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Peculiarity of Evaporating Liquid-Surface with Reference to Turbulent Heat Transfer*

By Yoshiro KATTO** and Hiroyuki AOKI***

Experiments have been carried out on a surface of water exposed to the flow of air of the range from 3 to 40 m/sec in the velocity and from 30° to 175° C in the temperature. Measured items include not only the heat transfer coefficient, the profiles of velocity and temperature, but also the critical velocity of air for the stability of water surface.

Results of heat transfer coefficient measured within the stable region of water surface are nearly in accord with the correlation proposed by Smolsky and Sergeev, and the heat transfer coefficient becomes greater than that of a solid surface with a rise of the temperature of air. However, the profiles of velocity and temperature measured in the boundary layer are almost in accord with the 1/7th power law, and it seems likely that they are not affected by evaporation within the range of the present study.

Increasing the velocity of air flow, there appears a point at which an abrupt increase in loss of water begins. The velocity of air at the critical point agrees well with the following, well-known value if the evaporation from the liquid surface is weak.

$$\sqrt{2\left(1 + \frac{\rho_g}{\rho_l}\right) \left[\frac{\sigma g (\rho_l - \rho_g)}{\rho_l} \right]^{1/4}}$$

However, as the temperature of air is raised to cause an increase of evaporation, the deviation of the experimental result from the theoretical value becomes noticeable. It should be concluded, therefore, that the liquid surface subject to evaporation has some peculiarity with respect to the critical velocity for its stability too.

1. Introduction

It is well known that the heat transfer coefficient on a wall exposed to a gas stream is mostly reduced with gas injection into the boundary layer through the wall. On a liquid surface exposed to a flow of heated gas, vapor effusion takes place due to evaporation so that the same principle should hold unless no other singular phenomenon appears. Under the condition of adiabatic evaporation, however, it should be kept in mind that the modification of the boundary layer due to the mass injection is generally small under ordinary conditions.

Since the heat necessary for evaporation is transferred to a liquid surface from the heated gas by convection, the following two equations hold at the surface.

$$\dot{m}L = \lambda \left(\frac{\partial T}{\partial y} \right)_w ; \quad \dot{m} = - \frac{1}{1 - w_w} \rho D \left(\frac{\partial w}{\partial y} \right)_w$$

where $\dot{m}$ is the mass of vapor produced by evaporation per unit area and time, L the latent heat, λ the thermal conductivity of gas, T the temperature of gas, y the vertical distance from the surface, w the fractional mass concentration (mass of vapor per unit mass of mixture), ρ the density of gas, D the diffusion coefficient, and the suffix W expresses the liquid surface. Now let us assume, for simplicity, that the energy transport due to molecular diffusion can be neglected and Lewis-Semenov number is equal to unity; in other words, the temperature and concentration profiles within the boundary layer coincide. Then, eliminating $\dot{m}$ from the equations afore-mentioned, provides with ease

$$\frac{c_p(T_\infty - T_w)}{L} = \frac{w_w - w_\infty}{1 - w_w} = \frac{(w_w - w_\infty)}{(1 - w_\infty) - (w_w - w_\infty)} \dots \dots (1)$$

where c_p is the specific heat at constant pressure of

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** Professor, Faculty of Engineering, University of Tokyo, Bunkyo-ku, Tokyo.

*** Engineer, Yawata Iron and Steel Co., Ltd., Yawata Works.

gas, T_∞ the temperature of free stream, T_w the temperature of liquid surface, w_∞ the concentration of vapor in free stream, and w_w the concentration of vapor at liquid surface. The equilibrium temperature of liquid surface, T_w , can be estimated using Eq. (1) for any given state of free stream: T_∞ and w_∞ . In most cases, the magnitude of $c_p(T_\infty - T_w)/L$ and consequently of $(w_w - w_\infty)$, the difference of vapor concentration across boundary layer, is kept low even when T_∞ becomes considerably high because the latent heat of evaporation, L , is generally great. The vapor effusion from the liquid surface is then inevitably weak so that it has little effect on the heat transfer coefficient.

