536. 71 : 621. 1. 013

New Equation of State for Steam up to 800°C and 1 000 bar*

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A new equation of state for steam was formulated for the range of temperatures from 0 to 800°C and of pressures from 0 to 1 000 bar.

This new equation, in which the specific volume is expressed as the function of temperature and pressure, has a relatively simple form of polynomials and is suitable for computer calculations.

This equation was transformed into a nondimensional canonical form (the equation of Gibbs function) from which various derived quantities such as the specific enthalpy, the specific entropy and the specific heat can be obtained by differentiation only.

Calculated values of the specific volume and the specific enthalpy agree with the values of the new Skeleton Tables (1963) established by the Sixth International Conference on the Properties of Steam held in New York.

The new equation of state and derived functions as well as tables of calculated property values are shown in this report.

1. Introduction

Among various fluids which are important as working fluids for industrial purposes, water (steam) is considered to have the most favorable characteristics. Importance of water and steam will be increased not only in the field of usual steam power generation but also in other new fields, such as atomic power generation.

Recently in accordance with progresses in the power engineering and the power industries, it is needed to know various properties of water and steam at much higher temperatures and pressures than before. Most of steam tables used up to the present have been based upon the Skeleton Tables for temperatures up to 550°C and pressures up to 300 kp/cm² established by the Third International Steam Tables Conference (1934). For example in the Revised Steam Tables of Japan Society of Mechanical Engineers⁽¹⁾ has been used the equation of state made by one of the present authors, I. Tanishita, which is based upon the International Skeleton Tables (1934). Because the available experimental values at that time were limited to a region up to 450°C and 300 kp/cm², the listed values

of those steam tables may not be reliable in the higher temperature and higher pressure region. On the other hand, the upper limits of temperature and pressure for recently built steam power plants have been much expanded in order to improve the thermal efficiency, and some of those plants were constructed to work with the steam conditions in excess of the limit of the existing steam tables. Therefore an intense demand for new steam tables valid for much higher temperatures and pressures has arisen and studies of various properties for high temperature and high pressure steam were carried out under international cooperation during more than ten years after the second World War.

In 1963, the Sixth International Conference on the Properties of Steam (ICPS) held in New York established the new Skeleton Tables⁽²⁾ as the fruits of those world-wide efforts. The effective region of this new Skeleton Tables (1963) is, as shown in Fig. 1, the range of temperatures up to 800°C and pressures up to 1 000 bar for the specific volume and the specific enthalpy, which is much more extensive than that of the previous Skeleton Tables (1934).

The Skeleton Tables (1963) give the standard values with respective "tolerances" only at the grid points for large intervals of temperature and pressure. In order to make a more detailed steam table or diagrams, it is necessary to formulate the equation of state which reproduces the standard values

* Received 17th November, 1967.

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within the tolerances stated in the Skeleton Tables. Properties such as the specific enthalpy, the specific entropy and the specific heat can be calculated by the equations derived from this equation of state. For industrial purposes such as designing power plants the equation of state can be used quite effectively with an electronic computer. And moreover if this equation of state for steam is an international one, it will be conveniently used for international trade in power plants. To meet these demands, the International Formulation Committee (IFC) was set up by the Sixth ICPS, in which members come from six countries, namely Czechoslovakia, Germany, Japan, UK, USA and USSR.

The present report describes the newly formulated equation of state for steam which reproduces the standard values of the Skeleton Tables (1963) within the respective tolerances.

2. Symbols and basic quantities

Following symbols, units and basic quantities are used in this report.

A : 1/10 J/bar cm³

c_p : specific heat at constant pressure J/g^oK

c_v : specific heat at constant volume J/g^oK

g : specific free enthalpy (Gibbs function) J/g

h : specific enthalpy J/g

k : isentropic exponent

m : isothermal exponent

P : pressure bar except otherwise stated

s : specific entropy J/g^oK

t : temperature in centigrade ^oC

T : Kelvin temperature ^oK

v : specific volume cm³/g

R : gas constant for steam $4.6151 \frac{\text{cm}^3 \text{ bar}}{\text{g}^{\circ}\text{K}}$

