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Wave-Form Effect on Fatigue Strength and Its Rheological Analysis*

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The stress wave under service is generally non-sinusoidal. The fatigue test by non-sinusoidal loading was made and compared with the sinusoidal stress wave fatigue test. Wave forms selected are 5 c/sec sinusoidal, square and triangular and 11.5 c/sec sinusoidal.

The test results show that, for test specimens of 0.3% carbon steel, especially for unnotched specimens, the fatigue strength differs significantly with different wave patterns, for the reason mainly of different amounts of repeating plastic strain. Then, an attempt was made to obtain an analytical expression of wave-form effect in terms of the relationship between stress and plastic strain during the fatigue test using a rheological model.

1. Introduction

It is uncommon to find a member subjected to such sinusoidal stress waves as in ordinary fatigue testing. The stress wave under service is generally non-sinusoidal. For the estimation of the fatigue life of a member subjected to non-sinusoidal stress waves, it will be necessary to confirm the influence of the wave form (wave-form effect) on the fatigue strength, before using the cumulative fatigue damage law or making a program fatigue test based on the frequency distribution of peak stress or stress amplitude values⁽¹⁾.

It is easy to understand that the influence of the wave form on the fatigue strength should be similar to the influence of the repeating speed on the fatigue strength. In the ordinary fatigue test region, however, it is said that the effect of the repeating speed can be neglected⁽²⁾, and little difference be found in the fatigue strength between sinusoidal and non-sinusoidal loading⁽³⁾, while experimental data so reported indicate that the fatigue strength is markedly different between 60 c/min and 1450 c/min⁽⁴⁾, showing that a definite conclusion has not yet been established on the problem.

In the study presented here, the Hydraulic Service Load Fatigue Testing Machine developed by the authors was used for axial fatigue tests by sinusoidal, triangular or square wave loading in

order to investigate the wave-form effect on the fatigue strength. The test results indicate that, for test specimens of 0.3% carbon steel, especially for unnotched specimens, the fatigue strength differs significantly with different wave patterns, for the reason mainly of different amounts of repeating plastic strain. Then, an attempt was made to obtain an analytical expression of the wave-form effect in terms of the relationship between stress and plastic strain during the fatigue test using a rheological model, a series combination of plastic deformation models each consisting of a spring, a slider and a dash-pot.

2. Testing apparatus and specimen

In the experiment, the Hydraulic Service Load Fatigue Testing Machine was used. This machine is of axial type having the maximum load capacity of ± 10 tons. As shown in Fig. 1, the output current

Fig. 1 Block diagram of the Hydraulic Service Load Fatigue Testing Machine

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of a servo-amplifier is applied to a servovalve to control the amount of working oil flowing into the cylinder of an actuator, and the load generated by the movement of the piston (or deformation of the test specimen) is detected and fed back to the preceding stage of the servo-amplifier, so as to obtain the load (or deformation) proportional to the input signal.

Fig. 2 shows the shape of test specimens: (a) an unnotched specimen and (b) a V-notched specimen with stress concentration factor of $\alpha=3$.

The input signal source used is Hewlett-Packard Low Frequency Function Generator type 202A.

This apparatus generates sinusoidal, triangular and square waves of small distortion in the frequency range of 0.008 c/sec to 1.2 kc. The variation of output voltage was less than 1% during a period for 10^7 cycles of repetition.

Even if an input signal of small distortion is applied from the generator mentioned above, the stress wave form is distorted to some extent by the limited frequency band of the testing machine. In the case of the present testing machine, the upper limit of the frequency for the unnotched specimen susceptible to deformation is about 30 c/sec, and 5 c/sec was taken as standard testing speed, because the square wave with a high content of high frequency component can be reproduced with relatively good precision at this speed. For the sinusoidal wave pattern only, tests were made at 5 c/sec and 11.5 c/sec to examine the speed effect. Fig. 3 shows examples of test wave form. All fatigue tests were made under the condition of completely reversed loading.