As a simple example, Fig. 1 shows a theoretical result as for the variation of Nusselt number, N_w with T_∞ in a laminar boundary layer composed on a water surface, which has been derived combining the well-known analysis of Hartnett and Eckert⁽¹⁾ on mass transfer cooling with Eq. (1) on condition that $w_\infty = 0$ and $P_r = S_c = 0.7$. It may be noticed that the heat transfer coefficient is nearly the same as that on a solid wall except in the region of extremely high T_∞ . If T_w and T_∞ are expressed by absolute temperature, $1 - (T_w/T_\infty)$ shown in Fig. 1 corresponds to a dimensionless number which is called Guhuman number in U. S. S. R.

The circumstances do not differ in the turbulent boundary layer, since the argument described so far can be applied as it is. After all, the modification due to the mass injection is ordinarily unimportant under the condition of adiabatic evaporation.

However, the liquid surface subjected to evaporation has a possibility to originate phenomena of the other kind forcing the apparent heat transfer coefficient to differ considerably from that on a solid wall.

Fig. 1 Heat transfer in laminar flow of air over water surface (atmospheric pressure)---Variation of average heat transfer coefficient with free-stream temperature

This has been suggested by many experimental studies carried out by Kinney and Sloop⁽²⁾, Nesterenko⁽³⁾, Lebedev⁽⁴⁾, Smolsky and Sergeyev⁽⁵⁾, Luikov⁽⁶⁾, Kikkawa and Nakatani⁽⁷⁾ and others. It should be also mentioned that the recent experimental study of Mawatari, Hiramoto and Okazaki⁽⁸⁾ who investigated the distributions of temperature and vapor-concentration in the boundary layer, seems to provide the data which suggest some peculiarity.

Now let us confine to the case of evaporation from a free liquid surface, because the evaporation from a surface of a capillary-porous body is much complicated including many factors. Smolsky and Sergeyev⁽⁵⁾ carried out very careful and accurate experiments on heat transfer with evaporation of liquids (water, acetone, benzol and butanol) from a free surface into a turbulent air stream. In their experiments, a nylon net was placed at a small distance below the liquid surface in order to eliminate wave formation, and the stream velocity was less than 15 m/sec. Then the results of average heat transfer coefficient were correlated by

$$N_u = 0.086 R_e^{0.8} P_r^{0.33} \left(1 - \frac{T_w}{T_\infty}\right)^{0.2} \quad (2)$$

where T_w and T_∞ in °K. Comparing Eq. (2) with the well-known correlation of turbulent heat transfer on a solid plate:

$$N_u = 0.037 R_e^{0.8} P_r^{0.33} \quad (3)$$

it yields

$$\frac{N_u}{N_{u_0}} = 2.32 \left(1 - \frac{T_w}{T_\infty}\right)^{0.2} \quad (4)$$

Equation (4) shows that N_u is greater than N_{u_0} if only T_∞ is greater than $1.015 T_w$; that is, N_u is almost always greater than N_{u_0} . Smolsky and Sergeyev⁽⁵⁾, and Luikov⁽⁶⁾ attempted to explain the fact of $N_u > N_{u_0}$ by introducing a hypothetical mechanism of volumetric evaporation, that is, additional evaporation of sub-microscopic liquid particles separated from a liquid surface and entrained to the boundary layer.

It should be mentioned here that N_u is certainly greater than N_{u_0} but does not increase infinitely. According to Eq. (4), even in the limit of $T_\infty \rightarrow \infty$ (N.B. T_w does not exceed the boiling point of liquid), N_u/N_{u_0} is equal to 2.32; that is, the maximum value of N_u is $2.32 N_{u_0}$.

The present study has not yet been advanced to the stage enabling us to discuss the mechanism yielding anomalous character in an evaporating liquid surface. But, if the liquid surface subject to evaporation has anomalous character, it might have a possibility to affect not only the heat transfer but also other physical phenomena such as the critical velocity of free stream at which the stability of liquid surface begins to be destroyed. In order to

examine it, experiments have been made on the surface of water exposed to the air stream within the range from 3 to 40 m/sec in velocity and from 30° to 175° C in temperature.