Reduced quantities

I : reduced gas constant $RT_{c1}/P_{c1}v_{c1}$

β : reduced pressure P/P_{c1}

Fig. 1 Range of the New Skeleton Tables

ε : reduced enthalpy $h/P_{c1}v_{c1}$

σ : reduced entropy $s T_{c1}/P_{c1}v_{c1}$

λ_p : reduced specific heat at constant pressure

$c_p T_{c1}/P_{c1}v_{c1}$

λ_v : reduced specific heat at constant volume

$c_v T_{c1}/P_{c1}v_{c1}$

θ : reduced temperature T/T_{c1}

χ : reduced specific volume v/v_{c1}

ζ : reduced free enthalpy

Quantities defined at the critical point

P_{c1} : 221.2 bar

T_{c1} : 647.30^oK

v_{c1} : 3.17 cm³/g

Subscripts

c1 : critical

0 : 0 bar

1 : compressed water region

2 : superheated steam region

3 : critical region

3. Review of previous investigations

There exist various equations of state for steam including that of van der Waals. After Callender (1901) and Eichelberg (1922), Mollier⁽³⁾ made an equation with two constants for his steam table in 1925. Davis⁽⁴⁾ derived an equation of state from the experimental results of Joule-Thomson effect in 1926. An equation based upon the Skeleton Tables (1930) was formulated by Sugawara for the first JSME steam tables. Using the Skeleton Tables of 1934 as the basic data, Keyes and others⁽⁵⁾ Koch⁽⁶⁾ and Tanishita⁽⁷⁾ formulated equations of state. Tanishita's equation of 1942 is the basic equation of the revised JSME steam tables and has the following form,

$$v = \frac{47.06}{P} T - \frac{0.668}{(T/100)^{2.7}} - \frac{437 + 6.31 \times 10^{-3} P - 1.81 \times 10^{-25} P^5}{(T/100)^{8.4}} - \frac{1.6 \times 10^{-10} P^5}{(T/100)^{30.5}} - \frac{3.26 \times 10^{-43} P^{25}}{(T/100)^{147}}$$

where units for P and v are kp/m² and m³/kg, respectively. This equation agrees with the Skeleton Table (1934) values.

In order to express a much more extensive region of the new Skeleton Tables (1963), several investigations have recently been reported, about which comparisons with present investigation will be discussed in the later section.

4. New equation of state for steam

In selecting the form of the equation, the following items were taken

into consideration.

(a) The new equation should agree with the Skeleton Tables (1963) for up to 800°C and 1000 bar.

(b) The form of the equation should be suitable for deriving functions such as the enthalpy, the entropy and the specific heat.

(c) The equation should have a simple form which is suitable for computer calculations.

(d) The equation should show favorable characteristics on the boundaries with the equations for the water and the critical regions.

(e) The loss of the number of significant figures in the course of calculations should be the least. Taking into consideration the above items, the form of the equation of state was determined as a polynomial in which specific volume was expressed as a function of temperature and pressure. After the elaborate work to determine details of the equation and to examine various characteristics, the following equation of state has finally been formulated.

$$v = \frac{RT}{P} \left[1 + \left\{ \sum_{j=0}^4 b_{1j} \left(\frac{T}{100} \right)^{-6+j} \right\} P + \left\{ \sum_{j=0}^4 b_{2j} \left(\frac{T}{100} \right)^{-9+j} \right\} P^2 + \left\{ \sum_{j=0}^4 b_{3j} \left(\frac{T}{100} \right)^{-14+j} \right\} P^3 + \left\{ \sum_{j=0}^4 b_{4j} \left(\frac{T}{100} \right)^{-20+j} \right\} P^4 + \left\{ \sum_{j=0}^4 b_{5j} \left(\frac{T}{100} \right)^{-24+j} \right\} P^5 + \left\{ \sum_{j=0}^4 b_{6j} \left(\frac{T}{100} \right)^{-50+j} \right\} P^6 + \left\{ \sum_{j=0}^4 b_{7j} \left(\frac{T}{100} \right)^{-77} \right\} P^{10} \right] \dots (1)$$

where b_{ij} are constants as shown in Table 1.

