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Crystallographic Study of Yield Condition of Polycrystalline Metals*

(2nd Report, hcp Metal)

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The yield condition of a polycrystalline hexagonal close-packed metal was investigated in connection with the slip characteristics of hcp crystal, as the continuation of the previous study on a body-centred cubic metal. Schmid factors were calculated for every crystal orientation both in tension and torsion and plotted on stereographic projection. Averaging the distribution of Schmid factors graphically, the ratio τ/σ of the torsional yield stress to the tensile yield stress was obtained, where the Sachs' model of polycrystalline aggregates was adopted. Value of τ/σ was also determined directly from integration of the formula for the Schmid factor with respect to all of possible crystal orientations. The obtained value of τ/σ was 0.590, which was slightly larger than that of bcc metal (0.573) as reported in the previous paper. Also the value is higher than 0.577 that is given from the von Mises' law.

1. Introduction

In discussion on deformation and strength of material under multi-axial stress condition, the yield conditions expressed by mathematical formulae are generally used in the field of mechanical engineering⁽¹⁾. The propriety of these yield conditions is confirmed experimentally, and these conditions are sufficiently applicable to practical design. However, it does not necessarily follow that these mathematical yield conditions are valid, when trying to throw light on the deformation or the strength of metal substantially in connection with the change in micro structure of the material. It is because these mathematical yield conditions are neither derived on the basis of physical behavior of metal nor related to the change in microscopic structure.

The authors are investigating the correlation between the change in microstructure and the macroscopic behavior of metals in a series of studies. The yield condition in the macroscopic sense is considered to be closely related with the fact that metallic material is a polycrystalline aggregate composed of variously oriented grains. For instance, the effect of crystal lattice structure on it may be expected. The authors, therefore, investigate the yield condi-

tion of metals from the view point of polycrystalline matter.

In the first report⁽²⁾, consideration of yielding was made on bcc metal and it became clear that the ratio of yield stress of torsion to tension

Table 1 Examples of hcp metal

Metal	z	c/a
Be	4	1.57
Mg	12	1.62
Ti	22	1.59
Zn	30	1.86
Cd	48	1.89

 z : atomic number

Fig. 1 Slip directions on(0001) plane of hcp crystal

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obtained from the slip characteristics of bcc crystals is close to that obtained from the von Mises' law, using the Sachs' model assuming that a polycrystalline metal is composed of parallel grains of variously oriented crystals. Further, it was suggested that the numbers of slip planes and slip directions were important as a factor affecting the yield condition. Therefore, in this study, the relation between the slip in crystals and the yield condition is investigated in the case of hcp (hexagonal close packed) metal, which is known to have remarkably small numbers of slip plane and slip directions, and also discussions are made on the crystallographic basis of the yield condition.

The crystalline structure of hcp metal and its typical examples are shown in Fig. 1 and Table 1, respectively. In Fig. 1, a and c are the atomic distances along the a and c axes of hcp metal, respectively. The basic slip plane of hcp metal is (0001) plane (this is called the basal plane). The slip directions are three, $a_1[2\bar{1}10]$, $a_2[\bar{1}2\bar{1}0]$, $a_3[\bar{1}\bar{1}20]$, as shown in Fig. 1.

Very few investigations have dealt with hcp metal from the standpoint of plasticity of crystals⁽³⁾. Among hcp metals, however, such metals as titanium and beryllium are often used as strength member, and furthermore hcp metals have been widely used in the field of metallurgical study⁽⁴⁾. Thus, it seems

also important to discuss the yield condition of hcp metal by itself.

It is known that in the deformation of several hcp metals at a certain temperature range slips on the other planes than the basal (0001) plane occur, which are called, say, the prismatic slip or the pyramid slip, and also deformation by twinning is frequently observed. However, since the main purpose of the present study is to discuss the relation between the slip and the yield condition, an idealized hcp metal that slips on only (0001) plane is treated in this report.

In the following, the yield condition of hcp metal is firstly obtained by graphical method adopting stereographic projection, and then the procedure and result of calculating it by integration are described.

2. Graphic calculation

As in the case of bcc metal discussed in authors' first report⁽²⁾, it is necessary to introduce the Schmid factor (orientation factor) μ in considering the slip in each crystal. Let τ_s be the shearing stress in the slip direction on the slip plane, and σ and τ the applied stresses of tension and torsion, respectively, then we have

$$\tau_s = \mu_{\text{Tension}} \sigma, \quad \tau_s = \mu_{\text{Torsion}} \tau \quad \dots \dots \dots (1)$$

where μ_{Tension} and μ_{Torsion} are the Schmid factors of tension and torsion, respectively. As shown in Fig. 1, hcp crystal has one slip plane and three slip directions.