2. Experimental apparatus

2.1 Apparatus

Since accurate experiments had already been carried out by Smolsky and Sergeyev⁽⁵⁾ for the heat transfer at least, a much simpler apparatus was used in the present study.

The most essential parts are shown in Fig. 2. (a) is a duct with a rectangular cross-section (90 × 120 mm) and a length of 500 mm, into which air enters from the left-hand side with uniform distributions of velocity and temperature and flows out towards the right. Along the center line of the upper wall of the air duct is opened a slit of 6 mm in width, through which Pitot tubes (size of tip: 0.4 × 1.2 mm) and thermocouples (copper-constantan of 50 μ in diameter) are inserted to measure the velocity and temperature profiles of air stream. The slit is also of use to keep the static pressure of air within the duct at nearly the same value as the atmospheric pressure.

At the bottom wall of the air duct is a rectangular opening (60 × 400 mm), through which a water-vessel (b) is inserted so as to fix its upper edge 2 mm higher than the bottom surface of the duct facilitating the generation of turbulent boundary layer on the surface of water. The water-vessel is connected with an overflow-vessel by means of flexible pipes, and the level of water in the former is regulated by changing the height of the latter. Inside of the water-vessel is partitioned into three

Size of water surface: 60 mm (width) × 400 mm (length)
Fig. 2 (a) Air duct, and (b) water vessel

parts with two plates set up in parallel with the direction of air flow, and the measurement is conducted in the central portion in order to eliminate the end effects. The water-vessel can be covered with a wire gauge being given tension to keep flatness. Water leakage from the water-vessel due to surface tension along the wire gauge is prevented perfectly by putting an adequate binding agent along the circumferential edge of the vessel.

The air duct connected with the water-vessel in the afore-mentioned way is thermally insulated from outside and is connected with the right-hand exit of the wind tunnel shown in Fig. 3. Air is supplied from a blower and heated by means of the direct combustion of city gas, which is supplied through a gas-compressor and a large surge-tank to the burners set up in the wind tunnel. The adjustment of the combustion rate of gas is facilitated by using burners which are divided into several groups, and each burner is of the type adequate for maintaining the stability over the wide range of combustion rate.

In order to obtain uniformity of velocity and temperature distributions of air stream at the exit of the tunnel, great attention was paid to the design and adjustment of various parts of the wind tunnel (especially, the arrangement of burners, the throttles of tunnel, the settling wire gauges, and the shape of converging part at the exit of the tunnel) as well as to the thermal insulation.

2.2 Performances

At the entrance of air duct, the velocity of air stream was found to be uniform over the rectangular cross-section of the duct except the vicinity of each corner where the velocity was about 2 % higher than the other part. The uniformity of temperature was of slightly inferior quality with a trend that the circumferential part was a bit colder, but it was not so serious as to raise a discussion.

When a smooth copper plate of the same size and of the same area as the water-vessel was placed in the air duct in substitution for the water-vessel, it was found that the velocity distribution within the boundary layer composed over the plate was such as shown in Fig. 4, where θ is the momentum thickness of boundary layer, y the vertical distance

Fig. 3 Wind tunnel

from the plate, u_∞ the free stream velocity, and x the horizontal distance along the plate measured from the front edge. For the total length of the plate of 400 mm, Fig. 4 shows experimental results obtained at three positions of $x=80$ mm ($u_\infty=8.1$ m/sec), $x=200$ mm ($u_\infty=9.0$ m/sec) and $x=350$ mm ($u_\infty=8.1, 17.3$ and 31.6 m/sec). A curve entered in Fig. 4 is the theoretical velocity distribution of $1/7$ th power law. According to Fig. 4, it may be concluded that a turbulent boundary layer develops along the copper plate. In addition, examining the variation of the boundary layer thickness along the plate, it is revealed that the apparatus shown in Fig. 2 is useful, approximately at least, to study the turbulent boundary layer over a flat plate submerged in a uniform flow.