By this equation, the loss of the number of

Table 1 Constants b_{ij} in Eq. (1)

b_{10}	$-1.486\ 321\ 46 \times 10^2$	b_{50}	$-7.999\ 467\ 01 \times 10^8$
b_{11}	$+7.978\ 192\ 79 \times 10^1$	b_{51}	$+4.756\ 782\ 10 \times 10^8$
b_{12}	$-1.659\ 445\ 73 \times 10^1$	b_{52}	$-1.043\ 176\ 17 \times 10^8$
b_{13}	$+8.660\ 733\ 85 \times 10^{-1}$	b_{53}	$+1.005\ 810\ 50 \times 10^7$
b_{14}	$-1.245\ 572\ 18 \times 10^{-2}$	b_{54}	$-3.600\ 354\ 20 \times 10^5$
		b_{55}	$+1.425\ 740\ 66 \times 10^{-30}$
b_{20}	$-6.894\ 455\ 44 \times 10^2$	b_{60}	$+6.861\ 451\ 00 \times 10^{18}$
b_{21}	$+2.311\ 651\ 52 \times 10^2$	b_{61}	$-3.814\ 223\ 98 \times 10^{18}$
b_{22}	$-2.300\ 632\ 01 \times 10^1$	b_{62}	$+7.824\ 482\ 64 \times 10^{17}$
b_{23}	$+8.510\ 362\ 49 \times 10^{-2}$	b_{63}	$-7.033\ 016\ 72 \times 10^{16}$
b_{24}	$+5.949\ 471\ 94 \times 10^{-2}$	b_{64}	$+2.339\ 939\ 79 \times 10^{15}$
		b_{65}	$-1.974\ 838\ 87 \times 10^{36}$
b_{30}	$-1.286\ 112\ 13 \times 10^5$	b_{70}	$+2.004\ 424\ 29 \times 10^8$
b_{31}	$+7.344\ 436\ 22 \times 10^4$	b_{71}	$+1.811\ 303\ 69 \times 10^8$
b_{32}	$-1.461\ 915\ 32 \times 10^4$	b_{72}	$-4.222\ 064\ 37 \times 10^{-1}$
b_{33}	$+1.178\ 879\ 17 \times 10^3$	b_{73}	$+3.083\ 224\ 11 \times 10^{-1}$
b_{34}	$-3.132\ 989\ 71 \times 10^1$	b_{74}	$-9.193\ 268\ 72 \times 10^{-3}$
b_{40}	$+2.792\ 979\ 16 \times 10^7$	b_{80}	$+4.976\ 708\ 92 \times 10^9$
b_{41}	$-1.260\ 893\ 70 \times 10^7$		
b_{42}	$+1.580\ 938\ 64 \times 10^6$		
b_{43}	$-3.285\ 410\ 12 \times 10^4$		
b_{44}	$-2.680\ 005\ 82 \times 10^8$		

significant figures in the calculating procedure is only three for the most severe conditions, which means that if the calculation is carried out using a computer of nine digits, then the results will be given with at least six effective figures.

The term $b_{55}(T/100)^{12}P^5$ in Eq. (1) is added to slightly modify the enthalpy value at 600°C and 1000 bar, and the term $b_{65}(T/100)^{-77}P^{10}$ is for slight correction of the values of the specific volume at 300°C and 75 bar. Both terms show their effects only in very narrow limited regions and in most other regions sufficiently precise results can be obtained without those terms.

Equation (1) is valid in the superheated steam region which is shown as ② in Fig. 2. Four equations, each for the sub-regions ①, ②, ③ and ④ in Fig. 2, are developed at Keio University to express the whole region of the Skeleton Tables (1963). To get the favorable characteristics on the boundary, the valid region of this equation is adjusted to overlap with that of the critical region equation and Eq. (1) is valid also in a region outside the boundary given by the equation β_L . β_L is the boundary equation which separates the region of Eq. (1) from that of the critical region equation⁽⁸⁾.

5. Derived functions

From the equation of state in which the specific volume is expressed as a function of temperature and pressure, as $v=v(P, T)$, the following equations for the specific enthalpy h , the specific entropy s and the specific heat at constant pressure c_p can easily be derived from the general thermodynamic relations,

$$h = h_0 - AT^2 \int_0^P \left[\frac{\partial(v/T)}{\partial T} \right]_P dP \dots (2)$$

$$s = s_0 - A \int_0^P \left(\frac{\partial v}{\partial T} \right)_P dP \dots (3)$$

