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On the Improvement in the Suction Performance of High-Head Axial-Flow Pumps*

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The optimum diameter of axial-flow pumps has been theoretically studied taking the suction performance into consideration. Two high-head axial-flow pumps, composed of two tandem impellers, were tested to study their characteristic performances and suction performances.

The main results of the study are as follows.

The optimum flow coefficient decreases with the cavitation coefficient of the blade section. The pump, which demonstrates better suction performance and efficiency, consists of a lightly loaded 1st stage impeller with high stagger angle and a heavily loaded 2nd stage impeller of somewhat larger flow coefficient.

1. Introduction

For the design of a high-head low-specific speed axial-flow pump, the blade setting angle is very small and the solidity is very large. Therefore mutual interference of adjacent blades in an impeller cascade as well as suction performance for cavitation become subjects of discussion.

L.B. Stripling⁽¹⁾ improved the suction performance of a volute pump with a helical inducer, and M. Oshima⁽²⁾ achieved good suction performance by installing a throttle plate at the entrance of a volute pump with an inducer. However, the authors did not find a single paper which investigated on the suction performances of an axial-flow pump with an inducer.

In this report, the suitable pump diameter for suction performance is theoretically studied and two high-head axial-flow pumps are built. The tests are made on the relation between the characteristic performance and the suction performance.

2. Nomenclature

The following nomenclature is used in the paper.

a : area of blade projected to plane perpendicular to pump axis
 D : diameter
 d : blade thickness
 g : acceleration of gravity

H : total head

$H_i (=L_e/(rQ))$: input head

ΔH_i : head loss in pump

h_{ic} : additional head loss due to cavitation

Δh : difference between static pressure head at impeller inlet and saturated vapor pressure head

K : cavitation zone

k_s : cavitation number

L_e : effective shaft power

l : chord length of blade element

l/t : solidity

$NPSH$: net positive suction head

n : speed of rotation

P : total pressure

p : static pressure

p_v : saturated vapor pressure

Q : flow-rate

t : blade spacing

u : circumferential velocity of impeller

v_m : meridional velocity of fluid

w : relative velocity of fluid

z : number of blades

γ : specific weight of water

$\zeta (=d_e/D_a)$: thickness diameter ratio

η_e : internal efficiency

$\lambda (=d_a/d_e)$: blade thickness ratio

ν : boss ratio

τ : cavitation parameter

$\phi (=v_m/u)$: flow coefficient

$\psi = H/(u^2/2g)$: head rise coefficient

$\psi_i = H_i/(u^2/2g)$: input head coefficient

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Subscripts

- a : blade tip, c : cavitation condition
 e : blade root, g : guide vane
 m : 2nd impeller, nc : noncavitation condition
 opt : optimum condition of noncavitation
 p : 1st impeller
 t : value taking blade thickness into consideration
 v : value taking theoretical flow-rate into consideration
 $4'$: 2nd impeller inlet
 Other symbols and subscripts are defined when they appear in the text.

3. Relation between optimum pump diameter and cavitation parameter

When the effect of blade thickness is considered on axial-flow pumps, the relation between $NPSH$ and the cavitation number k_{sat} becomes

$$NPSH = (P_1 - p_v)/\gamma \left\{ \begin{aligned} &= (P_1 - p_v)/\gamma + (v_{mt}^2/2g) \\ &= k_{sat}(w_{at}^2/2g) + (v_{mt}^2/2g) \end{aligned} \right\} \quad \dots\dots\dots(1)$$

where P_1 and p_1 are the total pressure and the static pressure at the impeller inlet, respectively. Thus the meridional velocity v_{mt} (see Fig. 1) based on theoretical flow-rate is

$$v_{mt} = Qv / \left\{ \frac{\pi}{4} D_a^2 (1 - \nu^2) - \frac{z(d_a + d_e)D_a(1 - \nu)}{4 \sin \theta} \right\} \quad (2)$$

