539. 431. 3. 016. 3 : 621. 833

Bending Fatigue Strength of Case-Hardened Gears*

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In this paper, the authors investigated the relationship between case-hardening conditions and the bending fatigue strength of a gear tooth from the view points of the residual stress generated on the root fillet and the increase of the materials strength (hardness) by case-hardening.

The main results obtained from this experiment are summarized as follows.

(1) By proper selection of case-hardening conditions, the endurance limit of a gear tooth can be raised by about 100 % of that of an unhardened gear.

(2) The contribution of the residual compressive stress generated on the root fillet to the fatigue strength reaches 40~50% of the increment of the endurance limit by case-hardening.

(3) There exists an optimal case thickness for the bending fatigue strength of case-hardened gears and an excessively large case thickness causes a decrease in the endurance limit.

(4) A formula for the calculation of the endurance limit of a gear tooth is developed, assuming that the residual stress influences on the fatigue strength of metals as mean stress and H_V - σ_u (hardness-endurance limit under pulsating stress) and H_V - σ_T (hardness-true breaking strength) diagrams are presented on the basis of the existing data. And the calculated values according to this formula and the diagrams show a good agreement with the experimental results.

1. Introduction

Gears which are hardened by such means as flame-, induction-, and case-hardening show not only a greater surface durability, but also a higher bending fatigue strength than unhardened ones. The tooth bending fatigue strength increases due to the generation of the residual compressive stress and to the increase in hardness at the tooth root fillet part resulting from heat-treatment. In this paper, the relationship between the bending fatigue strength of gear tooth and case-hardening conditions is discussed quantitatively from the view points of residual stress resulting from heat-treatment, hardness and case thickness, applying the same method as used in the previous paper⁽¹⁾, in which the bending fatigue properties of flame- and induction-hardened gears were investigated.

2. Nomenclature

σ_T : true breaking strength
 σ_{r0} : residual stress (originally present)
 σ_r : ($=0.85 \sigma_{r0}$): residual stress (mean residual

stress in consideration of fading of residual stress by pulsating loading)

σ_{Nb} : nominal stress on critical section due to bending component of applied load

σ_{Nc} : nominal stress on critical section due to compressive component of applied load

τ_N : nominal shearing stress due to shearing force

σ_t : true stress at load application side (tension side)

σ_m : mean stress

σ_a : stress amplitude

σ_{wp} : endurance limit under alternating stress (with lower residual stress)

σ_u : endurance limit under pulsating stress (with lower residual stress)

σ_{up} : endurance limit under pulsating stress

σ_{uop} : theoretical endurance limit under pulsating stress without contribution of residual stress (calculated)

σ_{upth} : theoretical endurance limit under pulsating stress with contribution of residual stress (calculated)

σ_A, σ_A' : endurance limit under pulsating stress of unhardened gears M and K respectively.

3. Experimental procedure

3.1 Test gears

Dimensions of test gears are shown in Table 1

* Received 16th February, 1967.

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and the mechanical properties and chemical compositions of their materials in Table 2 and their heat-treatment conditions in Table 3. Before case-hardening operations, the side faces of test gears were plated with copper so as to be hardened uniformly in the tooth width direction.

3.2 Bending tests

(1) Static bending tests A test gear was fixed at the gear holder with a shaft and two teeth. Then a tooth was statically loaded by a universal

Table 1 Dimensions of test gears

Tooth profile	Standard
Module m	4
Pressure angle α	20°
Number of teeth z	18
Tooth width b	10 mm

Table 2 Mechanical properties and chemical compositions of materials

	Yielding point kg/mm ²	Tensile strength kg/mm ²	C	Si	Mn	P	S	Cr	Mo
Gear M (SCM21)	75.4	88.3	0.15	0.18	0.73	0.02	0.03	1.03	0.19
Gear K (S15CK)	37.0	57.0	0.13	0.26	0.42	0.02	0.03	—	—