Fig. 2 Valid range of the new equation of state

$$c_p = c_{p0} - AT \int_0^P \left(\frac{\partial^2 v}{\partial T^2} \right)_P dP \quad (4)$$

where h_0 , s_0 and c_{p0} denote the specific enthalpy, the specific entropy and the specific heat at constant pressure at zero bar pressure, i.e. for the ideal gas state; h_0 , s_0 and c_{p0} are the functions of temperature only, and have the following relations:

$$c_{p0} = \frac{dh_0}{dT} = T \frac{ds_0}{dT} \quad (5)$$

$$s_0 = b_{s0} + \int_0^T \frac{1}{T} \left(\frac{dh_0}{dT} \right) dT \quad (6)$$

Ordinarily, as c_{p0} in above equation the experimentally obtained value is used and h_0 and s_0 are determined from the values of h and s defined at a reference temperature.

For the present case, the expression of c_{p0} was obtained from that of h_0 which was determined by the least square fitting of the standard values of the Skeleton Tables (1963) at zero pressure, and adjusted to agree with the values of Wagman and co-workers⁽¹⁰⁾ based on experiment.

The following equation of h_0 was obtained,

$$h_0 = b_{70} + b_{71} \left(\frac{T}{100} \right) + b_{72} \left(\frac{T}{100} \right)^2 + b_{73} \left(\frac{T}{100} \right)^3 + b_{74} \left(\frac{T}{100} \right)^4 \quad (7)$$

The constant b_{7j} in Eq. (7) as well as b_{s0} in Eq. (6) was determined so as to keep the differences $g_1 - g_2$ within the limit recommended by the IFC held at Prague in 1965 (Prague value), where g_1 and g_2 are the specific free enthalpies $g = h - Ts$ at saturation pressures calculated by the equation of state for the compressed water⁽⁹⁾ and by Eq. (1) respectively. Values of constants in Eqs. (6) and (7) are given in Table 1. By the way, in the equation for the compressed water region, constants which are independent of temperature have been determined in order to express the entropy and the internal energy of saturated water as zero at the triple point (0.01 °C). Also the differences of the specific free enthalpy on the boundary with the equation for the critical region are so adjusted as to be less than the Prague Value, 0.2 J/g.

The equation for c_{p0} is given as,

$$c_{p0} = b_{71} + 2b_{72} \left(\frac{T}{100} \right) + 3b_{73} \left(\frac{T}{100} \right)^2 + 4b_{74} \left(\frac{T}{100} \right)^3 \quad (8)$$

Comparisons of the calculated results by Eq. (8) with those by Wagman and co-workers⁽¹⁰⁾ show good agreement as shown in Fig. 3. Values by Wagman and co-workers are those calculated from the experimental results of spectrum and are supposed to be one of the most reliable studies. In Fig. 3 are also shown the calculated values by a German formulation⁽¹¹⁾.

6. Conversion to the non-dimensional canonical form

Equation (1) is formulated for the absolute units such as bar, °K, g and cm. But for various purposes it is convenient to denote the equation in the non-dimensional form. So Eq. (1) was transformed to a non-dimensional form with reference to the quantities at the critical point and was converted to the so-called "canonical form".

For the equation of the present form, $v = v(P, T)$, the canonical function is given as the equation of the reduced free enthalpy (Gibbs function). If the canonical function is once obtained, equations of other derived functions can easily be given by differentiation process without any integration process.

6.1 Canonical function

The following canonical function is obtained from Eq. (1) as the equation of the reduced free enthalpy (Gibbs function).

$$\begin{aligned} \zeta = I\theta \ln \beta + & \left(\sum_{j=0}^4 B_{0j} \theta^j + B_{05} \theta \ln \theta \right) + \left(\sum_{j=0}^4 B_{1j} \theta^{-5+j} \right) \beta \\ & + \left(\sum_{j=0}^4 B_{2j} \theta^{-8+j} \right) \beta^2 + \left(\sum_{j=0}^4 B_{3j} \theta^{-13+j} \right) \beta^3 \\ & + \left(\sum_{j=0}^4 B_{4j} \theta^{-19+j} \right) \beta^4 + \left(\sum_{j=0}^4 B_{5j} \theta^{-23+j} + B_{55} \theta^{13} \right) \beta^5 \\ & + \left(\sum_{j=0}^4 B_{6j} \theta^{-49+j} + B_{65} \theta^{-76} \right) \beta^{10} \quad (9) \end{aligned}$$

Constants B_{ij} in Eq. (9) are given in Table 2.