Further, because for many pumps d_a/d_e is approximately equal to ν , supposing $d_a/d_e = \nu$ and $\sin \theta = \phi_a$ for $\theta \leq 20^\circ$, Eq. (2) reduces to

$$v_{mt} = Qv / \left\{ \frac{\pi}{4} D_a^2 (1 - \nu^2) - \frac{z d_e (1 - \nu^2) D_a}{4 \phi_a} \right\} = \frac{4Qv}{\pi(1 - \nu^2)D_a^2} + \frac{z d_e n}{60} \quad \dots\dots\dots(2)'$$

The relative velocity w_{at} is expressed as

$$w_{at} = \left[\left(\frac{\pi D_a n}{60} \right)^2 + \left\{ \frac{4Qv}{\pi(1 - \nu^2)D_a^2} + \frac{z d_e n}{60} \right\}^2 \right]^{1/2} \quad \dots\dots\dots(3)$$

Fig. 1 Cross section of impeller blade

Substituting v_{mt} of Eq. (2)', w_{at} of Eq. (3) and $\xi = d_e/D_a$ into Eq. (1), $NPSH$ becomes

$$NPSH = \frac{1}{2g} \left[k_{sat} \left(\frac{\pi D_a n}{60} \right)^2 + \left\{ \frac{4Qv}{\pi(1 - \nu^2)D_a^2} + \frac{z \xi n}{60} D_a \right\}^2 (1 + k_{sat}) \right] \quad \dots\dots\dots(4)$$

In order to find the optimum pump diameter for the suction performance, $NPSH$ of Eq. (4) is differentiated with respect to the variable D_a and the result is put equal to zero, then

$$\left\{ 1 + \frac{1 + k_{sat}}{k_{sat}} \left(\frac{z \xi}{\pi} \right)^2 \right\} D_a^3 - \frac{1 + k_{sat}}{k_{sat}} \frac{240}{\pi^3 (1 - \nu^2)} \times \left(\frac{Qv}{n} \right) z \xi = \frac{1 + k_{sat}}{k_{sat}} \frac{115200}{\pi^4 (1 - \nu^2)^2} \left(\frac{Qv}{n} \right)^2 D_a^{-3} \quad \dots\dots\dots(5)$$

Eq. (5) is calculated for $z \xi = 0.17$ and $d_a/d_e = \nu$. The optimum diameter vs. the flow-rate variable at the cavitation number $k_{sat} = 0.05$ and the boss ratio ranging from 0.30 to 0.80 appears in Fig. 2, while the relationship for the boss ratio $\nu = 0.5$ and the cavitation number ranging from 0.01 to 0.30 appears in Fig. 3. It is seen that the optimum pump diameter $D_{a\ opt}$ increases with the boss ratio while it increases as the cavitation number k_{sat} decreases.

When k_{sat} and ν are constant, $D_{a\ opt}$ increases

Fig. 2 Optimum pump diameter $D_{a\ opt}$ versus flow-rate variable Q_v/n for various boss ratios ν Fig. 3 Optimum pump diameter $D_{a\ opt}$ versus flow-rate variable Q_v/n for various cavitation numbers K_{sat}

with the flow-rate variable (Q_v/n). Introducing $\phi_v = v_{mv}/u_a = [Q_v / \{(\pi/4)D_a^2(1-\nu^2)\}] \{60/(\pi n D_a)\}$, Eq. (5) becomes

$$2 \frac{1+k_{sat}}{k_{sat}} \phi_v^2 + \frac{1+k_{sat}}{k_{sat}} \frac{z\zeta}{\pi} \phi_v - \left\{ 1 + \frac{1+k_{sat}}{k_{sat}} \left(\frac{z\zeta}{\pi} \right)^2 \right\} = 0 \quad \dots\dots\dots (6)$$

For the variable $z\zeta = 0.14 \sim 0.25$, cavitation number k_{sat} is plotted against flow coefficient ϕ_v in Fig. 4. It is seen that the cavitation number k_{sat} increases with the flow coefficient ϕ_v and that the slope of performance curves depends upon the variable $z\zeta$.

The pump KI was designed with the conditions that the 1st stage impeller contributed 4 meters of head while the 2nd stage impeller contributed 6 meters of head, and the both stages of the impeller had the same flow coefficient. For the pump KII, the 1st stage impeller contributed only 2 meters of head and had a small flow coefficient, while the 2nd stage impeller contributed 8 meters of head and had a large flow coefficient.

