\circ\text{C}$, $\lambda=0.03 \text{ kcal/mhr}^\circ\text{C}$, $k/k'=10$, $r_1=30 \mu$

Contact pressure kg/cm ²	z_1 μ	z_2-z_1 μ	l_s cm	Contact pressure kg/cm ²	z_1 μ	z_2-z_1 μ	l_s cm
1	1	0	1.10	100	1	0	0.78
		1	1.11			1	0.78
		10	1.12			10	0.80
		50	1.20			50	0.88
		100	1.30			100	0.98
	10	0	10.8		10	0	4.22
		1	10.8			1	4.22
		10	10.8			10	4.24
		50	10.9			50	4.32
		100	11.0			100	4.42
	20	0	21.2		20	0	6.58
		1	21.2			1	6.58
		10	21.3			10	6.60
		50	21.3			50	6.68
		100	21.4			100	6.78
	30	0	31.7		30	0	8.83
		1	31.7			1	8.83
		10	31.7			10	8.85
		50	31.8			50	8.93
		100	31.9			100	9.03

Table 2

Equivalent length l_s for Fe-Fe in air with oxide films on the surfaces
 $k=44 \text{ kcal/mhr}^\circ\text{C}$, $\lambda=0.03 \text{ kcal/mhr}^\circ\text{C}$, $k/k'=10$, $r_1=30 \mu$

Contact pressure kg/cm ²	z_1 μ	z_2-z_1 μ	l_s cm	Contact pressure kg/cm ²	z_1 μ	z_2-z_1 μ	l_s cm
1	1	0	0.28	100	1	0	0.23
		1	0.28			1	0.23
		10	0.30			10	0.25
		50	0.38			50	0.33
		100	0.48			100	0.43
	10	0	2.76		10	0	1.98
		1	2.76			1	1.98
		10	2.78			10	2.00
		50	2.86			50	2.08
		100	2.96			100	2.18
	20	0	5.50		20	0	3.69
		1	5.50			1	3.70
		10	5.52			10	3.71
		50	5.60			50	3.79
		100	5.70			100	3.89
	30	0	8.25		30	0	5.36
		1	8.25			1	5.36
		10	8.27			10	5.38
		50	8.35			50	5.46
		100	8.45			100	5.56

The photograph shows many traces of a sandpaper, and in this case the thickness of oxide film is less than the surface roughness and little change of surface roughness is found between before and after oxidation.

Generally speaking, the oxidized and eroded surfaces are very irregular and rough and the effective height of gap formed by the two surfaces in contact is very difficult to estimate. We have been using the maximum height of roughness for practical convenience, but in these cases only the maximum height of roughness is not an enough value for calculating the thermal contact resistance. This problem should be considered not only on the case of oxidation or erosion but also on any case of irregular surface roughness such as the roughness extremely depending on the local position of surfaces. Figs. 18~24 show the extreme examples of surface roughness. Although the maximum heights of Figs. 18 and 19 are same, the voids produced by these surfaces and the deformations of these roughnesses under same contact pressure are quite different. Moreover, near the surface of Fig. 19 constricting and spreading heat flows are supposed to occur once; however, in the case of Fig. 18, twice. For this reason, the maximum height of roughness itself will not give an enough data for describing the state of surface roughness. Taking Fig. 18 for example, the small roughness

on the surface of each groove plays a more important role on the total resistance than the large roughness formed by grooves, because the resistance of constricting and spreading heat flows through roughness of grooves is larger. In Figs. 20 and 21, there exists one big hill or valley on a surface profile curve, and it is clear that this one big hill or valley of profile does give little effect on thermal contact resistance. Then in this case the exceptional roughness can be neglected. However, considering the fact that the real contact area is extremely small, even the exceptional hills should not be neglected, if the number increases. Therefore, in Fig. 22 which is only a small part of profile, maximum height of roughness should play a decisive role. Generally speaking the exceptional valleys as in Figs. 18 and 24 could not be taken into account in most cases, comparing with the exceptional hills. When the surfaces of metal are oxidized or eroded, big hills locally gather and are irregularly distributed on account of rust, together with the change of whole height of roughness. And if the big hills are very sparsely distributed, we should also take the roughness of larger portion or the roughness before oxidation into account. Figs. 25 and 26 show the maximum heights of roughness are 14μ and 9μ respectively and the maximum heights of roughness before oxidation were 6μ and 5μ respectively. The experimental data are shown in Fig. 28. Figs. 27~30 show the experi-

Fig. 16 Fe

× 45

Fig. 17 Fe

× 150

Fig. 18

Fig. 19

Fig. 20

Fig. 21

Fig. 22

Fig. 23

Fig. 24

mental data on Al, Fe, Br and Cu test pieces the surfaces of which are covered with oxidized or eroded layers. As for the roughness written in these figures, if the difference between before and after oxidation could be found and the actual gap formed by contact is supposed to be in-between, the roughness is shown in ranged figures. In case when a difference exists but the roughness of oxidized or eroded surfaces is comparatively uniform, we have adopted the maximum height of roughness. These results are very close to the data on unoxidized test pieces within the ranged height of roughness. However, in detail examination, the thermal contact resistance is greater than that of unoxidized case. Moreover, we should say the most characteristic behavior of oxidized case

compared with unoxidized case is that the variation of thermal contact resistance with contact pressure is slower. Figs. 31 and 32 are calculated examples for Al and Fe, using Eq. (60) when $z_2 - z_1 = 0$ and $k'/k = 1, 0.5$ and 0.1 . As already discussed, the effect of $z_2 - z_1$ up to 50μ which is considered to be a reasonable value for ordinary cases is not remarkable. This is the reason why we assumed $z_2 - z_1 = 0$ in the calculation of Figs. 31 and 32, which show that as k' increases, the value of thermal contact resistance increases and the variation with contact pressure becomes slower. This tendency is the same as that of experimental data. Since we do not have any idea about the thermal conductivity of oxide layer, the results of Figs. 27 ~ 30 can not be compared with the calculation. If the experi-

mental data agree well with the calculated values it might be said that it is possible to estimate the thermal conductivity of oxide layer. Now we assume $k = k'$ in Eq. (48) of the 1st report, calculating l_3 , then compare l_3 with l_5 .