The three slip directions in a grain of a given orientation have individual values of Schmid factors, separately. However, it is natural to consider that the slip takes place in the slip direction having the largest value of μ , so that in the following we will define the Schmid factor of the grain by this largest value of μ .

In order to consider the yielding of a polycrystalline material, in the next place, we make the same three assumptions that were used in the case of bcc metal⁽²⁾: the critical shearing stress is constant and independent of the direction of the crystal and of the loading stress condition, and the yielding of a polycrystalline metal is determined by the average value $\bar{\mu}$ of Schmid factors that is the mean of the Schmid factors for all possible orientations of the grains, which corresponds to the Sachs' model of polycrystals⁽⁵⁾.

Applying the stereographic pro-

Fig. 2 Standard (0001) stereographic projection of zinc (hcp, $c/a=1.86$), (C.S. Barrett⁽⁶⁾)

jection, a graphical method is introduced to find the average $\bar{\mu}$ of Schmid factors for all possible orientations, under the assumption that the distribution of grain orientations in polycrystals is uniform. It would be useful to show the distribution of Schmid factors μ on the stereographic projection for finding the mean value $\bar{\mu}$ and also as a basis of experimental investigation of crystal plasticity by X-ray, for example.

Handling of hcp crystal structure on stereogra-

$$\cos \gamma = \frac{h_1 h_2 + k_1 k_2 + \frac{1}{2}(h_1 k_2 + h_2 k_1) + \frac{3a^2}{4c^2} l_1 l_2}{\sqrt{\left(h_1^2 + k_1^2 + h_1 k_1 + \frac{3a^2}{4c^2} l_1^2\right) \left(h_2^2 + k_2^2 + h_2 k_2 + \frac{3a^2}{4c^2} l_2^2\right)}} \quad (2)$$

where a and c are the lattice spacings shown in Fig. 1. [in the expression of Miller indices (h, i, k, l) of hcp crystal, $h+k=i$]

The Schmid factor μ_{Ten} for tension is given⁽⁵⁾ by

$$\mu_{\text{Ten}} = \cos \theta_0 \cos \theta_d \quad (3)$$

where θ_0 and θ_d are the angles from the specimen axis to the slip plane normal and the slip direction, respectively. On the other hand, the Schmid factor μ_{Tor} of the crystal lying on the circumference point P of a cylindrical torsion specimen shown in Fig. 3 is expressed in the form⁽⁶⁾:

$$\mu_{\text{Tor}} = \sin \theta_0 \cos \theta_d \sin(\psi_0 - \lambda) + \cos \theta_0 \sin \theta_d \sin(\psi_d - \lambda) \quad (4)$$

and,

$$\cot \theta_d = -\tan \theta_0 \cos(\psi_d - \psi_0) \quad (5)$$

where ψ_0 , ψ_d and λ are the circumference angles on the plane perpendicular to the specimen axis measured from an arbitrary radius, and correspond respectively to the slip plane normal N, the slip direction D and the position P of the crystal on the specimen, as illustrated in Fig. 3.

In the next place two sets of loci are drawn in the stereographic triangle at an interval of four degrees measured respectively from (1010) and (0001) poles⁽²⁾, and those crystals which are oriented to the intersection points of the two loci are dealt with. In the case of tension, μ_{Ten} is calculated for each of the three slip directions of hcp metal from Eqs. (2) and (3), and the largest of them is taken as the value of μ_{Ten} for the grain in that orientation. In the case of torsion, μ_{Tor} are obtained from Eqs. (2), (4) and (5), for every probable value of λ and for each of the three slip directions, and then the largest values of them are integrated with respect to λ to obtain the average value $\bar{\mu}_{\text{Tor}}$ ($\bar{\mu}_{\text{Tor}}$ represents the mean value of μ_{Tor} with respect to λ). The practical method of calculation is given in the appendix.

On the basis of Schmid factors μ thus obtained for every lattice point of the 4°-mesh of the loci in the

stereographic triangle, the contour lines of μ are drawn in the triangles for tension and torsion, which are shown in Figs. 4 and 5, respectively.