The design specifications, the main design parameters and the main dimensions of the test pumps are summarized in Tables 1, 2 and 3. The blade design parameters are summarized in Table 4. The blade profiles⁽³⁾ for the 1st impeller of the pump KII are shown in Fig. 5. Photographs of the test impellers are shown in Fig. 6.

4. Test facility and test procedures

A schematic diagram of the facility is shown in Fig. 7. The pump speed was observed with an electronic counter and the shaft power was measured with a D.C. dynamometer. The volume flow-rate measured with a venturi flowmeter at the outlet side of the pump.

The test high-head axial-flow pumps are called KI type and KII type. A cross section of the pump is shown in Fig. 8. The pump KI can be tested without the 1st impeller when necessary. The impeller casing and a part of tank are made of plexiglass, so that cavitation zones in the neighbourhood of blades can be observed.

Fig. 4 Cavitation number K_{sat} versus flow coefficient ϕ_v for variables $z\zeta = 0.25, 0.14$

Table 2 Main design parameters of pumps

Main design parameters	Pump type Classification of impeller	KI		KII	
		KI_m	$KI_p + KI_m$	KII_m	$KII_p + KII_m$
Volumetric efficiency	η_v	0.950	0.960 0.940	0.970	0.980 0.960
Boss ratio	ν	0.600	0.600 0.600	0.775	0.310 0.775
Flow coefficient	ϕ_v	0.0851	0.0840 0.0858	0.1333	0.0583 0.1347
Theoretical head coefficient	ϕ_{th}	0.4258	0.1879 0.2800	0.4176	0.1078 0.3356
Manometric efficiency of blade element	η_{Mbe}	0.735	0.708	0.795	0.747
Blade pressure-distribution coefficient	S_t^*	1.197	1.115	1.325	1.115
Cavitation parameter	τ_a	0.060	0.050	0.118	0.048
Net positive suction head	$\frac{(m)}{NPSH}$	1.818	1.509	3.561	1.449

Table 1 Design specifications of test pumps

Specifications		Flow-rate	Total head	Speed of rotation	Flow coefficient	Head rise coefficient	Specific speed	Dimensionless specific speed	
Classification	Symbols								
Pump type	of impeller	$Q \text{ m}^3/\text{sec}$	$H \text{ mAq}$	$n \text{ rpm}$	ϕ_a	ϕ_a	$n_s (\text{m}, \text{m}^3/\text{sec}, \text{rpm})$	σ	
KI	1st impeller	0.0167	10	3600	0.0809	0.3313	72.5	164	1.04
	2nd impeller							121	0.77
KII	1st impeller	0.0167	10	3600	0.1383	0.3313	72.5	276	1.75
	2nd impeller							97.5	0.619

Table 3 Main dimensions of test pumps

Pump type	Dimensions Symbols	Casing diameter	Rotor outer diameter	Boss diameter	Blade span	Tip clearance	Boss ratio	Tip clearance span ratio	Number of impeller vanes	Number of guide vanes
		D_0 mm	D_a mm	D_b mm	s mm	c mm	ν	c/s %	z	z_g
KI		130.0	129.0	78.0	25.5	0.5	0.60	1.96	4	6
KII	KII _p	130.0	129.0	40.0	44.5	0.5	0.31	1.12	3	6
	KII _m			100.0	14.5		0.78	3.35	4	

Notice: For KI pump, the axial spacing of two impellers and the number of moving vanes can be varied.