6.2 Derived functions

By general thermodynamic relations, various derived properties can be determined as follows:

Reduced entropy

$$\sigma = - \left(\frac{\partial \zeta}{\partial \theta} \right)_\beta \quad (10)$$

Reduced specific volume

$$\chi = \left(\frac{\partial \zeta}{\partial \beta} \right)_\theta \quad (11)$$

Reduced enthalpy

$$\varepsilon = \zeta + \theta \sigma \quad (12)$$

Reduced specific heat capacity at constant pressure

Fig. 3 Comparison of c_{p0}

$$\lambda_p = -\theta \left(\frac{\partial^2 \zeta}{\partial \theta^2} \right)_p \dots \dots \dots (13)$$

$$\text{Reduced specific heat capacity at constant volume } \lambda_x = \lambda_p + \theta \left(\frac{\partial^2 \zeta}{\partial \theta \partial \beta} \right)^2 / \left(\frac{\partial^2 \zeta}{\partial \beta^2} \right)_\theta \dots \dots \dots (14)$$

$$\text{Isentropic exponent } k = -\frac{\lambda_p}{\lambda_x} \frac{\chi}{\beta (\partial \chi / \partial \beta)_\theta} \dots \dots \dots (15)$$

$$\text{Isothermal exponent } m = -\frac{\chi}{\beta (\partial \chi / \partial \beta)_\theta} \dots \dots \dots (16)$$

Applying the relations of Eqs. (10)~(13) to Eq. (9), the following equations are obtained.

(1) Reduced entropy

$$\begin{aligned} \sigma = \frac{s}{P_{e1} v_{e1} / T_{e1}} = & -I \ln \beta - \left\{ \sum_{j=1}^4 j B_{0j} \theta^{j-1} + B_{05} (1 + \ln \theta) \right\} - \left\{ \sum_{j=0}^4 (-5+j) B_{1j} \theta^{-6+j} \right\} \beta \\ & - \left\{ \sum_{j=0}^4 (-8+j) B_{2j} \theta^{-9+j} \right\} \beta^2 - \left\{ \sum_{j=0}^4 (-13+j) B_{3j} \theta^{-14+j} \right\} \beta^3 - \left\{ \sum_{j=0}^4 (-19+j) B_{4j} \theta^{-20+j} \right\} \beta^4 \\ & - \left\{ \sum_{j=0}^4 (-23+j) B_{5j} \theta^{-24+j} + 13 B_{55} \theta^{12} \right\} \beta^5 - \left\{ \sum_{j=0}^4 (-49+j) B_{6j} \theta^{-50+j} - 76 B_{65} \theta^{-77} \right\} \beta^{10} \dots \dots \dots (17) \end{aligned}$$

(2) Reduced specific volume

$$\begin{aligned} \chi = \frac{v}{v_{e1}} = & I \frac{\theta}{\beta} \left(\sum_{j=0}^4 B_{1j} \theta^{-5+j} \right) + \left(\sum_{j=0}^4 2 B_{2j} \theta^{-8+j} \right) \beta + \left(\sum_{j=0}^4 3 B_{3j} \theta^{-13+j} \right) \beta^2 \\ & + \left(\sum_{j=0}^4 4 B_{4j} \theta^{-19+j} \right) \beta^3 + \left(\sum_{j=0}^4 5 B_{5j} \theta^{-23+j} + 5 B_{55} \theta^{13} \right) \beta^4 + \left(\sum_{j=0}^4 10 B_{6j} \theta^{-49+j} + 10 B_{65} \theta^{-76} \right) \beta^9 \dots \dots \dots (18) \end{aligned}$$

(3) Reduced enthalpy

$$\begin{aligned} \varepsilon = \frac{h}{P_{e1} v_{e1}} = & \left\{ B_{00} - B_{05} \theta + \sum_{j=2}^4 (1-j) B_{0j} \theta^j \right\} + \left\{ \sum_{j=0}^4 (6-j) B_{1j} \theta^{-5+j} \right\} \beta \\ & + \left\{ \sum_{j=0}^4 (9-j) B_{2j} \theta^{-8+j} \right\} \beta^2 + \left\{ \sum_{j=0}^4 (14-j) B_{3j} \theta^{-13+j} \right\} \beta^3 + \left\{ \sum_{j=0}^4 (20-j) B_{4j} \theta^{-19+j} \right\} \beta^4 \\ & + \left\{ \sum_{j=0}^4 (24-j) B_{5j} \theta^{-23+j} - 12 B_{55} \theta^{13} \right\} \beta^5 + \left\{ \sum_{j=0}^4 (50-j) B_{6j} \theta^{-49+j} + 77 B_{65} \theta^{-76} \right\} \beta^{10} \dots \dots \dots (19) \end{aligned}$$