Fig. 25* Fe

Fig. 26* Fe

Fig. 27

Fig. 28

Fig. 29

Fig. 30

* The inside pair of horizontal lines in Figs. 25 and 26 shows the maximum heights of roughness before oxidation

calculated by Eq. (60) for the cases of Al and Fe when $z_1=10\mu$. These calculated results are shown in Tables 3 and 4, and they are found to be same. If we calculate l_3 , putting $k=k'$ in Eq. (48) of the 1st report, l_3 is supposed to be equivalent length for k' . Therefore, in Tables 3 and 4, we have transformed them into l_3 for k .

It is more likely that this leads to a very important conclusion that the thermal contact resistance could be determined by the proportion of heat flows into both real contact spots and gap, and the substance of real contact spots, together with the fluid filling the gap is most important. In other words, in case of contact with oxide layer, the material of unoxidized portion has no effect upon the thermal contact resistance. Considering this conclusion as well as the previous discussion that the thickness of oxide layer itself has little effect upon the resistance, it is more likely that it will be unreasonable to treat the thermal contact resistance of oxidized metal as a function of thickness of oxidized layer. It should be a function of roughness, hardness and thermal conductivity of oxide layer. And if the layer is very thin and easily broken by the contact pressure, the layer has nothing to do with the thermal contact resistance and it will be the same as pure metal contact.

The experimental data previously published on the thermal contact resistance of metals with oxidized surfaces were so much scattered that nothing seems to come out. And their unreasonable results might be attributed to the afore-mentioned

reason. Although we have no data on hardness and thermal conductivity of oxide film, the experimental data fall within the calculated values of the ranged roughness. Taking Fig. 31 for example, this shows the calculated values for Al when $z_1=10\mu$, which agree well with the experimental data of Al with maximum height of roughness 9μ within the range $k'/k=0.5\sim 0.1$. And the experimental data for Fe of maximum height of roughness $9\sim 14\mu$ in Fig. 28 agree fairly well with the calculated values in Fig. 32 within the range $k'/k=0.5\sim 0.1$.

3. Conclusion

When the surfaces in contact are oxidized or eroded, the surface roughness becomes larger and more irregular than on unoxidized surfaces, and the surfaces are covered with oxide layer whose thermal conductivity is very poor. These two are the main causes to the fact that the thermal contact resistance is larger. If the oxidation covers the whole surface area and any localization is not to be found, the treatment of surface roughness might be the same as in an unoxidized case. On the contrary, if the rust makes big hills which are concentrated locally in a small area, or the roughness

Table 3

Al-Al in air $z_1=10\mu$ $r_1=30\mu$ $H_V=125$ $k_{Al}=178$ kcal/mhr°C, $\lambda=0.03$ kcal/mhr°C * l_3 is an equivalent length for Al		
Contact pressure p kg/cm ²	l_3 cm $z_2-z_1=0$ $k'=0.1 k_{Al}$ in Eq. (60)	l_3 cm* $k'=0.1 k_{Al}$ in Eq. (48) of 1st report
1	10.77	10.78
5	9.67	9.67
10	8.88	8.88
20	7.80	7.80
30	7.02	7.02
40	6.40	6.40
50	5.89	5.89
100	4.22	4.22

Table 4

Fe-Fe in air $z_1=10\mu$ $r_1=30\mu$ $H_V=221$ $k_{Fe}=44$ kcal/mhr°C, $\lambda=0.03$ kcal/mhr°C * l_3 is an equivalent length for Fe		
Contact pressure p kg/cm ²	l_3 cm $z_2-z_1=0$ $k'=0.1 k_{Fe}$ in Eq. (60)	l_3 cm* $k'=0.1 k_{Fe}$ in Eq. (48) of 1st report
1	2.76	2.76
5	2.62	2.62
10	2.53	2.53
20	2.41	2.42
30	2.33	2.34
40	2.27	2.27
50	2.21	2.21
100	1.98	1.98

Fig. 31

Fig. 32

is different in localities, we should take into account both the maximum height of roughness before and after erosion or oxidation accordingly. If we have already known the relation between the gap and the real contact area as shown in the 1st report, we will be able to calculate the gap under any contact pressure, even in the case of irregular roughness. But such a method could not be performed in each specified case from a practical point of view. Then it cannot be helped but to adopt the ranged roughness of before and after erosion, and within this region the experimental data are said to agree with the calculation. The thermal contact resistance of metals with oxidized surfaces in contact is larger than in an unoxidized case and its variation with contact pressure is very slow. This is more likely to be explained by its poor thermal conductivity of oxide layer. Now that the effect of thickness of oxide layer upon thermal contact resistance can be neglected, if we are able to know the thermal conductivity and hardness of

oxide layer, we could calculate thermal contact resistance by Eq. (48) of the 1st report instead of Eq. (60). This fact shows that the thermal contact resistance, or the resistance of constricting and spreading heat flows is determined by the proportion of heat flows in the real contact area and the gap, and the other substance except for that of real contact parts has nothing to do with the thermal contact resistance.

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