Fig. 2 Test specimens

The tested material was a 0.3% carbon steel and the test specimens were taken out of a round bar of 28 mm diameter in received condition. Chemical composition is shown in Table 1, and mechanical properties in Table 2.

3. Test results and discussions

3.1 Results of fatigue test of unnotched specimens

In the fatigue test of carbon steel specimens, the stress amplitude is usually below the yielding point. Such a test in its initial stage, first causes repeating elastic strain alone in general, but with an increasing number of cycles, it allows the plastic strain to yield and increase the strain amplitude

Table 1 Chemical composition of the tested material (%)

C	Si	Mn	P	S
0.30	0.27	0.45	0.018	0.018

Table 2 Mechanical properties of the tested material

Upper yield point kg/mm ²	Lower yield point kg/mm ²	Tensile strength kg/mm ²	Breaking strength on final area kg/mm ²	Reduction of area %
33.6	29.1	54.3	87	60.3

Sinusoidal wave 11.5 c/sec

Sinusoidal wave 5 c/sec

Triangular wave 5 c/sec

Square wave 5 c/sec

Fig. 3 Examples of testing wave forms

until it becomes nearly stabilized after the number of cycles corresponding to several percent of the fatigue life. Microscopic reasoning of this phenomenon is that the generation of the plastic strain in the early test stage means a process in which movable dislocations existing so far in a small number are multiplied and form cell structures in crystals, and the stabilization of the strain amplitude means a process in which dislocations on the cell wall gather close together without increasing their number and form nuclei of cracks; in this process the multiplication and the annihilation of dislocations are probably in equilibrium. When microscopic cracks grow into macroscopic cracks which injure the load bearing capability of the test specimen, the strain amplitude increases rapidly, resulting in a failure.

Fig. 4 shows the relationship between stress amplitude σ_a and strain amplitude ϵ_a after stabilized, together with the static stress-strain curve (this may be considered the first cycle in the test). From the figure, it may be seen that the σ_a - ϵ_a curve in the region of fatigue failure can be approximated by a straight line and that, below the fatigue limit, the plastic strain repeated is so small that the overall behavior is roughly elastic. Thus, the dynamic σ_a - ϵ_a curve under repeating stress may be approximated by two straight lines, line E which

Fig. 4 Dynamic σ_a - ϵ_a curves after stabilized (unnotched specimen)

* Such a relation was obtained also by Bairstow and Manson, and used by Kawamoto and Nishioka for the explanation of the stress distribution vs. fatigue strength relationship. E' and σ_p seem to vary with the material, test conditions, etc.⁽⁵⁾

is sloped at Young's modulus E ($=21\,000\text{ kg/mm}^2$) and line F which intersects line E at stress σ_p and has the slope E' as shown schematically in Fig. 5*. Turning back to Fig. 4, it is seen that the σ_a - ϵ_a curve varies with the wave form, being indicative of the wave-form effect on the σ_a - ϵ_a curve. Assuming a common value of E' of $5\,000\text{ kg/mm}^2$ for each wave form in consideration of insufficient experiment points and fitting the line F to each group of measurements gives Table 3 which indicates that σ_p varies with the wave form.

Then, by definitions given in Fig. 5, stress amplitude σ_a , strain amplitude ϵ_a , elastic strain amplitude $\epsilon_{e,a}$ and plastic strain amplitude $\epsilon_{p,a}$ can be denoted in terms of E , E' and σ_p as follows.

$$\epsilon_a = \frac{\sigma_p}{E} + \frac{\sigma_a - \sigma_p}{E'} = \frac{\sigma_a}{E'} + \left(\frac{1}{E} - \frac{1}{E'} \right) \sigma_p \quad \dots\dots\dots (1)$$

$$\epsilon_{p,a} = \epsilon_a - \epsilon_{e,a} = \epsilon_a - \frac{\sigma_a}{E} = \left(\frac{1}{E'} - \frac{1}{E} \right) (\sigma_a - \sigma_p) \quad \dots\dots\dots (2)$$

Plastic strain amplitude $\epsilon_{p,a}$ can be obtained by substituting the measured values of σ_a and ϵ_a in Eq. (2).