Table 3 Gear materials and heat-treatment conditions

Gear sign*	Heat-treatment methods and conditions		
	Heat-treatment	Case-hard. time	Heat-treatment conditions
A A'	Normalized	—	Temp. $\begin{matrix} 930^{\circ}\text{C} \\ \text{Time} \end{matrix}$ $\begin{matrix} \text{AC} \\ \text{Time} \end{matrix}$
B	Gas case-hardened	$\frac{1}{2}$ hr	Temp. $\begin{matrix} 930^{\circ}\text{C} \\ \text{Case-hard. time} \end{matrix}$ $\begin{matrix} 850^{\circ}\text{C} \\ 2\text{h} \end{matrix}$ $\begin{matrix} 180^{\circ}\text{C} \\ 5\text{h} \end{matrix}$ $\begin{matrix} \text{AC} \\ \text{Time} \end{matrix}$
C	"	$1\frac{1}{2}$ hr	
D	"	$3\frac{3}{4}$ hr	
B'**	"	$1\frac{1}{2}$ hr	Temp. $\begin{matrix} 930^{\circ}\text{C} \\ \text{Case-hard. time} \end{matrix}$ $\begin{matrix} 850^{\circ}\text{C} \\ 3\text{h} \end{matrix}$ $\begin{matrix} 180^{\circ}\text{C} \\ 5\text{h} \end{matrix}$ $\begin{matrix} \text{AC} \\ \text{Time} \end{matrix}$ $\begin{matrix} 850^{\circ}\text{C} \\ 1\frac{1}{2}\text{h} \end{matrix}$ $\begin{matrix} \text{WQ} \\ \text{Time} \end{matrix}$ $\begin{matrix} 180^{\circ}\text{C} \\ 2\text{h} \end{matrix}$ $\begin{matrix} \text{AC} \\ \text{Time} \end{matrix}$
C'**	"	$1\frac{1}{2}$ hr	
D'	"	$3\frac{3}{4}$ hr	
E E'	Powder case-hardened	9 hr	Temp. $\begin{matrix} 930^{\circ}\text{C} \\ 9\text{h} \end{matrix}$ $\begin{matrix} 850^{\circ}\text{C} \\ 1\frac{1}{2}\text{h} \end{matrix}$ $\begin{matrix} \text{WQ} \\ \text{Time} \end{matrix}$ $\begin{matrix} 180^{\circ}\text{C} \\ 2\text{h} \end{matrix}$ $\begin{matrix} \text{AC} \\ \text{Time} \end{matrix}$
F F'	Through hardened	—	Temp. $\begin{matrix} 850^{\circ}\text{C} \\ 1\frac{1}{2}\text{h} \end{matrix}$ $\begin{matrix} \text{WQ} \\ \text{Time} \end{matrix}$ $\begin{matrix} 180^{\circ}\text{C} \\ 2\text{h} \end{matrix}$ $\begin{matrix} \text{AC} \\ \text{Time} \end{matrix}$

* Gears marked by signs A~F are made of SCM21 steel and gears marked by signs A'~F' of S15CK steel.

** Gears B' and C' are the same in case-hardening time but they are cut out of different bars of S15CK steel.

testing machine at the place as shown in Fig. 1 ① and load-deflection curves were obtained for various gears.

(2) Bending fatigue tests Bending fatigue tests were carried out using the bending fatigue testing machine of hydraulic type⁽²⁾, which was developed by the authors. The position of load application was the same as in the static bending test (Fig. 1 ①).

The frequency of load repetition was about 1 000 c/min.

3.3 Measurement of hardness gradient and observation of hardened layers

A side face of gear tooth was ground by about 2 mm and finished with emery papers and in relation to this face the hardness gradient of the hardened layer was measured with a Micro-Vickers hardness tester.

