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Service Load and Fatigue Strength*

(Report 6, Crack Initiation and Propagation under Repeated Varying Load)

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To clarify the behavior of fatigue strength under service load from the view point of the crack propagation, fatigue tests under successively varying load were performed.

In this paper, a prediction method for the crack propagation curve is proposed by applying the superposition law to the crack propagation of the annealed carbon steel specimens and the induction hardened carbon steel specimens, both of which are loaded under sinusoidally varying amplitude.

Experimental results show that the Minor's law is applicable to the crack initiation for three materials: 0.17% and 0.36% carbon annealed and 0.17% carbon induction hardened steels.

On the other hand, the superposition law is applicable to the induction hardened specimens, but is not applicable to the annealed ones and the results are situated in the dangerous side.

Either, the effect of the period of varying load is not recognized in the present three materials.

1. Introduction

It is very difficult to predict the fatigue behavior under varying load, and there are still many obscure points about its intrinsic nature. For this reason, many studies have been performed to clarify these uncertain points, but most researchers have been studying about the behavior of the final fracture of specimens. Recently, the micro structural observation, for example, the observation of the crack propagation has come to be adopted to make clear the fatigue behaviors under varying load^{(1)~(3)}.

Most of these papers are, however, concerned with the fatigue crack initiation and propagation under stepwise varying load and the detailed behaviors are not clarified yet. The present research group has also discovered many facts about the crack initiation and propagation under stepwise varying load^{(3)~(4)}.

As a result of these findings, the following

conclusion is reached: Identifying the crack propagation process is the most important subject to make clear the behaviors and estimate the life of fatigue under varying load.

In order to clarify the intrinsic nature of fatigue behaviors and estimate the fatigue life, more detailed and extensive experimental researches on the initiation and propagation of fatigue crack are being planned.

As mentioned above, there are a few studies on the crack initiation and propagation behaviors under stepwise varying load tests, but there are very few studies on the fatigue behaviors under continuously varying load.

To make clear the behaviors of the crack under continuously varying load is, however, indispensable for understanding the behaviors of the fatigue crack propagation under various kinds of varying load fatigue test and also is important from the viewpoint of the investigation of fatigue behaviors under service conditions, for example, the studies of the fatigue strength of car axles.

Therefore, in this paper the fatigue crack initiation and propagation behaviors under sinusoidally varying load are examined to find a clue to the fatigue behavior under continuously varying load and also to investigate the difference between the behaviors

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under stepwise varying load and continuous one. Then the superposition law is applied to the crack propagation and the expected crack growth curves are obtained by a simple calculation method for the purpose of investigating the test results. In addition, the effects of the period of loading cycles are examined.

2. Experimental procedure

For these fatigue tests, the following three materials are used: 0.17% and 0.36% carbon annealed steels and 0.17% carbon induction hardened one (the maximum temperature equals 1100°C and the heating duration equals 6.5 seconds). The mechanical properties and the chemical compositions of the annealed steels are shown in Tables 1 and 2, and the dimensions of the specimens used are shown in Fig. 1.

Testing apparatus is an Ono type rotating bending fatigue testing machine with the varying load imposing equipment composed of oil pressure and electric controlled systems⁽⁵⁾. Its photograph is shown in Fig. 2: The signs from 1 to 5 show a specimen, a slip-ring, a load cell, a spring and an oil pressure

Table 1 Mechanical properties of annealed testing materials

	Upper yield point σ_{20} kg/mm ²	Lower yield point σ_{su} kg/mm ²	Tensile strength σ_B kg/mm ²	Actual break- ing point σ_T kg/mm ²	Elonga- tion φ %	Con- traction ψ %
0.17% C steel	25.5	24.2	43.9	85.5	36.5	62.4
0.36% C steel	28.0	27.3	53.2	83.7	42.1	23.3

Table 2 Chemical compositions of testing materials (%)

	C	Si	Mn	S	P
0.17% C steel	0.17	0.20	0.56	0.016	0.021
0.36% C steel	0.36	0.27	0.53	0.011	0.014

	d mm	ρ mm
0.17% C steel (annealed)	11	0.35
0.36% C steel (annealed)	10	0.35
0.17% C steel (Induction hardened)	10	0.40

Fig. 1 Dimensions of test specimens (mm)

pipe respectively. As the maximum permissible load at the load cell for this apparatus equals 50 kg, a suitable weight is put on the upper position of the load cell for the experiment of the induction hardened specimens which need a load heavier than 50 kg.