If the results of fatigue test are plotted in

Table 3 Dynamic σ_a - ϵ_a curves
 $E=21\,000\text{ kg/mm}^2$, $E'=5\,000\text{ kg/mm}^2$

Wave patterns		$\sigma_p\text{ kg/mm}^2$
Sinusoidal wave	11.5 c/sec	21.6
Triangular wave	5 c/sec	20.3
Sinusoidal wave	5 c/sec	19.8
Square wave	5 c/sec	18.7

Fig. 5 Schematic representation of a dynamic σ_a - ϵ_a curve

terms of stress amplitude σ_a and strain amplitude ϵ_a after stabilized against the number of cycles to failure, Fig. 6 and Fig. 7 are obtained. From σ_a - N curve of Fig. 6, it is seen that the fatigue strength is decreased from sinusoidal 11.5 c/sec to triangular 5 c/sec, sinusoidal 5 c/sec and square 5 c/sec in the same order as σ_p in Table 3, indicating a significant influence of the wave form on the fatigue strength.

The wave-form effect is seen also on the ϵ_a - N curve although not so remarkable as on the σ_a - N curve.

The process of fatigue failure divides roughly into two stages, nucleation of cracks and propagation of cracks. It is said in general that the crack nucleation depends largely on the movement of dislocations, that is, the amount of plastic strain repeated macroscopically, while the crack propagation depends mainly on the magnitude of stress because it is due to the repeating plastic strain at crack

Fig. 6 σ_a - N curves (unnotched specimen)

Fig. 7 ϵ_a - N curves (unnotched specimen)

tips which is produced by stress concentration. In the case of axial fatigue test of unnotched specimens where macroscopic cracks occur in the very end of the fatigue test, it may be considered that the plastic strain repeated during the fatigue test is the most important factor for determining the fatigue damage. As for the influence of the stress on the fatigue damage, it is considered that, in addition to the predominant role played by the stress in the propagation of macroscopic cracks, microscopic cracks have an inclination to be opened by stress components perpendicular to the cracks, thus expediting the fatigue damage even before the stage of the macroscopic cracks. On such a material as 0.3% carbon steel which is not sensitive to notch effect, however, it is considered that the influence of the stress on the fatigue damage is considerably small as compared with the repeating plastic strain.

On the other hand, it is reported that plastic strain energy $W_p \cong \sigma_a \times \epsilon_{p,a}$ determines fatigue damage⁽⁶⁾.

In connection with this point, the fatigue test results are plotted in terms of $\epsilon_{p,a}$ and w_p in Fig. 8

Fig. 8 $\epsilon_{p,a}$ - N curves (unnotched specimen)

Fig. 9 W_p - N curves (unnotched specimen)

and Fig. 9, respectively.

In either figure, little difference with different wave forms is seen and a linear relation to the number of cycles to failure N is found when plotted to a double logarithmic scale.

For the $\varepsilon_{p,a}$ - N curve

$$\varepsilon_{p,a} N^{0.562} = 34.4 \quad (3)$$

and for the W_p - N curve

$$W_p N^{0.599} = 1.29 \times 10^3 \quad (4)$$

The validity of Eq. (3) suggests that the Manson-Coffin's equation for plastic fatigue^{(7) (8)}

$$\varepsilon_{p,a} N^\alpha = K \quad (5)$$

holds up to a high cycle region.

On the assumption that the $\varepsilon_{p,a}$ - N curve is free from the wave-form effect, let us derive equations for σ_a - N and ε_a - N curves. Substituting Eq. (2) in Eq. (5) gives

$$(\sigma_a - \sigma_p) N^\alpha = \frac{K}{\left(\frac{1}{E'} - \frac{1}{E}\right)} \quad (6)$$

for the ε_a - N curve. For the ε_a - N curve, substituting the following equation

$$(\sigma_a - \sigma_p) = \left(\varepsilon_a - \frac{1}{E} \sigma_p\right) E' \quad (7)$$

in Eq. (6) gives

$$\left(\varepsilon_a - \frac{1}{E} \sigma_p\right) N^\alpha = \frac{K}{\left(1 - \frac{E'}{E}\right)} \quad (8)$$