The hardness measuring positions lay on the line, which passed through the intersecting point of the

① Position of load application
② Position of hardness measurement
③, ④ wire strain gauges
Fig. 1

Fig. 2 Boring of gear for determining residual stresses on tooth root fillet

tooth fillet curve with the Hofer's critical section and was normal to the fillet curve (Fig. 1 ②).

The hardened layer finished with emery papers was macroetched, further, with 15% nitric acid and then observed with a microscope.

3.4 Measurement of residual stresses

Measurement of residual stresses existing on the tooth root fillets was carried out by boring-out method^{(1) (3) (4)}. Two wire strain gauges (gauge length: 1 mm, gauge width: 1 mm, electric resistance: 120Ω) were pasted on the root fillets of every two teeth (Fig. 1 ③, ④). The bore of the gear was machined away with a lathe from 28 mm to 59 mm in diameter as shown in Fig. 2.

The bored-out gear was cut off into two or three pieces and each of them etched off, further, by 15% nitric acid, so as to relieve the residual stress existing on the root fillet of gear tooth as much as possible.

Residual stresses were calculated by Eqs. (1) and (2) from the difference of readings of gauges before and after relieving. The negative value of the residual stress in this case shows the compressive one.

$$\sigma_x = -\frac{E}{1-\nu^2}(\epsilon_x + \nu\epsilon_y) \quad \dots\dots\dots (1)$$

$$\sigma_y = -\frac{E}{1-\nu^2}(\epsilon_y + \nu\epsilon_x) \quad \dots\dots\dots (2)$$

where

σ_x : residual stress in tooth height direction

σ_y : residual stress in tooth width direction

ϵ_x : strain in tooth height direction

ϵ_y : strain in tooth width direction

E : modulus of elasticity

ν : Poisson's ratio

The strain in the tooth width direction ϵ_y was assumed to be distributed uniformly through the tooth width.

4. Experimental results and discussion

4.1 Hardness distribution

Figs. 3 (a) and (b) show hardness gradient curves in the normal direction to the root fillet curve for gears of SCM21 (hereafter sometimes called gear M) and for gears of S15CK (hereafter called gear K) respectively.

For normalized gears (A, A'), hardness is uniform in its distribution and $H_V=130\sim150$.

The surface hardness of gas case-hardened gears (B, B', C, C', D, D') and powder case-hardened gears (E, E') is 700~800 and the core hardness nearly 300 H_V for gear M and 230 H_V for gear K.

For a through-hardened gear F there is a little difference in hardness between the surface and the core, with $H_V=450$ on the surface and $H_V=350$ in

the core. Gear F' is through-hardened uniformly to the core with $H_V\approx 250$.

Figs. 4 (a) and (b) show the relation between the case-hardening time and the surface hardness or the thickness of hardened layers. For both gears M and K, the thickness of hardened layers increases steeply with case-hardening time in the range less than two hours and gradually over two hours.

The surface hardness also increases steeply in

Fig. 3 Hardness distribution on normal section to tooth root fillet curve

a: surface hardness, b: case thickness
c: case thickness for 500 H_V

Fig. 4 Relation among case-hardening time, case thickness and surface hardness

the range less than two hours and remains nearly constant over two hours. Each value for the case-hardening time of nine hours in Figs. 4 shows that by powder case-hardening method.

The thickness of hardened layers is denoted by the distance from the surface to the point where hardness becomes equal to that of the core. The thickness of hardened layers for $500 H_V$ is also shown together.

$500 H_V$ corresponds nearly to $50 R_c$ and this is usually taken as a standard in the determination of thickness of hardened layers.

As is seen from Figs. 4, the case-hardenableity of gear M is better than that of gear K and the case depth for gear M is larger than that of gear K for the same case-hardening time.

The surface hardness for various gears is shown in Table 4, column 1.

Macroetched side faces of gear teeth are shown in Fig. 5.

4.2 Results of residual stress measurement

The average residual stress between the two teeth of each gear, which is determined by boring-out method, is shown in Table 4, column 5.