In order to get the mean value $\bar{\mu}$ of Schmid factors over all possible orientations, the contour lines or Figs. 4 and 5 are graphically integrated and averaged, postulating that the crystal orientations are uniformly distributed. Thus,

$$\left. \begin{aligned} \bar{\mu}_{\text{Ten}} &= 0.31_8 \\ \bar{\mu}_{\text{Tor}} &= 0.53_8 \end{aligned} \right\} \quad (6)$$

(Owing to the characteristics of stereographic projection, the actual distribution of crystal orientation does not correspond linearly with its density distribution on the projection plane⁽²⁾. This difference, however, was corrected graphically in the present calculation).

The yield stress ratio τ/σ of hcp polycrystalline metal is obtained from Eqs. (1) and (6) as

$$\tau/\sigma = \bar{\mu}_{\text{Ten}}/\bar{\mu}_{\text{Tor}} = 0.59_1 \quad (7)$$

Fig. 3 Angular relations of slip plane and slip direction in torsion

It follows that the value of τ/σ for hcp metal is slightly larger than the value 0.573 for bcc metals.

It is not possible at present to confirm this result experimentally, since so far we can not have experimental data concerning the yield condition of hcp metals. Experimental proof of interaction between the macroscopic and the microscopic side of the yield phenomena is left for the future investigations.

3. Calculation by integration

The reason why the graphical method should be introduced to obtain the mean value $\bar{\mu}$ of Schmid factors for fcc or bcc crystal lies in the fact that a rather complicated operation is needed in selecting the maximum value of μ from among several possible slip planes or slip directions in the crystal of each orientation. However, since hcp crystal has only one slip plane, it is expected that the mean value $\bar{\mu}$ could be obtained by a relatively simple method of integration.

3.1 $\bar{\mu}_{\text{Ten}}$

In the case of tension, it is necessary, at first, to select the maximum value of μ_{Ten} from among the three slip directions shown in Fig. 1. For this purpose, the angle α between the maximum gradient direction of slip plane and slip direction is introduced, as shown in Fig. 6. The angle θ_d in Fig. 6 is given by

$$\cos \theta_d = \sin \theta_0 \cos \alpha \quad \dots \dots \dots (8)$$

Substituting Eq. (8) into Eq. (3), we obtain

$$\mu_{\text{Ten}} = \sin \theta_0 \cos \theta_0 \cos \alpha \quad \dots \dots \dots (9)$$

From the assumption that the orientations of grains are distributed randomly, it follows that the density of grain orientation is uniform on the surface of unit sphere shown in Fig. 6, where the grain is put at the origin. When the crystal axis is taken as co-ordinate axis, the orientations of the specimen axis (i.e. the stress axis) lying within the angular increment of $d\theta_0$, which is inclined to the co-ordinates by θ_0 , are distributed over the hatched surface area $2\pi \sin \theta_0 d\theta_0$ of the unit sphere in Fig. 6. Accordingly, it is necessary to take this area into account as a weight when integrating μ_{Ten} with respect to α and θ_0 . (The weight $\sin \theta_0$ is generally used in the integration by the spherical co-ordinates.) Finally, from Eq. (9),

Fig. 6 Angular relationship between crystal axes and stress axis in tension

Fig. 4 Curves of " $\mu_{\text{Ten}} = \text{constant}$ " for hcp crystals

Fig. 5 Curves of " $\mu_{\text{Ten}} = \text{constant}$ " for hcp crystals

Fig. 7 Angular relationship between crystal axes and stress axes in torsion

$$\begin{aligned}\bar{\mu}_{\text{Ten}} &= \frac{2\pi \int_0^{\pi/2} \int_0^{\pi/6} \sin \theta_0 \mu_{\text{Ten}} d\alpha d\theta_0}{2\pi \int_0^{\pi/2} \int_0^{\pi/6} \sin \theta_0 d\alpha d\theta_0} \\ &= 1/\pi = 0.318 \dots\dots\dots (10)\end{aligned}$$

(In the above equation, the integral range of α is taken from zero to $\pi/6$, in order to select the range of highest μ_{Ten} values. Although the range of α is generally $\pi/3$ since there are three slip directions in hcp crystal, it is sufficient to take $\pi/6$ in the present case because of the symmetry of Eq. (9) with respect to the sign of α .) The resultant value of Eq. (10) strictly agrees with the former graphical result [Eq. (6)].