For such data as shown below

$$\left. \begin{aligned} E &= 21\,000 \text{ kg/mm}^2 \\ E' &= 5\,000 \text{ kg/mm}^2 \\ \alpha &= 0.562 \\ K &= 34.4 \end{aligned} \right\} \quad (9)$$

Eq. (6) and Eq. (8) are written respectively as follows.

$$(\sigma_a - \sigma_p) N^{0.562} = 2.26 \times 10^3 \quad (10)$$

$$\left(\varepsilon_p - \frac{\sigma_p}{21\,000}\right) N^{0.562} = 4.51 \times 10^{-3} \quad (11)$$

By use of the values of Table 3 for σ_p , equations for σ_a - N and ε_a - N curves for the various wave patterns are obtained. Solid lines in Fig. 6 and Fig. 7 show the results of calculation of Eq. (10) and Eq. (11), respectively, suggesting that the calculation is in fairly good agreement with the experiment.

The results of fatigue test presented in this paper refer to the stress (or load) control, and disclose a considerably remarkable wave-form effect on the fatigue life. If, on the contrary, the strain (or deformation) of the test specimen is controlled in the fatigue test, the difference in the plastic strain with different wave forms may not be, as imaginable from Fig. 4 and Fig. 7, so remarkable, that is, the influence of the strain wave form on

the fatigue strength may not be so remarkable.

3.2 Results of fatigue test of notched specimens

Fig. 10 shows the results of fatigue test of a notched test specimen. Unlike the unnotched specimen, the notched specimen shows no difference in the fatigue life between sinusoidal 11.5 c/sec and square 5 c/sec loading.

4. Analysis of the wave-form effect on stress-plastic strain curves by mechanical models (rheological models)

As shown in Fig. 5, the stress-strain curve during fatigue test shows a hysteresis loop with plastic strain amplitude of $\varepsilon_{p,a}$. Assuming that the hysteresis loop is affected by the speed of deformation and results in different σ_a - ε_a curves for different wave forms, an analysis is made by use of rheological models.

If the deformation speed is very small, that is, in the case of quasi-static application of stress cycles, the following relation holds between stress and plastic strain at loading or in the region of A $\rightarrow$ B (C $\rightarrow$ D) of Fig. 5.

$$\varepsilon_p \cong a \sigma^\alpha \quad (12)$$

At unloading or in the region of B $\rightarrow$ C (D $\rightarrow$ A),

$$\varepsilon_p \cong 0 \quad (13)$$

For the material used in this study, it is found after the experiment by various values of stress⁽⁹⁾ that Eq. (12) takes the form

$$\varepsilon_p \cong a \sigma^4 \quad (14)$$

Fig. 11 shows stress and strain wave forms at relatively high stress amplitudes in triangular and square loading.

Typical examples of the influence of wave forms on the behavior of deformation of the test specimen during fatigue test are given in this figure: in triangular loading, the stress wave form shows a relatively slight deformation in contrast with the strain

Fig. 10 σ_a - N curves (notched specimen)

wave form which is bent at zero stress and shows much more strain developed on the loading side than on the unloading side, and in square loading, the stress wave form keeps a nearly constant value of stress while the strain wave form shows a continued increase in strain.

The behavior of deformation expressed by Eqs. (12) through (14) can be represented by a mechanical model consisting of springs and sliders as shown in Fig. 12 (a) ⁽⁹⁾.

Let us assume that force F is applied to the lower end of the model.

Deformation of the i -th element, X_i , is given by

$$X_i = \frac{F - F_i}{k_i} \quad (15)$$

for $F \geq F_i$, and

$$x_i = 0 \quad (16)$$

for $F < F_i$.