The varying stress in the course of the fatigue test is measured by the wire strain gages which are put on the flat part of the specimens.

As shown in Fig. 3, the stress amplitude pattern which is imposed on the specimens is only the sinusoidally varying one in this paper. And the stress wave pattern which is obtained by the fatigue testing machine is named primary wave and the envelope of stress amplitudes is named secondary one or loading cycles, and then the maximum stress amplitude of the primary wave is expressed as σ_{max} and the minimum of it as σ_{min} .

The tests are conducted as follows:

Fig. 2 Random fatigue testing machine

Fig. 3 Amplitude pattern of stresses

In the first place, in order to estimate the crack propagation under varying load, crack propagation diagrams under constant stress amplitudes are obtained for three materials. Next, holding the values of $\sigma_{\max}$ and $\sigma_{\min}$ constant and changing the period of loading cycles in 2 or 3 kinds, the fatigue tests are performed on three materials to get the crack propagation diagrams. Throughout these experiments the tests are always started from the minimum stress amplitude state, $\sigma_{\min}$.

The measurement of the crack depth is made as follows. The fatigued specimens are heated at about 450°C for one hour in an electric furnace. By this procedure the cracked parts of the specimens are colored black or brown by oxidization. Then, breaking the specimens and measuring the colored width of the fractured surface by a 50 magnification shadowgraph, the crack depth is obtained as mean width of the colored region. An example of the colored surface is shown in Fig. 4.

3. Expression of the test results

Several methods have been used to present these test results. The superposition law is the most general method to investigate the crack propagation problems⁽¹⁾. Therefore, this method is adopted in this paper. Up to this time, however, almost all the expected crack growth curves using the superposition law have been obtained schematically from the crack propagation diagrams under constant stress amplitude, but it is difficult to get by this method the expected curves for the present experiment which has a remarkable variation of the stress amplitude in the short period of repeated cycles. For this reason, the expected crack propagation diagrams using the superposition law are obtained from a

simple calculation method, by which the test results are investigated.

Before explaining this calculation method, the schematical method to obtain the expected crack growth curves is described as follows.

Changing the stress amplitude from σ_1 to σ_2 after propagating the crack to the length of λ_1 , at σ_1 , subsequent crack propagation expected from the superposition law is traced on the λ - N diagram of σ_2 as indicated with the thick line from C to D in Fig. 5 (a). Applying the above method to the experiment of multiple repeated stress at many stress levels given in Fig. 5 (b), the expected crack propagation is traced on the λ - N curves of Fig. 5 (d) as shown AB, CD, EF, GH, ... and so on. Therefore, plotting the relation between the summation of the increment of crack depth, $\sum \Delta \lambda_j$, and the whole number of cycles, $\sum n_j$, successively beginning from $j=1$, the expected crack growth curve is obtained. Approximating the stress amplitude pattern in Fig. 5 (c) to that of Fig. 5 (b), this method is applicable to the stress amplitude pattern in Fig. 5 (c).

As this method is hard to apply to the case of having a small increment of crack depth per one load step, the following calculation method is adopted. First, the crack propagation diagrams under constant stress amplitude are given experimentally, then measuring the slope of these curves, the relation between the crack propagation rate $d\lambda/dN$ and the crack depth λ is obtained. The λ - N and the

Fig. 4 The aspect of crack growth

Fig. 5 Explanation diagrams for the expected crack growth curve schematically using the superposition law to the crack growth under varying stress amplitude

λ - $d\lambda/dN$ curves in this case show in Figs. 7, 8 and 9 and Figs. 10, 11, and 12 respectively.

The stress amplitudes situated between $\sigma_{\max}$ and $\sigma_{\min}$ are divided into k stress levels which are numbered 1, 2, 3, ..., i ..., k in the order of higher stress amplitude level and σ_i and ΔN_i^{*1} are the symbol of the mean stress in the i level and the number of primary stress cycles contained in one loading cycle respectively (cf. Fig. 3).