Table 4 Blade design parameters
(a) 1st impeller

Pump type		KI					KII				
Blade section	Name	a	b	c	d	e	a	b	c	d	e
	Diameter D mm	129.0	117.0	104.0	91.0	78.0	129.0	112.8	93.6	69.6	40.0
Radius ratio ξ		1.000	0.907	0.806	0.705	0.605	1.000	0.874	0.726	0.540	0.310
Chord length l mm		153.2	137.8	122.5	107.2	91.9	40.0	40.0	40.0	40.0	40.0
Blade spacing t mm		204.2	183.8	163.4	142.9	122.5	130.9	118.2	98.1	72.9	41.9
Solidity l/t		0.750	0.750	0.750	0.750	0.750	0.310	0.337	0.408	0.549	0.954
Blade setting angle θ		6°18'	7°16'	8°38'	10°10'	12°25'	5°24'	6°00'	7°30'	10°42'	21°19'
Blade section		circular arc profile (f/l %)					NACA 65- ($C_u=1.0$)				
		1.50	2.00	2.50	3.00	3.50	(4A ₂ I _{sb})08	(4A ₂ I _{sb})09	(5A ₂ I _{sb})10	(5A ₂ I _{sb})11	(12A ₂ I _{sb})12

(b) 2nd impeller

Pump type		KI					KII				
Blade section	Name	a	b	c	d	e	a	b	c	d	e
	Diameter D mm	129.0	117.0	104.0	91.0	78.0	129.0	122.4	115.4	108.0	100.0
Radius ratio ξ		1.000	0.907	0.806	0.705	0.605	1.000	0.949	0.895	0.838	0.775
Chord length l mm		91.9	85.0	77.6	69.7	61.3	100.0	97.8	95.4	92.8	90.0
Blade spacing t mm		102.1	91.9	81.7	71.5	61.3	101.3	96.2	90.6	84.8	78.6
Solidity l/t		0.900	0.925	0.950	0.975	1.000	0.987	1.017	1.052	1.094	1.146
Blade setting angle θ		5°14'	7°10'	9°54'	12°56'	19°42'	12°32'	13°14'	14°38'	15°53'	17°43'
Blade section		NACA 16- ($C_u=1.0$)					NACA 16- ($C_u=1.0$)				
		505	506	507	508	1010	105	(1.2)06	207	(2.5)08	(3.5)09

Notice: Each value in table (b) indicates design value with 1st impeller and 2nd impeller.

(c) guide vane

Pump type		KI					KII				
Blade section	Name	a	b	c	d	e	a	b	c	d	e
	Diameter D mm	130.0	117.0	104.0	91.0	78.0	129.0	122.4	115.4	108.0	100.0
Radius ratio ξ		1.000	0.900	0.800	0.700	0.600	1.000	0.949	0.895	0.837	0.775
Chord length l_g mm		200.0	200.0	200.0	200.0	200.0	100.0	100.0	100.0	100.0	100.0
Blade spacing t_g mm		51.1	46.0	40.8	35.8	30.6	67.5	64.1	60.4	56.6	52.4
Solidity l_g/t_g		3.915	4.345	4.900	5.585	2.080	1.480	1.560	1.667	1.766	1.908
Blade setting angle θ_g		52°52'	56°51'	56°09'	54°40'	53°28'	73°42'	73°09'	73°30'	73°48'	76°19'
Blade section		Circular arc profile					NACA 65- (24A ₄ K ₆)04 ($C_u=1.0$)

Fig. 5 Blade profiles for 1st impeller of pump KII

(a) Impeller of pump KI

(b) Impeller of pump KII

Fig. 6 Photographs of test impellers of pumps

Fig. 7 Schematic diagram of test facility

(a) Pump KI

(b) Pump KII

Fig. 8 Cross-sectional view of high-head axial-flow pumps

The performance tests were carried out for the pump KI with and without the 1st impeller as well as for the pump KII. First, the overall performance tests of each pump were made at 2 400, 3 000 and 3 600 rpm.

The suction performance tests were conducted by reducing suction head at a constant pump speed and a constant flow-rate. Then, the inception and the growth of cavitation in the pumps were observed by a stroboscope and photographed by means of microflashes.

5. Cavitation performance of the test pumps

5.1 Cavitation zones and cavitation coefficients

The inception and the growing processes of cavitation were clarified by means of flow visualizations. The cavitation zones were classified as shown in Fig. 9.