Setting

$$F_1 \leq F_2 \leq \dots \leq F_i \leq \dots \leq F_n \quad (17)$$

and

$$F_j \leq F < F_{j+1} \quad (18)$$

means the condition that sliders F_1 through F_j slip and F_{j+1} through F_n do not. Then, the entire elongation of the mechanical model, x , is given by

$$x = \sum_{i=1}^j x_i = \sum_{i=1}^j \frac{F - F_i}{k_i} \quad (19)$$

The term x is considered to represent a plastic deformation of the model. If values of k_i and F_i on the right side are selected properly and the number of elements is taken sufficiently large, it is possible to approximate the right side of Eq. (14) by the right side of Eq. (19).

Performing this approximation gives

$$\left. \begin{aligned} \epsilon_p &= x \\ a\sigma^t &= \sum_{i=1}^j \frac{F - F_i}{k_i} \end{aligned} \right\} \quad (20)$$

Then, consider the dynamic stress-strain hy-

steresis curve. In this case, a dynamic mechanical model consisting of the quasi-static mechanical model of Fig. 12 (a) added with speed-depending dash-pot elements as shown in Fig. 12 (b) may be applied. Similarly to the previous case, the equation for the i -th element can be derived.

$$k_i x_i + \eta_i \frac{dx_i}{dt} = F - F_i \quad (21)$$

Dividing both sides by k_i gives

$$x_i + \tau_i \frac{dx_i}{dt} = \frac{F - F_i}{k_i} \quad (22)$$

where $\tau_i = \eta_i / k_i$ is the delay time. Under the condition that sliders F_1 through F_j slip and F_{j+1} through F_n do not, summing up Eq. (22) over $i=1 \dots j$ gives

(a) Quasi-static model (b) Dynamic model
Fig. 12 Mechanical models

(i) Stress wave

(ii) Strain wave

(a) Triangular wave 5 c/sec

(i) Stress wave

(ii) Strain wave

(b) Square wave 5 c/sec

Fig. 11 Typical wave-form effect on the plastic deformation during fatigue test (unnotched specimen)

$$\sum x_i + \sum \tau_i \frac{dx_i}{dt} = \sum \frac{F - F_i}{k_i} \dots (23)$$

At this point, make a simplifying assumption that the delay time is same for all the elements and is τ , that is

$$\tau = \tau_i \dots (24)$$

Then, Eq. (23) is simplified as follows.

$$\sum x_i + \tau \frac{d}{dt} (\sum x_i) = \sum \frac{F - F_i}{k_i} \dots (25)$$

Further on the assumption that k_i and F_i have the same distribution as in the quasi-static case, Eq. (25) can be converted by the same procedure as used to obtain Eq. (20) as follows.

$$\varepsilon_p + \tau \frac{d\varepsilon_p}{dt} = a\sigma^4 \dots (26)$$

Eq. (26) is the basic equation for dynamic stress-strain hysteresis curves. Solving this equation for each type of wave form determines the form of the dynamic stress-strain hysteresis curve under loading.

For the unloading side, Eq. (13) can be used. On the loading side shown by solid lines in Fig. 13, plastic strain amplitudes developed in each 1/2 cycle of the loading wave form can be calculated from Eq. (26).

(a) Square wave

Take for example the case of Fig. 13 (a) where step stress input σ_a is sustained for a time $2t_0$ ($=T/2$). The calculation of the plastic strain is shown below using Eq. (26)

$$\varepsilon_p + \tau \frac{d\varepsilon_p}{dt} = a\sigma_a^4 \dots (27)$$

Solving this equation gives

$$\varepsilon_p = c_1 e^{-t/\tau} + c_2 + a\sigma_a^4 \dots (28)$$

From conditions, $\varepsilon_p = 0$ at $t = 0$ and $\varepsilon_p = a\sigma_a^4$ at $\tau \rightarrow 0$, c_1 and c_2 are determined, then Eq. (28) is

(a) Square wave 5%/sec

(c) Sinusoidal wave 5%/sec

(b) Triangular wave 5%/sec

(d) Sinusoidal wave 11.5%/sec

Fig. 13 Tested wave patterns

written as

$$\varepsilon_p = a\sigma_a^4 (1 - e^{-t/\tau}) \dots (29)$$

At $t = 2t_0$, plastic strain amplitude $\varepsilon_{p,a}$ is given by

$$2\varepsilon_{p,a} = a\sigma_a^4 (1 - e^{-2t_0/\tau}) \dots (30)$$

(b) Triangular wave

The stress wave form of Fig. 13 (b) is expressed by

$$\sigma = \left(\frac{\sigma_a}{t_0}\right)t \quad (0 \leq t \leq t_0) \dots (31)$$