3.2 $\bar{\mu}_{\text{Tor}}$

The calculation of mean Schmid factor for torsion is more complicated because of the biaxial condition of stress state. It is necessary to rewrite Eq. (4) with the angle α , similar to the previous section. The variables θ_d and ψ_d are expressed in terms of θ_0 and α . Since there is only one slip plane in hcp crystal, ψ_0 can be taken arbitrarily. For simplicity, ψ_0 is chosen equal to zero. Substituting Eqs. (5) and (8) into Eq. (4), μ_{Tor} is given by the elimination of θ_d and ψ_d as

$$\mu_{\text{Tor}} = \cos 2\theta_0 \cos \alpha \sin \lambda + \cos \theta_0 \sin \alpha \cos \lambda \dots (11)$$

Next, it is necessary to integrate Eq. (11) with respect to the three variables of λ , α and θ_0 , for the purpose of getting the average value of μ_{Tor} over all possible grain orientations. However, the two following cautions should be taken in the course of integration. (1) If there are several slip planes or slip directions, the ones subjected to the maximum shear stress are operative. (2) The distribution of the orientation of grains is uniform. (This condition is equivalent to the situation that the orientation of the crystal axis or the stress axis is distributed uniformly on the surface of unit sphere in the stress or the crystal co-ordinates, respectively.)

Among the three integrations related to λ , α and θ_0 , λ concerns the stress axis (the axis of specimen), and α the crystal axis (Fig. 7). Let's denote here the two stress axes by X and Y (where X is the axis of test specimen, but not the principal axis

$$\int_{\alpha_1}^{\alpha_1 + \pi/3} \mu_{\text{Tor}} d\alpha = \sqrt{\cos^2 2\theta_0 \sin^2 \lambda + \cos^2 \theta_0 \cos^2 \lambda} \dots\dots\dots (12)$$

The next process is to integrate μ_{Tor} with respect to λ . The integral range is taken as $0 \leq \lambda \leq \pi/2$, owing to the symmetry of Eq. (12) with respect to λ . Consequently, the mean value $\bar{\mu}_{\text{Tor}}$ is expressed together with the above weight and Eq. (12) as,

$$\begin{aligned}\bar{\mu}_{\text{Tor}} &= \frac{\int_0^{\pi/2} \int_0^{\pi/2} \int_{\alpha_1}^{\alpha_1 + \pi/3} \sin^2 \theta_0 \cos \theta_0 \mu_{\text{Tor}} d\alpha d\lambda d\theta_0}{\int_0^{\pi/2} \int_0^{\pi/2} \int_{\alpha_1}^{\alpha_1 + \pi/3} \sin^2 \theta_0 \cos \theta_0 d\alpha d\lambda d\theta_0} \\ &= \frac{18}{\pi^2} \int_0^{\pi/2} \int_0^{\pi/2} \sin^2 \theta_0 \cos \theta_0 \sqrt{\cos^2 2\theta_0 \sin^2 \lambda + \cos^2 \theta_0 \cos^2 \lambda} d\lambda d\theta_0 = \frac{18}{\pi^2} I \dots\dots\dots (13)\end{aligned}$$

of stress), and two crystal axes by N and D (which are the normal to slip plane and the slip direction, respectively).

Although the order of integration is originally λ , α and θ_0 , here, the integration α corresponding to the slip direction is done at first, followed by λ and θ_0 , for the convenience of selecting the active slip direction as mentioned in the note (1). [Equation (11) is symmetric relating to α and λ , and in consequence, the same result is obtained irrespective of the order of the integration. Care, however, should be taken, when integrating, to keep condition of $\mu_{\text{Tor}} \geq 0$.]

A certain weight has to be added in the integration to assure the uniform distribution of the orientation, mentioned in the note (2). Let's consider the integration with respect to α , that is, the rotation of crystal axis D . The orientations of the stress axes X and Y should be distributed uniformly on the unit sphere whose co-ordinate axis is the crystal axis. Accordingly, the weight for Y -axis becomes $\cos \theta_0$ from the definition of θ_0 , that is, taking $\lambda = \psi_0 = 0$ (Fig. 7). The weight for X -axis is $\sin \theta_0$ similar to the case of tension.

Next is the integration with respect to λ (the rotation of stress axis Y), and at this time it is sufficient for uniform distribution of crystal axis to take only N -axis into account, because D -axis has already been integrated by α . Namely, the weight for the integration by λ is simply $\sin \theta_0$. Consequently, as for the case of torsion, the total weight in the integration with respect to α , λ and θ_0 turns out to be $\sin^2 \theta_0 \cos \theta_0$.