As is seen from these experimental results, the residual compressive stress resulting from heat-treatment is largest for gas case-hardened gears, smaller for powder case-hardened gears with a larger case depth, and smallest for through-hardened gears.

Residual stress of gear K is, on the whole, larger than that of gear M for the same case-hardening time.

This may be because the thickness of hardened layers of the former is smaller than that of the latter.

4.3 Bending test results

(1) Static bending test results Load-deflection curves for various gears are shown in Figs. 6 (a) and (b).

The deflection given by these curves does not show that of a gear tooth accurately, which includes, though very small, the deflection of the two teeth of the gear holder.

(2) Bending fatigue test results Figs. 7 (a) and (b) show S-N curves for gears M and

Fig. 5 Photos of macroetched side faces of gear teeth

Table 4 Experimental results and calculated values

Test gears		1	2	3	4	5	6	7	8	9
Material	Sign	Surface hardness on tooth root fillet	True breaking strength	Endurance limit under pulsating stress	Endurance limit under pulsating stress (experimental)	Residual stress originally present	Endurance limit (without contribution of residual stress)	Grade of increment of endurance limit due to residual stress	Endurance limit under pulsating stress (calculated)	$\frac{\sigma_{upth}}{\sigma_{up}}$
		H_V	σ_T kg/mm ²	σ_u kg/mm ²	σ_{up} kg/mm ²	σ_{r0} kg/mm ²	σ_{uop} kg/mm ²	E_r %	σ_{upth} kg/mm ²	
SCM21	A	150	105	37	45(σ_A)	—	—	—	—	—
	B	750	185	78	107	-66.9	81.8	40.6	102.0	0.95
	C	850	193	80	107	-69.8	81.9	40.5	104.6	0.98
	D	800	190	79	107	-67.8	82.1	40.2	102.9	0.96
	E	750	185	78	83	-46.6	68.4	38.4	94.7	1.14
	F	450	157	67	73	-16.3	67.1	21.1	72.9	1.00
S15CK	A'	130	102	35	50($\sigma_{A'}$)	—	—	—	—	—
	B'	750	185	78	93	-67.1	71.1	50.9	102.0	1.10
	C'	800	190	79	93	-67.1	71.5	50.0	102.7	1.10
	D'	800	190	79	107	-71.9	81.0	45.6	104.4	0.98
	E'	800	190	79	85	-53.8	68.5	47.1	98.0	1.15
	F'	250	123	48	60	-20.1	52.7	73.0	54.7	0.91

K respectively. In these figures the ordinate shows the normal tooth load and the true stress at the root fillet of the load application side (tension side) σ_t , calculated by the following equations⁽⁵⁾ (see Fig. 1).

$$\sigma_t = \left(1 + 0.08 \frac{s}{\rho}\right) (0.66\sigma_{Nb} + 0.40\sqrt{\sigma_{Nb}^2 + 36\tau_N^2} + 1.15\sigma_{Nc}) \dots\dots\dots (3)$$

$$\left. \begin{aligned} \sigma_{Nb} &= \frac{6Pl \sin \theta}{s^2b} \\ \sigma_{Nc} &= -\frac{P \cos \theta}{sb} - \frac{6Py \cos \theta}{s^2b} \\ \tau_N &= \frac{P \sin \theta}{sb} \end{aligned} \right\} \dots\dots\dots (4)$$

where

P : normal tooth load, θ : angle between center line of tooth and load application line, l : distance from loading point to critical section, s : tooth thickness on critical section, ρ : radius of fillet curve, b : tooth width, y : distance between loading point and center

line of tooth, σ_{Nb} : nominal stress on critical section due to bending component of applied load, σ_{Nc} : nominal stress on critical section due to compressive component of applied load, τ_N : nominal shearing stress due to shearing force, which is assumed to be distributed uniformly through critical section. The endurance limit under pulsating load σ_{up} for various gears is shown in Table 4, column 4.

Fig. 8 shows the endurance limit of hardened gears in percentage of the endurance limit of unhardened gears A and A'.