Then, Eq. (26) is written as

$$\varepsilon_p + \tau \left(\frac{d\varepsilon_p}{dt}\right) = a\left(\frac{\sigma_a}{t_0}\right)^4 t^4 \dots (32)$$

Solving this equation gives

$$\varepsilon_p = c_1 e^{-t/\tau} + c_2 + a\left(\frac{\sigma_a}{t_0}\right)^4 (t^4 - 4\tau t^3 + 12\tau^2 t^2 - 24\tau^3 t + 24\tau^4) \dots (33)$$

From conditions, $\varepsilon_p = 0$ at $t = 0$ and $\varepsilon_p = a(\sigma_a/t_0)^4 t_0^4$ at $\tau \rightarrow 0$, c_1 and c_2 are determined, then plastic strain amplitude at $t = t_0$ is given by

$$2\varepsilon_{p,a} = a\sigma_a^4 \left\{ 1 - 4\left(\frac{\tau}{t_0}\right) + 12\left(\frac{\tau}{t_0}\right)^2 - 24\left(\frac{\tau}{t_0}\right)^3 + 24\left(\frac{\tau}{t_0}\right)^4 - 24\left(\frac{\tau}{t_0}\right)^4 e^{-t_0/\tau} \right\} \dots (34)$$

(c) Sinusoidal wave

First, solve Eq. (26) for the case of 5 c/sec sinusoidal loading. The stress wave form of Fig. 13 (c) is expressed by

$$\sigma = \sigma_a \sin \omega t, \quad 0 \leq \omega t \leq \frac{\pi}{2} \dots (35)$$

Then, Eq. (26) is written as

$$\varepsilon_p + \tau \frac{d\varepsilon_p}{dt} = a\sigma_a^4 \sin^4 \omega t \dots (36)$$

Solving this equation gives

$$\varepsilon_p = c_1 e^{-t/\tau} + c_2 + \frac{a\sigma_a^4}{8} \left[3 - \frac{4}{1 + (2\tau\omega)^2} (\cos 2\omega t + 2\tau\omega \sin 2\omega t) + \frac{1}{1 + (4\tau\omega)^2} (\cos 4\omega t + 4\tau\omega \sin 4\omega t) \right] \dots (37)$$

From conditions, $\varepsilon_p = 0$ at $t = 0$ and $\varepsilon_p = a\sigma_a^4 \sin^4 \omega t$ at $\tau \rightarrow 0$, c_1 and c_2 are determined, then by eliminating ω from $\omega t_0 = \pi/2$ at $t = t_0$, plastic strain amplitude is given by

$$2\varepsilon_{p,a} = a\sigma_a^4 \left[1 - \frac{\pi \left(\frac{\tau}{t_0}\right)^2}{\left\{ 1 + \left(\pi \frac{\tau}{t_0}\right)^2 \right\} \left\{ 1 + \left(\frac{2\pi\tau}{t_0}\right)^2 \right\}} \times \left\{ 1 + 10 \left(\frac{\pi\tau}{2t_0}\right)^2 + 6 \left(\frac{\pi\tau}{2t_0}\right)^2 e^{-t_0/\tau} \right\} \right] \dots (38)$$

For the case of 11.5 c/sec sinusoidal loading, in place of t_0 in Eq. (37), $t_0' [(5/11.5)t_0 = 0.435t_0]$ may be used, as shown in Fig. 13 (d).