Then the λ - $d\lambda/dN$ diagram corresponding to σ_i stress amplitude is drawn among the experimentally obtained ones, whose schematical diagram is shown in Fig. 6. Using this curve, a small increment of the crack depth at λ_j^{*2} is given as $\left(\frac{d\lambda}{dN}\right)_{\sigma=\sigma_i, \lambda=\lambda_j} \cdot \Delta N_i$ in the case of imposed number of cycles ΔN_i at the stress amplitude σ_i . Then dividing the λ axis into some intervals and numbering the sections 1, 2, 3, ..., j , ... in order of short crack length, namely from $\lambda=0$, the crack propagation rate at an arbitrary section of crack depth and stress amplitude is shown as $\left(\frac{d\lambda}{dN}\right)_{\sigma=\sigma_i, \lambda=\lambda_j}$. As this is the crack increment per one cycle of stress amplitude, the increment of the crack depth per one loading cycle at the number j section of the crack depth is equal to

$$\sum_{i=1}^k \left\{ \left(\frac{d\lambda}{dN} \right)_{\sigma=\sigma_i, \lambda=\lambda_j} \cdot \Delta N_i \right\}.$$

Therefore, the number of loading cycles necessary

Fig. 6 Schematical diagram indicating the relation between the rate of crack propagation $d\lambda/dN$ and the crack depth λ to calculate the expected crack growth curve

*1 As shown in Fig. 3, ΔN_i is the whole number of cycles of the primary wave whose stress amplitude belongs to the number i section and the numbers of cycles in the gradually decreasing and increasing parts of the stress amplitude pattern are equal to each other.

*2 This is the mean crack length in the number j section.

*3 The width of the intervals is selected in such extent that the variation of $d\lambda/dN$ may be small and more than one loading cycle may be contained in this section. In this paper, Δa_j is constant for all intervals.

to increase the crack depth Δa_j^{*3} , which is the width of the section, is equal to

$$m_j = \Delta a_j / \sum_{i=1}^k \left\{ \left(\frac{d\lambda}{dN} \right)_{\sigma=\sigma_i, \lambda=\lambda_j} \cdot \Delta N_i \right\}.$$

Now, take the frequencies of stress amplitudes and loading cycles as ω_1 and ω_2 respectively, and the cyclic number of stress amplitude which is necessary to increase the crack depth, Δa_j , will be equal to

$$N_j = \frac{\omega_1}{\omega_2} \cdot \Delta a_j / \sum_{i=1}^k \left\{ \left(\frac{d\lambda}{dN} \right)_{\sigma=\sigma_i, \lambda=\lambda_j} \cdot \Delta N_i \right\}.$$

Then, plotting the relation between $\sum_{p=1}^j N_p$ and

$\sum_{p=1}^j \Delta a_p$ in order from $p=1$, the crack growth curve which is expected from the superposition law is obtained.

In this case, the number of cycles at $\lambda=0$ is taken as that of stress amplitude at which the cumulative cyclic ratio of crack initiation becomes unity. Namely, take the number of cycles for the crack initiation at σ_i as N_i , and the cyclic ratio of the one loading cycle will be $\sum_{i=1}^k \Delta N_i / N_i$. Therefore, the number of loading cycles to make the above value unity is equal to

$$1 / \sum_{i=1}^k \Delta N_i / N_i$$

Then, expressing this value with the cyclic number of stress amplitude,

$$N = \frac{\omega_1}{\omega_2} / \sum_{i=1}^k \frac{\Delta N_i}{N_i}$$

Taking N_i as the number of cycles to fracture, this method is also applicable to the final fracture. The calculated crack propagation curves are drawn in Figs. 13, 14 and 15 and examples of calculation are shown in Tables 3, 4 and 5.