It can be seen from this figure that compared with an unhardened gear, a properly hardened gear attains an endurance bending strength, which is 100% or more higher.

Furthermore, gear M shows, on the whole, a higher endurance limit than gear K and gas case-hardened gears also show a higher value than powder case-hardened gears with a larger case depth.

The paths of fractures for gears B and E are shown in Figs. 9 (a) and (b). In both cases, paths of fractures start from near the intersecting point of the root fillet curve with the critical section by Hofer's method.

Fig. 6 Static bending test results

Fig. 8 Endurance limit of various gears in percentage of endurance limit of unhardened gear (A or A')

Fig. 7 S-N curves for various gears

Figs. 10 (a) and (b) show the relation among the thickness of hardened layers, the endurance limit, the surface hardness and the residual stress existing on the root fillet part for gears M and K respectively.

It is found from these figures that there exists an optimal case depth covering a certain range for the bending fatigue strength of case-hardened gears and that an excessively large case depth causes a decrease in the endurance limit.

Both the surface hardness and the residual stress

Fig. 9 Paths of fractures

a: endurance limit, b: surface hardness
c: residual stress in tooth height direction
d: residual stress in tooth width direction

Fig. 10 Relation among thickness of hardened layer, surface hardness, residual stress and endurance limit

become largest at the optimal case depth. This may mean that the bending fatigue strength of gear tooth depends highly on the surface hardness and the residual stresses on the tooth root fillet.

4.4 Influence of residual stress on bending fatigue strength of gear tooth

Since residual stress is generally considered to act on the bending fatigue strength of metals as mean stress on the endurance limit diagram^{(6) (7)}, let the influence of residual stress on the bending fatigue strength of gear tooth be discussed from this point of view.

Assuming that the strength of metals is mainly determined by hardness and that the relation between hardness H_v and endurance limit under alternating stress σ_{wp} and the relation between hardness H_v and true breaking strength σ_T are given by Fig. 11^{(1) (8) (9)}, the endurance limit diagram can be obtained as shown in Fig. 12.

From this diagram H_v - σ_u curve is determined and shown in Fig. 11. The true breaking strength σ_T and the endurance limit under pulsating load σ_u corresponding to the surface hardness are determined from Fig. 11 and shown in Table 4, columns 2 and 3 respectively.

If the residual compressive stress originally present does not change and fade by pulsating loading, the straight line $\sigma_a = \sigma_m$ of Fig. 12 can be shifted

Fig. 11 Relation among hardness, endurance limit under alternating stress, endurance limit under pulsating stress and true breaking strength

Fig. 12 Relation between hardness and endurance limit under pulsating stress

to the left by the value of residual stress σ_r , as shown in Fig. 13.

The intersecting point of this straight line with the endurance limit line gives the half value of the endurance limit under pulsating stress of the specimen $(1/2)\sigma_{up}$.

If the endurance limit without a contribution of residual stress is represented by σ_{uop} , the following relation can be obtained from Fig. 13.

$$\sigma_{up} = \left(1 - \frac{\sigma_r}{\sigma_T}\right) \sigma_{uop} \quad \dots\dots\dots (5)$$

from which

$$\sigma_{uop} = \frac{1}{1 - \frac{\sigma_r}{\sigma_T}} \sigma_{up} \quad \dots\dots\dots (6)$$

According to the hitherto existing experimental results^{(3) (10)}, the residual stress originally present σ_{r0} changes and fades during the duration of pulsating load and the residual stress σ_r becomes smaller by about 30% of σ_{r0} at the number of 10^7 load cycles. Assuming that the residual stress fades linearly due to the pulsating loading and that the mean residual stress $0.85 \sigma_{r0}$ acts on the bending fatigue strength as mean stress through the whole duration of pulsating load, the theoretical endurance limit σ_{uop} , in which the increment of the endurance limit due to residual stress is not taken into consideration, is calculated by Eq. (6) and shown in Table 4, column 6.