Next, Fig. 5 indicates the velocity distribution within the boundary layer composed over the water-vessel covered with a wire gauge (mesh: 100) and filled with water. The experimental results shown in Fig. 5 are the data obtained at the positions of $x=60, 120, 180, 240, 300$ and 360 mm for the free stream velocity of $u_\infty=9.4$ m/sec. The curve entered in the figure is the velocity distribution of $1/7$ th power law. Except a negligibly small variation, no difference is found between Figs. 4 and 5. In the latter case, water fills the space within the wire gauge up to the upper surface, and the surface thus

produced may be regarded hydrodynamically the same as a smooth solid wall. In connection with this, it might be of interest to mention that quite a different situation appears for the boundary layer when the water-vessel is covered with a wire gauge but is kept empty without water. In this case, through the wire gauge, air enters into the empty vessel from the free stream at the front part and then flows out of the vessel at the rear part.

3. Turbulent heat transfer

3.1 Measurement of heat transfer

Whether or not the experimental apparatus used in the present study gives the result similar to that obtained by Smolsky and Sergeyev as to heat transfer, is a very important problem. Accordingly, experiments of heat transfer were carried out on each water surface with and without a wire gauge, both being exposed to the air stream of 60° to 175°C in temperature. As for the velocity of free stream, if it exceeds critical values, a noticeable instability appears on the liquid surface as will be described in the next chapter. Consequently the velocity was kept below the critical value.

Apparent quantity of evaporated water was determined by measuring both the quantity which was supplied to the overflow-vessel and that which over-flowed: that is, the difference between the two

Fig. 4 Velocity distribution in the turbulent boundary layer (copper plate)

Fig. 5 Velocity distribution in the turbulent boundary layer (water surface covered with a wire gauge)

quantities was supposed to have evaporated from the water surface. Of course, when the thermal equilibrium was established after waiting for a long time, the measurement was started using a precise measuring device including a physical balance. Calibration showed that the accuracy of this method was very great.

As is shown in Fig. 6, the equilibrium temperature of water surface, T_w , in the present study, was determined as a function of the free stream temperature, T_∞ , mainly due to the fact that the steam concentration in free stream, w_∞ , could not be altered on a large scale independently of T_∞ . In order to make this situation clear, Fig. 6 also shows the values of w_∞ estimated theoretically from the measured humidity of air in the laboratory and the quantity of steam to be generated through the perfect combustion of city gas used. The latter's effect on w_∞ is not great within the range of the present experiment, and w_∞ only increases a little as the combustion rate is increased.

Total amount of heat transferred to the water surface is calculated from the quantity of evaporated water (latent heat of evaporation is given as a function of T_w). The portion attributed to radiant heat exchange between the water surface and its surroundings (solid walls of wind tunnel and air duct) is not great being less than 15% in the present study, and it can be estimated rather accurately because the radiant system is not complicated in the present case. Thermal insulator is interposed between the air duct and the water-vessel, and qualitative examination of thermal conduction reveals that heat transferred to the water-vessel by conduction through solid parts is negligibly small. After all, amount of heat transferred by convection to the water surface from the air stream can be determined with reasonable exactness.

3.2 Results

Nusselt numbers thus obtained for the water surface covered with a wire gauge are shown in Fig. 7, where the physical properties of air at the temperature of $(T_\infty + T_w)/2$ have been used to cal-

Fig. 6 Equilibrium temperature of water surface, T_w , and concentration of steam in the free-stream, w_∞

culate N_u and R_e (N. B. w_w is not great being less than 0.07 or so within the range of the present study). For comparison, the case of a solid surface given by Eq. (3) with $P_r=0.7$ is also shown in Fig. 7. Although no data are presented here for the sake of saving the space, almost the same result has been obtained for the free water surface without a wire gauge.

It may be seen from Fig. 7 that as the temperature of free stream, T_∞ , rises, the heat transfer coefficient on a water surface becomes higher than that on a solid surface with a tendency to increase until the former becomes nearly twice the latter. This is the same tendency as that of Eq. (4), qualitatively at least. It may be seen also from Fig. 7 that the ratio of Nusselt number of water surface to that of solid surface, N_u/N_{u0} , is independent of R_e and it is a function of T_∞ alone (N. B. as is shown in Fig. 6, the temperature of water surface, T_w , is nearly subject to T_∞).