The cavitation tests was made by reducing the cavitation parameter τ_a or the cavitation number k_{sa} and the change of the pump characteristics was observed, where

$$\tau_a = \left(\frac{P_4' - p_v}{\gamma} \right) / \left(\frac{u_a^2}{2g} \right) = NPSH / \left(\frac{u_a^2}{2g} \right) \quad \dots (7)$$

$$= \left\{ \Delta h + \frac{(\phi_a u_a)^2}{2g} \right\} / \left(\frac{u_a^2}{2g} \right)$$

$$k_{sa} = \left(\frac{P_4' - p_v}{\gamma} \right) / \left(\frac{w_4' a^2}{2g} \right) = \Delta h / \left(\frac{w_4' a^2}{2g} \right) \quad \dots (8)$$

$$= \Delta h / \left\{ \frac{u_a^2}{2g} (1 + \phi_a^2) \right\}$$

The relation between τ_a and k_{sa} is given by

$$\tau_a = k_{sa}(1 + \phi_a^2) + \phi_a^2 \quad \dots (9)$$

Table 5 shows the main characteristic values of

Fig. 9 Classification of cavitation zone

the test pumps. The characteristic performances of the test pumps are made dimensionless by comparing them with the corresponding values at the optimum flow condition. The relations among these variables are presented in Fig. 10. The influence of cavitation on the characteristic performance of the pumps is summarized in Fig. 11. The inception lines of cavitation zones are shown by chain lines in Fig. 11. K_1 and K_3 are influenced by the condition of the operation. K_3 can not always be distinguished from K_1 clearly; when K_1 extended from blade leading edge to approximately 20 percent of the chord length on the upper surface it was regarded as the inception of K_3 .

Table 5 Main characteristics of test pumps

Pump type		KI		KII
Characteristics		KI _m	KI _p +KI _m	KII _p +KII _m
Values at the optimum operating efficiency	Flow coefficient ϕ_{aopt}	0.082	0.082	0.170
	Head rise coefficient ψ_{aopt}	0.190	0.220	0.210
	Power coefficient π_{eaopt}	0.0310	0.036 6	0.055 0
	Internal efficiency η_{eopt}	0.540	0.520	0.625
	Input head coefficient ϕ_{iaopt}	0.352	0.423	0.336
	Dimensionless specific speed σ_{opt}	0.795 8	0.713 0	0.840 1
	Specific speed n_{sopt} (m, m ³ /sec, rpm)	125.5	112.4	132.5
Values at the shut off	Head rise coefficient ϕ_{as}	0.475	0.572	0.740
	Power coefficient π_{eas}	0.046 0	0.056 0	0.128 5
Head rise ratio ϕ_{as}/ϕ_{aopt}		2.500	2.600	3.524
Power ratio π_{eas}/π_{eaopt}		1.484	1.530	2.336

Fig. 10 Overall performances for test pump

Fig. 11 Head rise coefficient ϕ_a versus cavitation number k_{sa} for various flow coefficients

Fig. 12 Photographs of the cavitation pump KI_m

It may be thought that K_7 rises from local separation on the upper surface, and that pK_8 and gK_2 rise from separation on the lower surface. Fig. 11 shows that the head coefficients are affected a little by K_2, K_3 and K_4 cavitation, but they break down abruptly as mK_7 grows. The pump KII demonstrates better suction performance in a wide range of flow-rates. The results of Stripling's experiment are compared with the results of the pump KII in Fig. 11 (c). Figs. 12, 13 and 14 show the appearance of cavitation in the pumps KI_m, KI_p + KI_m and KII_p + KII_m, respectively. The cavitation development at a small cavitation number is so complex that its cavitation zone can not be distinguished from others. The pK_8, mK_4 and gK_2 are shown in Photos. (a) at a small cavitation number k_{sa} . The mK_4 develops to the lower surface, and it may be interpreted that the 1st impeller contributes almost the head and the 2nd impeller obstructs partly the flow.

5.2 Effects of solidities and blade sections

Fig. 15 shows the relation between the head rise ratio ϕ_a/ϕ_{anc} and the variable $\tau_a\{(l/t)_p + (l/t)_m\}_a$; the variable implies the effect of pressure loading for a given head rise. It is seen from Fig. 15 that the cavitation performance curves of the pumps KI_m, KI_p + KI_m and KII_p + KII_m nearly overlap each other, and that still more the existing discrepancy arises from the influence of the shape of blade section.