Representing the endurance limit under pulsating stress of the unhardened gear A by σ_A and gear A' by $\sigma_{A'}$, the grade of the increase in the endurance limit due to the residual stress resulting from heat-treatment E_r is given by Eq. (7).

$$E_r = \frac{\sigma_{up} - \sigma_{uop}}{\sigma_{up} - \sigma_A(\sigma_{A'})} \times 100\% \quad \dots\dots\dots (7)$$

In this case the residual stresses existing on the tooth root fillet of the gears A and A' are assumed to be negligibly small. The rest of the increment of the endurance limit [is considered to be due to the improvement of the material or the increase in hardness.

The extent of the increase in the endurance limit due to the residual stress for various gears is shown in Table 4, column 7 and also in Fig. 14.

Fig. 13 Residual stress and endurance limit

Taking σ_u corresponding to the surface hardness of the tooth root fillet as σ_{uop} of Eq. (5) and σ_r as $0.85 \sigma_{r0}$, the endurance limit under pulsating stress σ_{up} can be obtained from Eq. (5) and is shown in Table 4, column 8 in terms of σ_{upth} .

Furthermore, the ratio of σ_{upth} to σ_{up} is shown in Table 4, column 9. It is found from the data shown in this column that calculated endurance limit values agree well with experimental ones.

This may show the possibility of predicting the endurance limit under pulsating load for case-hardened gears on the basis of the surface hardness and the residual stress on the root fillet.

4.5 Position of crack initiation

Assuming that a crack initiates at the point where the pulsating stress value exceeds the endurance limit of the material⁽⁸⁾, let it be discussed

Fig. 14 Increase of endurance limit due to residual stress and improvement of materials resulting from heat-treatment

Fig. 15 Actual stress due to applied load on normal section to root fillet curve and theoretical endurance limit under pulsating stress

whether a crack initiates from the surface or from below the surface for case-hardened gears.

The true breaking strength σ_T and the endurance limit σ_u corresponding to each point on the hardness distribution curves of Fig. 3 are determined from Fig. 11.

Regarding σ_u as σ_{uop} of Eq. (5) and taking σ_r as $0.85 \sigma_{r0}$ in consideration of the fading of the residual stress, σ_{up} of each point is calculated from Eq. (5) and shown in Fig. 15.

Pasting wire strain gauges on the tooth root fillet of a gear tooth model of carbon steel plate (4.8 times as large as test gear), the stress distribution on the normal section to the root fillet curve due to the applied load are measured and shown in Fig. 15.

In this figure no stress distribution curves intersect with endurance limit curves. Hence it is considered that a crack initiates from the surface for every test gear used in this experiment, so far as there are no defects below the surface.

5. Conclusions

The main results obtained from this experiment are summarized as follows.

(1) By proper selection of case-hardening conditions, a case-hardened tooth attains an endurance bending strength, which is 100% or more higher, than with unhardened gears.

(2) Gears of SCM21 show, on the whole, a higher endurance limit than those of S15CK and gas case-hardened gears also show a higher endurance limit than powder case-hardened ones with a higher case depth.

(3) There exists an optimal case depth for the bending fatigue strength of case-hardened gears

and an excessively larger case depth causes a decrease in the endurance limit.

(4) The contribution of the residual compressive stress on the root fillet resulting from case-hardening to the endurance limit reaches 40~50% of the increment of the endurance limit by case-hardening.

(5) When the surface hardness and the residual stress value on the tooth root fillet are given, the endurance limit of case-hardened gears can be calculated with a fairly good accuracy from Fig. 11 and Eq. (5).

(6) In every gear tested in this experiment, a crack is thought to initiate from the surface. Depending on case-hardening conditions, however, a crack may possibly initiate from below the surface and this leads to the decrease in the endurance limit.

In the choice of case-hardening conditions, therefore, special attention should be paid to the case-depth.

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