Then, the all-inclusive result of heat transfer

Fig. 7 Nusselt number for water surface covered with a wire gauge

Fig. 8 Nusselt number for water surface

in the present study is shown in Fig. 8, and is compared with Eq. (4) of Smolsky and Sergeev. With respect to Fig. 8, it should be noticed that the range of abscissa, $1-(T_w/T_\infty)$, does not exceed unity even at the limit of $T_\infty \rightarrow \infty$. Anyhow, the authors' result differs from Eq. (4) to some extent, but it may be considered that there is an approximate agreement between the two. In addition, N_w/N_{w_0} in either case tends to a limiting value in the vicinity of two as T_∞ increases.

In some of the experiments afore-mentioned, the velocity and temperature profiles within the boundary layer were also measured. However, comparing the results with those shown in Figs. 4 and 5, any noticeable difference could not be found. This situation agrees with the presumption which has been described in Chapter 1. It should be mentioned here that the rate of evaporation of water measured within the range of the present experiment is extremely small so that the effusion velocity of vapor from the water surface is also small.

4. Critical velocity of air stream for stability of water surface

As the velocity of air stream is increased, there appears a point at which a heavy waste of water from the water surface begins. The order of the waste rate in the region of air velocity higher than the critical point is extraordinarily great as compared with that in the region treated in the preceding section.

4.1 Case of very weak evaporation

At the beginning, experiments were carried out in the case of very weak evaporation using an air

Fig. 9 Rate of waste of water from free surface of water vs. free-stream velocity

stream of room temperature. Figure 9 shows the variation of the waste rate with the air stream velocity for the case when the free water surface without a wire gauge is exposed to the air stream and the level of water is kept at 0, 3, 5, 7 and 9 mm respectively below the upper edge of the water-vessel. The experiment with zero mm water level is considerably difficult and its result is apt to become unreliable. Fortunately, however, it may be noticed from Fig. 9 that the level of water does not influence the critical velocity so much. Consequently, the ideal value at zero mm water level can be approximately deduced from the afore-mentioned data.

As is well known, a horizontal interface between gas and liquid becomes unstable when the relative velocity between the two phases increases and arrives at

$$u_g - u_l = \sqrt{2 \left(1 + \frac{\rho_g}{\rho_l} \right) \left[\frac{\sigma g (\rho_l - \rho_g)}{\rho_g^2} \right]^{1/4}} \quad \text{..... (5)}$$

where u_g is the velocity of gas, u_l the velocity of liquid, ρ_g the density of gas, ρ_l the density of liquid, σ the surface tension, and g the gravitational acceleration. Putting $u_g = u_\infty$ and $u_l = 0$, for simplicity, in the present case, Eq. (5) gives a critical velocity, which is around 7 m/sec for the experimental condition of the present study and is shown by a vertical line in Fig. 9. This theoretical value is in accord approximately with the experiment of zero mm in water level.

The experimental results obtained under the same condition as before for the water surface with a wire gauge is shown in Fig. 10. In this case, it may be noticed that the critical velocity is about

Fig. 10 Rate of waste of water from water surface covered with a wire gauge vs. free-stream velocity

15 m/sec (approximately twice as high as in the case of free water surface) independently of mesh size of wire gauge within the range of mesh 50 to 200. This may be attributed to the state that water covers the wire gauge so as to form a continuous, thin, liquid film extending over the wire gauge: in other words, the liquid surface is always rather free and the wire gauge which exists underneath the liquid film has the function to suppress the generation of instability in the liquid film.

In order to confirm that the criterion for critical condition is related with the velocity of free stream alone, experiments were made altering the longitudinal length of water surface. The water-vessel was divided into two parts and only the front part was filled with water. According to the result, no variation was found in the critical velocity of free stream at all. This means that the critical condition is independent of Reynolds number.