In the following, analytical expressions obtained above will be applied in the fatigue-

test: Eqs. (30), (34) and (38) show the plastic strain amplitudes developed by square, triangular and sinusoidal loading, respectively, of such a material which shows the deformation curve of Eq. (14) when subjected to quasi-static loading. Each of these equations can be expressed in a general form

$$2\varepsilon_{p,a} = a\sigma_a^4(1-\Delta) = 2\varepsilon_{p,0}(1-\Delta) \quad (39)$$

where $\varepsilon_{p,0}$ is the plastic strain amplitude developed by quasi-static application of stress amplitude σ_a , and Δ is the correction factor determined by the type of wave form. Fig. 14 shows calculated values of Δ against τ/t_0 taken as the abscissa.

In the fatigue test, just like the yielding in the static test, movable dislocations existing in the early test stage in a small number are multiplied gradually as the stress is repeated, and after having reached a critical value, the multiplication is rapidly enhanced; a considerable degree of plastic strain is repeated until failure under repeating application of stress. It is considered that the stress amplitude, just below which the number of dislocations, or macroscopically the plastic strain reaches the critical value, is the fatigue limit, and the process of dislocation multiplication results in an increased strain amplitude in the early stage.

In this connection, the behavior of deformation of a material during fatigue test may be discussed in two phases: early stage before the number of dislocations reaches the critical value and strain amplitude stabilized stage after the multiplication of dislocations has been almost completed. The deformation resistance of the material in the stabilized stage is considerably small as compared with the early stage. This phenomenon may be interpreted by rheological reasoning that, once the slider so far held by static frictional force slips, a dynamic frictional force occurs and reduces the deformation

resistance; or the number of deformable plastic elements is larger in the stabilized stage.

The wave-form effect on the dynamic stress-plastic strain hysteresis curve may be considered on the following assumptions.

(1) In both the early stage and the stabilized stage, the plastic strain is related to the stress as follows.

$$\varepsilon_p = a\sigma^4 \quad (40)$$

for quasi-static loading and

$$\varepsilon_p = 0 \quad (41)$$

for unloading. It is needless to say that "a" has different values for the early test stage and the stabilized stage.

(2) The mechanical model of the deformation behavior of a material shown in Fig. 12 refers to a condition of zero stress after removal of load from the negative stress side. The deformation resistance of each slider, F_p , takes into account the residual force.

(3) Fatigue limit σ_w corresponds to such a stress amplitude that the plastic strain amplitude $\varepsilon_{p,a}$ analysed above has a constant value $\Delta\varepsilon_p$.

(4) In quasi-static stress repetition, the following relation exists between stress amplitude σ_a and plastic strain amplitude $\varepsilon_{p,a}$.

$$B\varepsilon_{p,a} = \sigma_{a,0} - \sigma_{w,0} \quad (42)$$

where $\sigma_{w,0}$ is the quasi-static fatigue limit and B is a constant determined by material and test conditions.

On these assumptions, relations of stress amplitude σ_a to plastic strain amplitude $\varepsilon_{p,a}$ for various types of wave forms will be derived. First, from Eq. (39)

$$\sigma_a = \left(\frac{2\varepsilon_{p,a}}{a} \right)^{1/4} \left(\frac{1}{1-\Delta} \right)^{1/4} \quad (43)$$

Since fatigue limit σ_w is the stress amplitude when $\varepsilon_{p,a}$ is $\Delta\varepsilon_p$, the quasi-static expression is as follows.

$$\sigma_{w,0} = \left(\frac{2\Delta\varepsilon_p}{a} \right)^{1/4} \quad (44)$$

Thus, the fatigue limit for each type of wave form is given from Eq. (43) and Eq. (44) as follows.

$$\sigma_w = \left(\frac{2\Delta\varepsilon_p}{a} \right)^{1/4} \left(\frac{1}{1-\Delta} \right)^{1/4} = \sigma_{w,0} \left(\frac{1}{1-\Delta} \right)^{1/4} \quad (45)$$

Term $\{1/(1-\Delta)\}^{1/4}$ in Eq. (45) shows the wave-form effect.