4. Experimental results and investigation

4.1 Crack propagation under constant stress amplitude

The crack propagation diagrams of three materials are obtained as Figs. 7, 8 and 9, and the crack depth, λ , versus crack propagation rate, $d\lambda/dN$, diagrams are shown in Figs. 10, 11 and 12 which are derived from the former λ - N curves. From these figures the following facts are confirmed: i) In the case of 0.17% carbon annealed steel specimens, $d\lambda/dN$ decreases in the short crack length range, but increases in the long crack length range. ii) In the case of 0.36% carbon annealed steel specimens, $d\lambda/dN$ monotonously increases from the crack initiation point to the final fracture. iii) In the case of 0.17% carbon induction hardened specimens, $d\lambda/dN$ decreases with the increase of crack depth. The differences of these crack propagation rates occur

for the following reasons^{(3) (6)}. Namely, as the cracked surfaces of induction hardened specimens do not open under the repeated stress due to the existence of compressive residual stress, the reduction of the cross section bearing the load and the stress concentration around the crack tip do not occur. Therefore, a certain point in the specimen suffers a con-

Table 3 Example of calculation for 0.17% carbon annealed steel specimen

$$\sigma_{\max} = 30 \text{ kg/mm}^2, \quad \sigma_{\min} = 15 \text{ kg/mm}^2 \\ \lambda_j = \lambda_1 = 0.05 \text{ mm}, \quad \Delta a_j = \Delta a_1 = 0.1 \text{ mm} \\ \omega_2 = 0.05 \text{ c/sec}$$

σ_i kg/mm ²	ΔN_i	$\left(\frac{d\lambda}{dN}\right)_{\lambda=\lambda_j}^{\sigma=\sigma_i}$	$\left(\frac{d\lambda}{dN}\right)_{\lambda=\lambda_j}^{\sigma=\sigma_i} \Delta N_i$
σ_1 28.75	129	$9.30 \times 10^{-5} \text{ mm/c}$	$1.20 \times 10^{-2} \text{ mm}$
σ_2 26.25	59	6.70 "	0.40 "
σ_3 23.75	47	4.60 "	0.22 "
σ_4 21.25	48	2.30 "	0.11 "
σ_5 18.75	57	0.93 "	0.05 "
σ_6 16.25	140	0.38 "	0.05 "

$$\sum_{i=1}^6 \left\{ \left(\frac{d\lambda}{dN} \right)_{\lambda=\lambda_1}^{\sigma=\sigma_i} \Delta N_i \right\} = 2.03 \times 10^{-2} \text{ mm}$$

Table 4 Example of calculation for 0.36% carbon annealed steel specimen

$$\sigma_{\max} = 35 \text{ kg/mm}^2, \quad \sigma_{\min} = 15 \text{ kg/mm}^2 \\ \lambda_j = \lambda_1 = 0.05 \text{ mm}, \quad \Delta a_j = \Delta a_1 = 0.1 \text{ mm} \\ \omega_2 = 0.05 \text{ c/sec}$$

σ_i kg/mm ²	ΔN_i	$\left(\frac{d\lambda}{dN}\right)_{\lambda=\lambda_j}^{\sigma=\sigma_i}$	$\left(\frac{d\lambda}{dN}\right)_{\lambda=\lambda_j}^{\sigma=\sigma_i} \Delta N_i$
σ_1 33.75	129	$4.30 \times 10^{-5} \text{ mm/c}$	$5.54 \times 10^{-3} \text{ mm}$
σ_2 31.25	49	2.70 "	1.32 "
σ_3 28.75	37	1.90 "	0.70 "
σ_4 26.25	36	1.30 "	0.47 "
σ_5 23.75	36	0.74 "	0.27 "
σ_6 21.25	46	0.45 "	0.21 "
σ_7 18.75	50	0.22 "	0.11 "
σ_8 16.25	112	0.09 "	0.10 "

$$\sum_{i=1}^8 \left\{ \left(\frac{d\lambda}{dN} \right)_{\lambda=\lambda_1}^{\sigma=\sigma_i} \Delta N_i \right\} = 8.72 \times 10^{-3} \text{ mm}$$

Table 5 Example of calculation for 0.17% carbon induction hardened steel specimen

$$\sigma_{\max} = 65 \text{ kg/mm}^2, \quad \sigma_{\min} = 45 \text{ kg/mm}^2 \\ \lambda_j = \lambda_1 = 0.025 \text{ mm}, \quad \Delta a_j = \Delta a_1 = 0.05 \text{ mm} \\ \omega_2 = 0.05 \text{ c/sec}$$