Based on these considerations, the integration is practically performed. At first, the above Eq. (11) is integrated with respect to α . The integral range of α should be taken as $\pi/3$, for the hcp crystal has three slip directions (Fig. 1). Let's define this range from α_1 to $\alpha_1 + \pi/3$, which gives the largest values of μ_{Tor} . Eq. (11) indicates that the value of μ_{Tor} changes cyclically with α , then

$$\mu_{\text{Tor}}(\alpha_1) = \mu_{\text{Tor}}(\alpha_1 + \pi/3)$$

Substituting α_1 into Eq. (11), the integration by α is represented as

The value I is obtained from numerical calculation as,

$$I=0.29572 \quad (14)$$

As the result,

$$\bar{\mu}_{\text{Tor}}=0.5393 \quad (15)$$

This value is also in good agreement with that estimated by the graphical method [Eq. (6)]. The results suggest that the previous graphical method has fairly good accuracy in calculating the mean value $\bar{\mu}$ of Schmid factors.

3.3 τ/σ

The yield stress ratio τ/σ is given from the integrated values of Eqs. (10) and (15) as,

$$\tau/\sigma=\bar{\mu}_{\text{Ten}}/\bar{\mu}_{\text{Tor}}=\pi/18I=0.5902 \quad (16)$$

which agrees with the graphical value of Eq. (7), as well.

4. Discussion

The resultant value of the yield stress ratio τ/σ of hcp metal is 0.590, which is a little larger than that of 0.577 due to the von Mises' law. This value is also greater than 0.57₈⁽⁵⁾ and 0.57₈⁽²⁾, the values of τ/σ obtained from the slip characteristics of fcc and bcc metal, respectively. (The latter two values are obtained from graphical calculation.) The difference between those values is slight from the practical point of view and may be said to be in the order of experimental error. However, it is significant, in such analytical treatment as our study, that such difference occurs under the same assumption, when the difference of crystalline structure is taken into account. Namely, these results predict that the numbers of slip plane and slip direction are essentially related to the yield condition and also indicate the tendency that the value of τ/σ increases, in general, according to the decrease in the numbers of slip plane and slip direction. It seems necessary to investigate the yield condition of the crystals having various numbers of slip plane and slip direction, in order to make this point clear. Adopting the calculation with integration developed in this study, it is expected that the yield condition can be easily obtained, in so far as the numbers of slip plane and slip direction are mathematically simple. Further discussions related on these points, however, will be reported in another paper.

The above-mentioned result is obtained considering only the basal slip on (0001) plane. However, actual hcp metal has the possibility of slip on the other plane and deformation by twinning, in such cases as deformation of large strain or at a certain temperature range. For example, the crystal oriented so as to take $\mu_{\text{Ten}}=0$, shown in Fig. 4, could not deform in case of tension unless another slip than

basal slip or twinning occurs. [Twinning of hcp metal occurs on (10 $\bar{1}$ 2) plane, in [10 $\bar{1}$ 1] direction. The slip direction of hcp metal is always $\langle 11\bar{2}0 \rangle$ direction regardless of the active slip plane.] The deformation due to twinning is basically a shear deformation and considered to resemble the slip. It seems that the slip other than basal slip or twinning has similar effect with the increase in the numbers of slip planes and slip directions, so long as the yield condition is concerned. However, it will be necessary in future to discuss the subject in detail together with the influences of various other lattice defects.

Nevertheless, since a relatively simple slip is thought to be dominant in the region of small strain near the yield point as shown by the microscopic observation, the result obtained here is considered to represent the yield condition of hcp metal sufficiently well. However, further experiments to elucidate these points should be conducted.

5. Conclusions

The following conclusions may be drawn from the above consideration;

(1) The Schmid factors for tension and torsion were obtained in every orientation of hcp crystal, and the results were presented as the distributions in the stereographic triangle.

(2) Adopting the Sachs' parallel model of polycrystalline aggregates, the yield stress ratio τ/σ of torsion to tension is calculated as 0.590 for hcp metal. This is slightly larger than the value 0.577 predicted by the von Mises' law.

(3) A method was established to calculate the yield stress ratio by integrating the Schmid factors for all crystal directions.

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Appendix

The procedure of calculating the values of μ_{Ten} and $\bar{\mu}_{\text{Tor}}$ for each orientation of grains, mentioned in the section 2, is as follows.

At first, two loci are drawn in the stereographic triangle, having 4°-intervals in the angles ϕ and θ_0 measured respectively from (1010) and (0001) poles. Those grains oriented to the cross points of the two loci are dealt with as representative ones.