The input head and the head loss in a pump are affected by the cavitation conditions. The losses of head in a pump under noncavitation conditions and cavitation conditions become Eqs. (10) and (11), respectively. The additional head loss due to cavitation may be described by Eq. (12) and its nondimensional form becomes Eq. (13).

$$\Delta H_{inc} = H_{inc} - H_{nc} \quad (10)$$

$$\Delta H_{ic} = H_{ic} - H_c \quad (11)$$

$$h_{ic} = \Delta H_{ic} - \Delta H_{inc} \\ = (H_{ic} - H_c) - (H_{inc} - H_{nc}) \quad (12)$$

$$\frac{h_{ic}}{(u_a^2/2g)} = \left\{ \begin{aligned} &(H_{ic} - H_c) \\ &- (H_{inc} - H_{nc}) \end{aligned} \right\} / (u_a^2/2g) \quad \dots (13)$$

$$= (\psi_{anc} - \psi_{ac}) - (\psi_{ienc} - \psi_{iac})$$

Fig. 16 shows the relation between the additional head loss due to cavitation and the variable $\tau_a \{ (l/t)_p + (l/t)_m \}_a$ for the various flow coefficients.

It is recognized that the additional head loss increases with a decreasing flow coefficient of the test pumps.

5.3 Influence of extent of cavitation zones on performances

The front views of the cavitation zones on the 1st impeller are illustrated in Fig. 17, at the best efficiency point of the pump KII. The extent of cavitation zones was graphically obtained on photographs. The relation between the area ratios a_c/a and the cavitation numbers k_{sa} at the various flow coefficients is shown in Fig. 18. The performance was not seriously reduced in the development of

cavitation shown in Fig. 17.

6. Conclusion

The following conclusion may be drawn from this study on the suction performance of the high-head axial-flow pumps.

(1) The relation between the flow coefficient and the cavitation number was shown dimensionless using the parameters; boss ratio ν , blade thickness ratio λ and variables $z\zeta$.

(2) K_7 was observed over the zone stretched from the whole upper surface of section b to the adjacent blade and K_8 was observed near mid-chord of the blade section b on the lower surface, at a small cavitation number.

(3) Ratio of head rise under cavitation conditions to head rise under noncavitation conditions was expressed as a function of the variable $\tau_a \{ (l/t)_p + (l/t)_m \}_a$ for each pump at the flow coefficient.

Fig. 13 Photographs of the cavitation pump K1p+K1m

(a) Photos. at the large flow

(b) Photos. at the optimum-operating flow

(c) Photos. at the small flow

Fig. 14 Photographs of the cavitation pump KIIp+KIIm

(4) The additional head loss due to cavitation and the ratio of cavitation area a_c/a were shown as a function of the cavitation number k_{sa} .

Fig. 15 Cavitation performance: Head ratio $\psi_a/\psi_{a,nc}$ versus variable $\tau_a\{(l/t)_p + (l/t)_m\}_a$

(5) The pump, which demonstrates good suction performance and efficiency, consists of a lightly loaded 1st stage impeller with high stagger angle and a heavily loaded 2nd impeller of somewhat large flow coefficient.

Fig. 16 Cavitation performance: Additional loss due to cavitation $h_{ic}/(u_a^2/2g)$ versus variable $\tau_a\{(l/t)_p + (l/t)_m\}_a$

$\phi_a=0.170, \psi_a=0.205$ $\phi_a=0.170, \psi_a=0.207$ $\phi_a=0.170, \psi_a=0.203$ $\phi_a=0.170, \psi_a=0.207$
 $k_{sa}=0.6087$ $k_{sa}=0.4645$ $k_{sa}=0.3456$ $k_{sa}=0.2932$

Fig. 17 Front views of cavitation 1st impeller for pump KII at the optimum-operating flow

Fig. 18 Cavitation area ratio a_c/a versus cavitation number k_{sa} for various flow coefficients

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Discussion

T. WATANABE: It is desirable to achieve a high pump-head in a simple axial-flow pump. It is significant that a head of 10 meters is obtained by the present axial-flow pumps. It may be a proper treatment that the authors designed the pump KII in which the 1st stage impeller had a small boss ratio and was located considerably far away from the 2nd stage impeller.