σ_i kg/mm ²	ΔN_i	$\left(\frac{d\lambda}{dN}\right)_{\lambda=\lambda_j}^{\sigma=\sigma_i}$	$\left(\frac{d\lambda}{dN}\right)_{\lambda=\lambda_j}^{\sigma=\sigma_i} \Delta N_i$
σ_1 63.75	122	$4.40 \times 10^{-4} \text{ mm/c}$	$5.37 \times 10^{-2} \text{ mm}$
σ_2 61.25	54	3.50 "	1.89 "
σ_3 58.75	36	2.80 "	1.00 "
σ_4 56.25	38	2.30 "	0.87 "
σ_5 53.75	37	1.90 "	0.70 "
σ_6 51.25	38	1.60 "	0.61 "
σ_7 48.75	51	1.20 "	0.61 "
σ_8 46.25	114	0.77 "	0.88 "

$$\sum_{i=1}^8 \left\{ \left(\frac{d\lambda}{dN} \right)_{\lambda=\lambda_1}^{\sigma=\sigma_i} \Delta N_i \right\} = 0.12 \text{ mm}$$

*4 The number of loading cycles per second.

stant repeated stress which is calculated from the elastic theory and its value decreases with the increase of crack depth in the case of bending fatigue tests⁽⁷⁾. On the other hand such phenomena do not occur in the case of annealed specimens and the crack propagation rate increases with the growth of crack depth.

4.2 The influence of loading cycles

Two or three frequencies of loading cycles are selected to investigate the effects of varied cycles. The results of the experiments on the 0.17% carbon annealed steel specimens are shown in Fig. 13. In this case $\sigma_{\max} = 30 \text{ kg/mm}^2$, $\sigma_{\min} = 15 \text{ kg/mm}^2$ and $\omega^2 = 0.5$ or 0.05 c/sec^* . On these results, the effect of ω_2 appears in the case of short crack length and at a definite number of cycles the crack depth for the

Fig. 7 Crack growth diagrams under various kinds of constant stress amplitude for 0.17% carbon annealed steel specimen

Fig. 8 Crack growth diagrams under various kinds of constant stress amplitudes for 0.36% carbon annealed steel specimen

Fig. 9 Crack growth diagrams under various kinds of constant stress amplitude for 0.17% carbon induction hardened steel specimen

larger ω_2 is greater than that for the shorter one. At the crack initiation point and the final fracture, however, no effect of the difference of ω_2 is observed in this case. Therefore, many experiments are necessary to clarify the effect of the difference of ω_2 more generally.

In the experiment of 0.36% carbon annealed steel specimens, $\sigma_{\max}=35 \text{ kg/mm}^2$, $\sigma_{\min}=15 \text{ kg/mm}^2$ and $\omega_2=0.5$, 0.05 or 0.005 c/sec are adopted as shown in Fig. 14. The effects of the period of loading cycles do not appear in this case.

In the experiment of the 0.17% carbon induction hardened specimens, $\sigma_{\max}=65 \text{ kg/mm}^2$, $\sigma_{\min}=45 \text{ kg/mm}^2$ and $\omega_2=0.5$ or 0.05 c/sec. The effects of the period of the loading cycles do not appear in this case, either.

This is the same result as the case of stepwise varying load fatigue test⁽⁹⁾. Therefore, at this point there is no qualitative difference between the cases of stepwise varying load fatigue test and continuously varying one^{*5}.

4.3 The investigation of the superposition law of crack propagation

The crack growth curves in Figs. 13, 14 and 15 are obtained from the calculation method of previous chapter on the assumption of the applicability of

cumulative damage law to the crack initiation and of the superposition law to the crack propagation.

In the case of 0.17% carbon steel specimens, Fig. 13, the superposition law of crack propagation is applicable to the short crack length range but is not applicable to the long crack length range and the result of the latter case has the same tendency as that of final fracture which will be described in section 4.4. Namely, the crack seems to propagate faster after its length exceeds about 0.6 mm. This behavior is distinguished from that in stepwise varying load tests⁽⁹⁾ and the lives which are obtained in the sinusoidally varying load test are shorter than those which are obtained in the stepwise varying one^{*6}. On the contrary, the cumulative damage law is applicable to the crack initiation of these sinusoidally varying load test.