A similar analysis may be applied to the stabilized stage of strain amplitude. Let stress amplitude σ_a required to develop plastic strain amplitude $\varepsilon_{p,a}$ in the quasi-static case, be denoted by $\sigma_{a,0}$, then

$$\sigma_a = \sigma_{a,0} \left(\frac{1}{1-\Delta} \right)^{1/4} \quad (46)$$

Therefore, from Eq. (45), Eq. (46) and Eq. (42),

Fig. 14 Result of calculation of Δ

$$\sigma_a - \sigma_w = (\sigma_{a,0} - \sigma_{w,0}) \left(\frac{1}{1-\Delta} \right)^{1/4} = \left(\frac{1}{1-\Delta} \right)^{1/4} B \varepsilon_{p,a} \quad \dots\dots\dots (47)$$

Eq. (47) shows the wave-form effect on the stress-plastic strain relationship. Comparison of Eq. (47) with the experiment is given in Fig. 15. Values of τ/t_0 , $\sigma_{w,0}$ and B are so selected as to be suitable for the experiment.

$$\left. \begin{array}{l} \frac{\tau}{t_0} = 0.14 \\ \sigma_{w,0} = 19.8 \text{ kg/mm}^2 \\ B = 4600 \text{ kg/mm}^2 \end{array} \right\} \dots\dots\dots (48)$$

Except for triangular loading, an approximate agreement is seen. It is not known why discrepancies exist in the case of triangular loading.

On the notched test specimen, local plastic deformation occurs in the notch root surrounded by elastic areas. A simplified rheological model of such a test specimen is shown in Fig. 16.

The notch root can be represented by a spring and a dynamic plastic model connected in series therewith. The surrounding area of elastic deformation can be represented by spring K. These two models are connected in parallel. When force F is applied, deformation x or $x(l_2/l_1)$ for each part occurs where $l_1: l_2$ is a lever ratio corresponding to the sharpness of the notch. If the notch root is locally limited as in a V-notched test specimen, the deformation of the test specimen under force F will be determined substantially by spring K and the notch root will be deformed in proportion to the magnitude of force F . This means that, in the fatigue test, a strain wave form proportional to the load wave form will be repeated in the notch root. From comparison between Fig. 6 and Fig. 7, it seems that the wave-form effect is rather difficult to appear on the notched specimen because the strain wave form has smaller influence on the fatigue strength than the stress wave form.

Fig. 15 $\sigma_a - \varepsilon_{p,a}$ curves

As described above, the wave-form effect on the fatigue strength varies with tested material, stress concentration, test conditions, etc. It is expected that this wave-form effect is more liable to appear on mild steel or the like which shows large plastic strain during fatigue test than on hard steel or the like which shows small plastic strain during fatigue test, and is more difficult to appear on any test specimen with larger stress concentration or stress gradient.

5. Conclusion

The stress wave under service is generally non-sinusoidal. The fatigue test by non-sinusoidal loading was made and compared with the sinusoidal stress wave fatigue test. Wave forms selected are 5 c/sec sinusoidal, square and triangular and 11.5 c/sec sinusoidal.

This test was carried out by a 10 ton Hydraulic Service Load Fatigue Testing Machine. Two types of test specimens, unnotched and V-notched, were used. The tested material is 0.3% carbon steel. The test results are summarized as follows.

(1) Unnotched specimens

- (a) Both fatigue limit and strength are affected by the wave forms.
- (b) Fatigue life is not affected by the wave form if plastic strain amplitude or plastic strain energy is same.
- (c) Difference in the deformation behavior of the material (wave-form effect) can be explained by the speed dependence of the deformation.
- (d) The wave-form effect on the deformation behavior of the material was analyzed by use of a plastic deformation model, a series connection of plastic elements each being a combination of a spring, a slider and a dash-pot.

Fig. 16 Mechanical model of notched specimen

(2) Notched specimens

The wave-form effect on the fatigue strength is more difficult to appear on the notched test specimen. The reason may be the plastic deformation in the notch root being restricted by the surrounding area of elastic deformation.

The wave-form effect is a critical problem in designing any machine whose running cycle is extremely prolonged as in the case of a pressure vessel. Since the fatigue failure occurs generally in the notched area, further investigation in future will be necessary of the wave-form effect on the fatigue strength of notched test specimens.

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