(4) Reduced specific heat capacity at constant pressure

$$\begin{aligned} \lambda_p = \frac{c_p}{P_{e1} v_{e1} / T_{e1}} = & - \left\{ B_{05} + \sum_{j=2}^4 j(j-1) B_{0j} \theta^{j-1} \right\} - \left\{ \sum_{j=0}^4 (5-j)(6-j) B_{1j} \theta^{-6+j} \right\} \beta - \left\{ \sum_{j=0}^4 (8-j)(9-j) B_{2j} \theta^{-9+j} \right\} \beta^2 \\ & - \left\{ \sum_{j=0}^4 (13-j)(14-j) B_{3j} \theta^{-14+j} \right\} \beta^3 - \left\{ \sum_{j=0}^4 (19-j)(20-j) B_{4j} \theta^{-20+j} \right\} \beta^4 \\ & - \left\{ \sum_{j=0}^4 (23-j)(24-j) B_{5j} \theta^{-24+j} + 156 B_{55} \theta^{12} \right\} \beta^5 - \left\{ \sum_{j=0}^4 (49-j)(50-j) B_{6j} \theta^{-50+j} + 5852 B_{65} \theta^{-77} \right\} \beta^{10} \dots \dots \dots (20) \end{aligned}$$

Constants B_{ij} in Eqs. (17)~(20) are of course the same as those in Eq. (9) and are shown in Table 2.

7. Calculated results and comparisons

Calculated results by Eqs. (1) and (2) or Eqs. (18) and (19), and comparisons with the values of the

Table 2 Constants B_{ij} in Eq. (9)

B_{00}	+2.858 546 57×10^1	B_{33}	+2.167 624 58×10^1
B_{01}	-3.744 703 13×10^1	B_{34}	-3.728 888 09×10^0
B_{02}	+2.522 851 45×10^{-1}	B_{40}	+4.270 496 71×10^0
B_{03}	-5.962 762 83×10^{-1}	B_{41}	-1.247 943 08×10^1
B_{04}	+7.672 320 56×10^{-2}	B_{42}	+1.012 830 86×10^1
B_{05}	-1.672 062 45×10^1	B_{43}	-1.362 439 13×10^0
B_{10}	-1.904 173 72×10^0	B_{44}	-7.193 973 97×10^{-1}
B_{11}	+6.616 128 81×10^0	B_{50}	-1.232 890 29×10^1
B_{12}	-8.907 751 36×10^0	B_{51}	+4.745 502 92×10^1
B_{13}	+3.009 298 94×10^0	B_{52}	-6.736 467 99×10^1
B_{14}	-2.801 464 64×10^{-1}	B_{53}	+4.204 325 82×10^1
B_{20}	-3.601 903 67×10^0	B_{54}	-9.741 616 67×10^0
B_{21}	+7.817 359 72×10^0	B_{55}	+3.481 013 96×10^{-9}
B_{22}	-5.036 055 37×10^0	B_{60}	+2.282 596 69×10^1
B_{23}	+1.205 860 32×10^{-1}	B_{61}	-8.213 438 62×10^1
B_{24}	+5.456 736 38×10^{-1}	B_{62}	+1.090 636 58×10^2
B_{30}	-8.719 202 96×10^0	B_{63}	-6.345 585 31×10^1
B_{31}	+3.223 012 76×10^1	B_{64}	+1.366 596 57×10^1
B_{32}	-4.152 707 90×10^1	B_{65}	-8.273 580 44×10^{-4}
		I	+4.260 321 15×10^0

Skeleton Tables (1963) are shown in Table 3 for the properties of saturated vapor, in Table 4 for the specific volume and in Table 5 for the specific enthalpy of steam. Saturation pressure in Table 3 has been calculated with the equation developed by authors⁽¹²⁾. As shown in these tables, the calculated results agree satisfactorily with the Skeleton Table values within the tolerances.