I would like to know the following points.

(1) I expect that the authors should continue experimental studies on KII type in order to find the best split of the total head for the 1st and the 2nd stage impeller and their optimum boss ratio.

(2) When the axial distance between the 1st stage impeller and the 2nd stage impeller is small like in the pump KI, it is preferable that two adjacent rows of blades be treated as a tandem cascade instead of two independent ones, because the cavitation performances may be affected by the relative position between two blade rows.

(3) The pump efficiencies are somewhat low. This may be due to the size of the pump. What efficiency of a pump may be achieved if it is large?

M. OSHIMA: (4) It is stated in connection with Fig. 4 that the cavitation number k_{sat} increases under the influence of the blade thickness with increasing flow coefficient. The statement should be that as the blade thickness increases, the optimum flow coefficient decreases i.e. the optimum pump diameter increases.

(5) As for Fig. 4, it is expressed that the tendency of the curve in Fig. 4 is reversed in the region of $\phi_v \leq 0.115$. Is this owing to the condition, $d_a/d_e = \nu$?

The ratio of the circumferential blade thickness to the blade spacing at the boss diameter is

$$\frac{zd_e/\sin\theta}{\pi D_e} = \frac{z\zeta}{\pi\nu\sin\theta}$$

This ratio becomes unity if $z\zeta = 0.17$, $\nu = 0.4$, and $\phi_v = 0.135$. This means that there is no flow area at the boss diameter. Should the available range of the flow coefficient be somewhat limited?

(6) The cavitation limit of this kind of pump is considered the inception point of head drop (3% decrease in total head). It is thought that the cavitation parameter referring to the limit can hardly be evaluated with the pressure distribution on the blade surface.

T. NAGATA: (7) How large are the values of the suction specific speeds of the pumps KI and KII?

How much superior is the pump with the 1st stage impeller to the only 2nd stage impeller with respect to the suction specific speed?

K. NAKASE: (8) I would like to know the relative dimensions of the 1st stage impeller blade and the 2nd stage impeller blade.

Authors' closure

The authors wish to thank the discussors for their interesting and enlightening comments on the paper.

The authors almost agree to Prof. Watanabe's comments, but would like to supplement the following three points:

(1) The head loading on the 1st stage impeller can be decided from the difference between the *NPSH* of the 2nd stage impeller and of the 1st stage impeller as stated in the papers by Ross^(a) and Oshima [see reference (2)].

The relations between the efficiency and the other parameters are theoretically obtained in the reference (3), but the authors believe that systematic experimental studies on the relations are very important.

(2) The cavitation characteristics of a tandem cascade are affected by the mutual interference of the two cascades, if they locate closely. However, no significant difference was observed on the characteristics of the pump KI when a series of experiments was made changing the relative circumferential position of the cascades, i.e. the 1st and the 2nd

(a) C.C. Ross and Gordon Banerian: *Trans. ASME*, Vol. 78, No. 8 (1956), p. 1715.

Append.-Fig. 1 Cross section of impeller blade

Append.-Fig. 2 Cavitation number k_{sat} versus flow coefficient ϕ_v for boss ratio $\nu=0.5$, blade thickness ratio $\lambda=0.5$ and variables $z\zeta=0.10, 0.15, 0.20$ Append.-Fig. 3 Range of available flow coefficient ϕ_{ve}

Append.-Fig. 5 Relative position between two blade rows

stage impeller. Therefore, it is suspected that the mutual interference of the two cascades is not significant with the present distance.