In the case of 0.36% carbon annealed steel specimens, Fig. 14, the cumulative damage law is also applicable to the crack initiation, because the experimentally obtained number of cycles to the crack initiation ($N=1.6 \times 10^3$) is nearly equal to the expected one. The experimentally obtained crack growth curve, however, does not coincide with the calculated one and leans to the dangerous side with an increase of the crack length, which is the same

Fig. 10 Relation between the rate of the crack propagation $d\lambda/dN$ (mm/c) and the crack depth λ (mm) for 0.17% carbon annealed steel specimen

Fig. 11 Relation between the rate of the crack propagation $d\lambda/dN$ (mm/c) and the crack depth λ (mm) for 0.36% carbon annealed steel specimen

Fig. 12 Relation between the rate of the crack propagation $d\lambda/dN$ (mm/c) and the crack depth λ (mm) for 0.17% carbon induction hardened steel specimen

*5 In the previous paper⁽⁸⁾, three kinds of fatigue tests whose stress amplitude patterns are shown in Fig. 16 were performed. The results are denoted in Fig. 17, taking the cyclic ratio of final fracture as ordinate and the period of loading cycles as abscissa. Present test results are also plotted in this diagram.

*6 Taking the number of cycles in high stress level, n_1 , and low stress level, n_2 , identically to the case of the many-fold-multiple-repeated-stress-in-two-stress-levels' fatigue test using the same 0.17% carbon steel specimens, the superposition law is applicable to the whole range of crack length and the experimentally obtained number of cycles to the final fracture is approximately equal to the expected one.

P.S. The expected crack growth curves calculated from the superposition law and the parallel law of crack propagation⁽⁹⁾ are approximately equal to each other in the case of $n_1=n_2=n$ as shown in the previous paper.

tendency as that of the final fracture illustrated in section 4.4.

Unlike in the results which are obtained from the test of the annealed steel specimens, the superposition law of crack propagation is applicable to the induction hardened specimens (cf. Fig. 15). There is, however, one case which shows that the superposition law is not applicable to the crack propagation of induction hardened specimens under stepwise varying load test⁽²⁾.

The reasons for these differences are considered follows. In the first place, as the number of stress cycles in low stress levels is very small in this case as shown in Table 5, the effect of the imposed varying load does not appear remarkably^{*7}. Therefore, it is necessary to perform such a continuously varying load test that the increment of the crack depth under high stress levels may be equal to that of the crack depth under the low stress levels and also to investigate which of the following two causes is the more effective factor to the applicability of the superposition law, i) the variation of the stress amplitudes, namely, stepwise varying load or continuously varying load, or ii) the frequency distribution

of stress amplitudes.

The second reason is the fact that the superposition law is not applicable to the crack propagation of the annealed steel specimens but is applicable to that of induction hardened one. In the former case, namely, in the experiment for the annealed steel specimens, the crack tip suffers very high stress or strain because of the stress or strain concentration at the crack tip and the reduction of the cross section which is bearing the load; and in the latter case, the crack tip does not suffer such high stress concentration as that of the former case, because the compressive residual stress restrains the opening of the cracked surface.

As these causes are considered to make a difference in the behaviors under the varying load test between the cases of induction hardened specimens and the annealed ones, it is necessary to perform further researches concerning the structural changes or the residual strain caused by the repeated stresses and so on.

4.4 Investigation of the final fracture

Usually, the linear cumulative damage law is adopted to investigate the final fatigue fracture under varying load test. According to the investigation of the processes of fatigue progress leading to the final fracture, it is difficult to consider that the damages suffered at any time during the fatigue tests are identical. For this reason, the superposition law of crack propagation is adopted to estimate the final fatigue fracture in this paper. It is, however, difficult to calculate the number of cycles to the final fracture using the method mentioned in the previous chapter. So the following procedures are used. Namely, λ - N curves under constant stress amplitudes are chosen, which are equivalent to the expected ones obtained by the calculation method based on the superposition law of the crack propa-

Fig. 13 Crack growth diagram under the repeated varying load for 0.17% carbon annealed steel specimen

Fig. 14 Crack growth diagram under the repeated varying load for 0.36% carbon annealed steel specimen

Fig. 15 Crack growth diagram under the repeated varying load for 0.17% carbon induction hardened steel specimen

*7 For example, the case of *6 is pointed out.

gation under varying load. And then, the numbers of cycles to the final fracture under these equivalent constant stress amplitudes are read and considered to be the expected values which are obtained from the superposition law of crack propagation*⁸.