Calculated values of the specific entropy and the specific heat at constant pressure, by Eqs. (3) and (4) or Eqs. (17) and (20), are shown in Tables 6 and 7 respectively. Those for the saturated vapor are shown in Table 3. Calculated values of the specific heat c_p are compared with experimental values by Сирота and co-workers⁽¹³⁾ as shown in Fig. 4. The discrepancies are less than 2%

except in regions very close to the boundaries, for example at 550 °C, 800 bar and 600 °C, 900~1000 bar.

Values of the specific heat at constant volume c_v , the isentropic exponent k and the isothermal exponent m are calculated with Eqs. (14), (15) and (16) as shown in Figs. 5, 6 and 7 respectively. Considerable deviations exist among the values of k obtained by this investigation, those by Dzung⁽¹⁴⁾ and those by Endres and Somm⁽¹⁵⁾, especially in the high temperature and high pressure region and at saturation pressures. The reliability of the values of k in this paper can be proved also by their reasonable agreement with those obtained by the equation for the critical region⁽⁸⁾ on the boundary.

But in order to examine the behavior of k more precisely, it is necessary to obtain k by some other ways, for example by the method of measuring the sonic velocity.

Equations of state for steam based upon the values of the Skeleton Tables (1963) have previously been reported by several investigators. Comparing with those equations, the equation of this investigation has a simpler form which is convenient to get the derived functions and also is suitable for industrial calculations.

8. Smoothness on the boundaries

Equations in this investigation are so made as to be valid in the superheated steam region (the sub-

Table 3 Calculated values for saturated steam

In the columns of the specific volume and the enthalpy, the numerator and the denominator of the righthand column show the difference from the standard value and the tolerance, respectively.

Temperature °C	Saturation pressure bar	Specific volume cm ³ /g		Enthalpy J/g		Entropy J/g °K	Specific heat at constant pressure J/g °K
0	0.006 108 01	206 265.09	-23/210	2 501.35	0/3	9.157 16	1.860
0.01	0.006 112 45	206 210.00	64/210	2 501.37	0/3	9.157 09	1.860
10	0.012 269 87	106 408.10	-14/110	2 519.58	1/3	8.901 03	1.869
20	0.023 365 62	57 823.90	-12/58	2 537.72	0/2	8.667 06	1.880
30	0.042 414 90	32 919.50	-10/33	2 555.74	0/2	8.452 80	1.894
40	0.073 750 11	19 540.11	- 6/19	2 573.61	0/2	8.256 09	1.910
50	0.123 352 7	12 041.99	- 3/12	2 591.28	-1/2	8.075 06	1.930
60	0.199 201 9	7 676.312	-13/77	2 608.71	0/2	7.908 03	1.954
70	0.311 620 0	5 044.987	- 3/50	2 625.85	0/2	7.753 52	1.981
80	0.473 581 2	3 408.505	2/34	2 642.64	0/2	7.610 23	2.013
90	0.701 088 4	2 360.917	0/24	2 659.04	-1/2	7.476 93	2.049
100	1.013 252 3	1 672.784	2/17	2 674.96	-1/2	7.352 59	2.091
110	1.432 660	1 209.791	- 3/12	2 690.33	-1/2	7.236 24	2.139
120	1.985 429	891.395 0	-32/89	2 705.08	-1/2	7.126 99	2.194
130	2.701 323	667.998 9	-32/67	2 719.13	-1/2	7.024 04	2.257
140	3.613 791	508.342 5	-32/51	2 732.38	-1.62/2	6.926 68	2.328
150	4.759 970	392.287 7	-28/39	2 744.76	-2/3	6.834 25	2.409
160	6.180 632	306.598 5	-25/31	2 756.18	-2/3	6.746 15	2.501
170	7.920 227	242.407 0	-21/24	2 766.55	-2/3	6.661 85	2.607
180	10.026 63	193.674 1	-18/19	2 775.79	-2/4	6.580 83	2.727
190	12.551 24	156.214 4	-14/16	2 783.79	-2/4	6.502 64	2.865
200	15.548 80	127.085 4	-10/13	2 790.46	-3/4	6.426 82	3.024
210	19.077 39	104.188 9	-76/104	2 795.68	-2/4	6.352 96	3.205
220	23.198 31	86.008 89	-53/86	2 799.34	-3/4	6.280 66	3.412
230	27.976 05	71.436 76	-35/71	2 801.31	-2/4	6.209 52	3.645
240	33.478 33	59.652 07	-22/60	2 801.43	-2/4	6.139 16	3.907
250	39.776 02	50.041 08	-15/50	2 799.54	-1/4	6.069 16	4.197
260	46.943 37	42.139 53	- 9/42	2 795.42	-1/4	5.999 10	4.520
270	55.057 83	35.592 90	- 6/36	2 788.80	-1/4	5.928 46	4.884
280	64.201 78	30.126 58	- 6/30	2 779.32	-1/4	5.856 57	5.302
290	74.460 72	25.527 82	- 9/30	2 766.50	0/4	5.782 65	5.799
300	85.926 93	21.628 70	-14/35	2 749.69	1/4	5.705 66	6.412
310	98.700 11	18.296 73	-19/35	2 728.09	1/5	5.624 36	7.194
320	112.890 9	15.426 82	-24/35	2 700.68	1/6	5.537 23	8.215
330	128.625 3	12.935 52	-31/35	2 666.27	0/6	5.442 54	9.561
340	146.051 7	10.756 65	-22/35	2 623.44	0/7	5.338 31	11.336
350	165.3 513	8.837 63	33/35	2 570.53	6/8	5.222 26	13.665