(3) As the conversion formula of efficiency on an axial-flow pump is not found, Rüttschi's formula^(b) on a volute pump is used. The efficiency of a pump ten times as large as the pump KII becomes

$$\eta_h = \frac{1 - 3.15/d_0^{1.6}}{1 - 3.15/d_{0m}^{1.6}} \eta_{hm} = 0.658$$

where η_h , η_{hm} , d_0 and d_{0m} are the hydraulic efficiency of the large pump, the hydraulic efficiency of the model pump, the specific size of the large pump and the specific size of the model pump, respectively.

The efficiency of the large pump is not high. This may be due to the facts that the tip clearance ratio c/s [see Table 3] of the 2nd stage impeller is large and the flow coefficient is small [see reference (3)].

(4)~(6) The authors agree with Mr. Oshima in his interpretation for Fig. 4. Concerning the comment pointed out by him that the flow area at the boss tends to zero, the authors

$$S = nQ^{1/2} (NPSH)^{-3/4} \text{ (m, m}^3/\text{min, rpm)}$$

Append.-Fig. 4 Cavitation performance: Head ratio versus suction specific speed

Relative circumferential position	A
6°	0.25t
45°	0.82t

(b) Stephen Kazarkiewicz and Adam. T. Troskolanski: *Impeller Pumps*, (1965), p. 618, D.R.P. Warszawa.

made a more detailed expression as follows. Axial velocity in the impeller passage is [see Append.-Fig. 1]

$$v_{mt} = Q_v / \left\{ \frac{\pi}{4} D_a^2 (1 - \nu^2) - \frac{z D_a (1 - \nu)}{4} \times \left(\frac{d_a}{\sin \theta_a} + \frac{d_e}{\sin \theta_e} \right) \right\} \dots \dots \dots (2)'$$

With the assumptions $\phi_{va} = \sin \theta_a$, $\phi_{ve} = \sin \theta_e$, free vortex flow pattern $\phi_{ve} = \phi_{va} / \nu$ and the definitions $d_a/d_e = \lambda$, $d_e/D_a = \zeta$, we obtain the optimum diameter $D_{a \text{ opt}}$ as

$$\left\{ 1 + \frac{1 + k_{sat}}{k_{sat}} \left(\frac{z\zeta}{\pi} \frac{\lambda + \nu}{1 + \nu} \right)^2 \right\} D_a^3 - \frac{1 + k_{sat}}{k_{sat}} \frac{240}{\pi^3 (1 - \nu^2)} \frac{Q_v}{n} \frac{\lambda + \nu}{1 + \nu} z\zeta \left\{ \dots \dots \dots (5) \right.$$

$$= \frac{1 + k_{sat}}{k_{sat}} \frac{115200}{\pi^4 (1 - \nu^2)^2} \left(\frac{Q_v}{n} \right)^2 D_a^{-3} \left. \right\}$$

Nondimensional optimum condition becomes

$$2 \frac{1 + k_{sat}}{k_{sat}} \phi_v^2 + \frac{1 + k_{sat}}{k_{sat}} \frac{z\zeta}{\pi} \phi_v - \left\{ 1 + \frac{1 + k_{sat}}{k_{sat}} \left(\frac{z\zeta}{\pi} \frac{\lambda + \nu}{1 + \nu} \right)^2 \right\} = 0 \dots \dots \dots (6)'$$

The relation between k_{sat} and ϕ_v is shown in Append.-Fig. 2 for the range of practical values of $z\zeta$, λ and ν . In Append.-Fig. 2, the flow coefficient increases with k_{sat} . The optimum coefficient ϕ_v decreases for large k_{sat} , and increases for small k_{sat} with increasing $z\zeta$. The available range of flow coefficient ϕ_{ve} is shown in Append.-Fig. 3.

The authors are not certain that these optimum values are not applicable when once cavitation occurs in an impeller, but the authors designate the optimum for cavitation inception considering the thickness of the blades.

(7) (8) To Mr. Nagata's and Mr. Nakase's questions, the authors add illustrations Append.-Figs. 4 and 5. The difference in the suction specific speed value [m, m³/min, rpm] between KII and KI pump is 290, and the difference between KI_m + KI_p and KI_m is 210. The arrangement of the cascade in Append.-Fig. 5 is the 1st stage impeller with four blades but the tests in this paper were carried out with the 1st stage impeller having two blades.