4.2 Case of evaporation increased

When the free stream temperature, T_∞ , was raised, it was found that the critical velocity becomes evidently lower than in the case of air stream of room temperature. For the free water surface without a wire gauge (water level: 3 mm), the reduction of critical velocity, $u_{\infty, \text{crit}}$, with the rise of T_∞ is shown in Fig. 11. The figure also shows a theoretical curve which is obtained with [Eq. (5) assuming that the temperature of water surface, T_w , is given by Fig. 6. It may be noticed from Fig. 11 that the discrepancy between the experimental results and Eq. (5) enlarges with the rise of free stream temperature. The reason for this discrepancy has not yet been made clear, but it may be regarded at least that this is a phenomenon peculiar to the liquid surface subject to evaporation. According to the experimental result of Fig. 11, it seems likely that the critical velocity decreases with the rise of T_∞ but it has a tendency to saturate near $u_{\infty, \text{crit}} = 5$ m/sec.

For the water surface covered with a wire

Fig. 11 Variation of critical free-stream velocity with the temperature of free-stream (free surface of water)

gauge, the experimental result is shown in Fig. 12. One can see the same tendency as in Fig. 11 for the variation of critical velocity with the rise of T_∞ . In addition, according to the result of Fig. 12 at least, it may be noticed that the critical velocity approaches almost the same saturated magnitude ($u_{\infty, \text{crit}} = 5$ m/sec) as in Fig. 11 when T_∞ becomes high. In connection with this, it may not be useless to mention a possibility that, as T_∞ increases, the affinity between water and wire gauge increases so that a comparatively thick layer of water is composed over the wire gauge. According to the observation at least, the wire gauge was seen to be wetted very well with water when T_∞ became high.

5. Conclusions

(1) When a foreign gas is injected through the wall into the boundary layer, great reduction takes place in the heat transfer coefficient generally with a slight injection. However, in the ordinary cases of adiabatic evaporation from a liquid surface exposed to a gas flow of high temperature, the effusion of vapor itself is so weak that the modification of the boundary layer as well as of the heat transfer is feeble.

(2) If some exaggeration is allowed in the expression in order to elucidate the situation, it may be stated that there is the same boundary layer on the water surface as that on the solid surface within the range of the present experiment. Both the heat transfer and the mass transfer which take place through the boundary layer have only one connection in the balance to be established between the transferred heat to the water surface and the quantity of evaporated water. In other points, they are independent and they are only superposed upon each other in the boundary layer.

(3) However, if the waste-rate of water from the water surface exposed to a turbulent flow of gas is measured and then the heat transfer coefficient is estimated assuming that the afore-mentioned waste

Fig. 12 Variation of critical free-stream velocity with the temperature of free-stream (water surface covered with a wire gauge)

of water is due to evaporation alone, the heat transfer coefficient is found to be higher than that on the solid wall.

(4) Is the heat transfer coefficient of the liquid surface exposed to a flow of gas really higher than that of the solid wall? Is the waste of water from the water surface generated due to the evaporation alone? The present study has not yet been advanced to the stage enabling us to give clear solutions to the questions afore-mentioned.

(5) If the liquid surface which is subject to evaporation has peculiarity, it may be allowed to expect the existence of peculiarity also in the other physical phenomena than the heat transfer coefficient (the apparent heat transfer coefficient at least). In the present study, the critical free-stream velocity for the stability of liquid surface has been examined giving the result that peculiarity appears also in the critical free-stream velocity when the water surface is subject to evaporation.

(6) At the present stage, the experimental facts alone are presented as to the matters described in the preceding items, and no reasonable explanation can be given to them. In addition, it has not yet been made clear whether the heat transfer and the critical free-stream velocity have a mutual relation or not. Further study is necessary to clarify the principal mechanism concerned with the subjects treated in the present paper.

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Nomenclature

- c_p : specific heat at constant pressure
 D : diffusion coefficient
 g : gravitational acceleration
 L : latent heat of evaporation
 $\dot{m}$: evaporation rate
 N_u : Nusselt number ($N_u = h_m x / \lambda$, where h_m is average heat transfer coefficient)
 P_r : Prandtl number
 R_e : Reynolds number ($R_e = u_\infty x / \nu$)
 T : temperature
 u : velocity
 w : fractional mass concentration (mass of vapor per unit mass of mixture)
 x : distance along a surface
 y : vertical distance from a surface
 θ : momentum thickness of boundary layer
 λ : thermal conductivity
 ν : kinetic viscosity
 ρ : density
 σ : surface tension
 Suffix. g : gas, l : liquid, W : surface, ∞ : free stream.

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