In the case of tension, the three angles θ_{a1} , θ_{a2} and θ_{a3} of θ_a , respectively corresponding to the three slip directions of hcp metal, are given from

Eq. (2) as,

$$\left. \begin{aligned} \cos \theta_{d1} &= \frac{\sqrt{3}}{2} \cos \phi - \frac{1}{2} \sqrt{1 - (\cos^2 \theta_0 + \cos^2 \phi)}, \\ \cos \theta_{d2} &= \sqrt{1 - (\cos^2 \theta_0 + \cos^2 \phi)}, \\ \cos \theta_{d3} &= -\frac{\sqrt{3}}{2} \cos \phi - \frac{1}{2} \sqrt{1 - (\cos^2 \theta_0 + \cos^2 \phi)} \end{aligned} \right\} \dots\dots\dots (17)$$

The Schmid factor μ_{Ten} for the given grain orientation is calculated from Eq. (3), substituting the largest value among $|\cos \theta_{d1}|$, $|\cos \theta_{d2}|$ and $|\cos \theta_{d3}|$ of Eq. (17).

In the case of torsion, as there is only one slip plane in hcp metal, it is possible to put $\phi_0 = 0$ for simplicity (cf. Section 3.2). Eliminating ϕ_d from Eqs. (4) and (5), we obtain

$$\left. \begin{aligned} \mu_{\text{Tor}} &= F \sin \lambda + G \cos \lambda, \\ F &= \frac{\cos 2\theta_0}{\sin \theta_0} \cos \theta_d, \\ G &= \cos \theta_0 \sin \theta_d \sqrt{1 - \cot^2 \theta_0 \cot^2 \theta_d} \end{aligned} \right\} \dots\dots\dots (18)$$

where F and G are deduced corresponding to each of the three values θ_{d1} , θ_{d2} , θ_{d3} defined by Eq. (17). Since the three values of μ_{Tor} in Eq. (18) change severally according to λ , it is necessary to choose the largest value of μ_{Tor} from among the three values and along the variable range of λ . The situation is shown schematically in Fig. 8. (The sign of μ_{Tor} is not important here, for it is reversed when the slip in the opposite direction is considered.) It

Fig. 8 Schematic relation between μ_{Tor} and λ , for each of the three slip directions of hcp metal

is possible in Eq. (18) to postulate that $F_1 \leq F_2 \leq F_3$ and $G_1 \geq G_2 \geq G_3$ at the same time, where F_1 , G_1 , etc. are the absolute values of F and G corresponding to the three slip directions. The variable range of λ is taken from 0 to π owing to the cyclic form of Eq. (18) with respect to λ . The three boundary values λ_1 , λ_2 and λ_3 in Fig. 8, at each of which one slip direction having the highest μ_{Tor} -value is replaced by the next one, are given by

$$\left. \begin{aligned} \tan \lambda_1 &= \frac{G_1 - G_2}{F_2 - F_1}, \quad \tan \lambda_2 = \frac{G_2 - G_3}{F_3 - F_2}, \\ \tan \lambda_3 &= -\frac{G_1 + G_3}{F_1 + F_3} \end{aligned} \right\} \dots\dots\dots (19)$$

The mean value of μ_{Tor} for λ is obtained considering the thick line in Fig. 8 as,

$$\bar{\mu}_{\text{Tor}} = \frac{1}{\pi} \left\{ \int_0^{\lambda_1} \mu_1 d\lambda + \int_{\lambda_1}^{\lambda_2} \mu_2 d\lambda + \int_{\lambda_2}^{\lambda_3} \mu_3 d\lambda - \int_{\lambda_3}^{\pi} \mu_1 d\lambda \right\} \dots\dots\dots (20)$$

where $\mu_i \equiv \mu_{\text{Tor}}(F_i, G_i)$. Substituting Eqs. (18) and (19) into Eq. (20), we obtain after simplification,

$$\bar{\mu}_{\text{Tor}} = \frac{1}{\pi} \{ \sec \lambda_1 (F_2 - F_1) + \sec \lambda_2 (F_3 - F_2) - \sec \lambda_3 (F_1 + F_3) \} \dots\dots\dots (21)$$

Consequently, the values $\bar{\mu}_{\text{Tor}}$ corresponding to every grain oriented to the cross points of the two loci can be calculated from Eqs. (19) and (21). (On the other hand, in the case of bcc metal reported previously⁽²⁾, $\bar{\mu}_{\text{Tor}}$ is obtained by graphical method because of the complexity of calculation.)

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