In the case of 0.17% carbon annealed steel specimens, comparing the expected λ - N curve in Fig. 13 and the λ - N curves in Fig. 7 the equivalent stress amplitude is obtained as 24.5 kg/mm². Thus, the expected number of cycles to the final fracture of this case is decided as 5.7×10^4 from the S - N curve A in Fig. 18. On the other hand, the experimentally obtained number of cycles is equal to 2.8×10^4 (this is the mean value of the number of cycles to fracture for four specimens), then the experimentally obtained fatigue life corresponds to 49% of the expected one. This result suggests that the difference between the experimentally and the theoretically obtained crack propagation rates becomes

Fig. 16 Amplitude patterns of stresses, types a, b and c

Fig. 17 The effect of the loading cycles

remarkable as the crack grows deeper.

In the case of 0.36% carbon annealed steel specimens, comparing Figs. 8, 14 and 18, the equivalent stress amplitude and the expected number of cycles to the final fracture are obtained as 28.5 kg/mm² and 3.7×10^4 respectively. As the experimentally obtained number of cycles to the final fracture is equal to 2.2×10^4 (this is the mean value of the number of cycles to fracture for eight specimens), the experimentally obtained fatigue life corresponds to 60% of the expected one.

Using the same method, the equivalent stress amplitude and the expected number of cycles to the final fracture are obtained as 57.5 kg/mm² and 1.8×10^4 respectively in the case of 0.17% carbon induction hardened steel specimens. (cf. Figs. 9, 15 and 18) As the experimentally obtained number of cycles to the final fracture is equal to 2.05×10^4 (this is the mean value of the number of cycles to fracture for five specimens), the experimentally obtained life corresponds to 114% of the expected one. Unlike the annealed steel specimens, the experimentally and expectedly obtained values coincide with each other. This is the same tendency as seen in the result of crack propagation processes.

For reference, the resulting cumulative cyclic ratios are obtained as 0.55, 0.65 and 0.93*⁹ for 0.17% and 0.36% carbon annealed steels and 0.17% carbon induction hardened steel specimens respectively.

5. Conclusions

The crack initiation and propagation under sinusoidally varying stress amplitude are investigated using 0.17% and 0.36% carbon annealed steels and 0.17% carbon induction hardened steel notched specimens. Then, the expected crack growth curves are obtained using a simple calculation method based

Fig. 18 S - N diagrams of the final fracture for three specimens

*⁸ It is not clear whether this is orthodox method or not. But this is adopted because a better method concerning the crack propagation is yet to be found. For reference, the cumulative cyclic ratios of the final fracture are indicated on p. 971, left, line 13.

on the superposition law of crack propagation and the following results are obtained.

1) In the case of 0.17% carbon annealed steel specimens, the effect of the period of loading cycles appears a little. But there are no such effects on the other materials.

2) The cumulative damage law is applicable to the crack initiation of these three materials.

3) In the case of 0.17% carbon annealed steel specimens, the superposition law of crack propagation is applicable in the short crack length range but is not applicable in the long crack length range and the experimentally obtained number of cycles to the final fracture corresponds to 49% of the expected one.

4) In the case of 0.36% carbon annealed steel specimens, the superposition law is not applicable and the crack propagates faster than the expected one. The experimentally obtained number of cycles to the final fracture corresponds to 60% of the expected one.

5) Unlike the annealed steel specimens, the superposition law is applicable to the case of 0.17% carbon induction hardened steel specimens and the experimentally obtained number of cycles to the final fracture corresponds to 114% of the expected one.

6) The cumulative cyclic ratios of the final

fracture are obtained as 0.55, 0.65 and 0.93 for the 0.17% and 0.36% carbon annealed steels and 0.17% carbon induction hardened steel specimens respectively.

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*9 The ratio of the number of cycles to fracture obtained by the experiment to the number of cycles to fracture obtained by the method stated in p. 967, left, line 6.