Fig. 4 Comparison of the specific heat at constant pressure

Table 4 Calculated results and comparisons of specific volume

v cm ³ /g																			t °C		P bar
100	150	200	250	300	350	375	400	425	450	475	500	550	600	650	700	750	800	850	900	950	1000
1695.33 -0.67/1	1936.61 0.61/1	2172.38 -1/2	2406.01 0/2	2638.65 0/2	2870.73 0/2	2986.64 0/2	3102.48 -1/2	3218.26 0/2	3334.00 0/2	3449.70 0/2	3565.36 0/2	3796.59 0/2	4027.74 0/2	4258.81 0/2	4489.82 0/2	4720.79 0/2	4951.73 0/2	5182.67 0/2	5413.61 0/2	5644.55 0/2	5875.49 0/2
424.949 -2/4	474.330 -1/4	522.468 0/4	569.986 -1/4	617.144 -1/4	664.629 0/4	712.114 0/4	759.601 -1/4	807.086 0/4	854.571 0/4	902.056 0/4	949.541 0/4	997.026 0/4	1044.511 0/4	1092.000 0/4	1139.485 0/4	1186.970 0/4	1234.455 0/4	1281.940 0/4	1329.425 0/4	1376.910 0/4	1424.395 0/4
205.900 -1/3	232.674 0/2	257.873 0/2	282.363 0/2	306.463 0/2	330.515 0/2	354.567 0/2	378.619 0/2	402.671 0/2	426.723 0/2	450.775 0/2	474.827 0/2	498.879 0/2	522.931 0/2	546.983 0/2	571.035 0/2	595.087 0/2	619.139 0/2	643.191 0/2	667.243 0/2	691.295 0/2	715.347 0/2
	87.046 0/2	98.898 0/1	109.698 0/1	114.898 0/1	120.011 0/1	125.055 0/1	130.045 0/1	134.989 0/1	139.886 0/1	144.731 0/1	149.526 0/1	154.271 0/1	158.966 0/1	163.611 0/1	168.256 0/1	172.901 0/1	177.546 0/1	182.191 0/1	186.836 0/1	191.481 0/1	196.126 0/1
	45.3805 4/7	51.9492 2/8	54.9209 2/8	57.7743 3/8	60.5422 1/9	63.2448 0/9	65.896 1/9	68.5052 1/10	71.066 1/10	73.576 1/10	76.037 1/10	78.448 1/10	80.809 1/10	83.120 1/10	85.381 1/10	87.592 1/10	89.753 1/10	91.864 1/10	93.925 1/10	95.936 1/10	97.897 1/10
	26.7600 4.9994/5	32.4637 2/7	34.7837 3/8	36.9357 1/8	38.9758 2/8	40.9358 1/8	42.8354 0/8	44.6876 0/8	46.4927 0/8	48.2513 0/8	49.9644 0/8	51.6320 0/8	53.2541 0/8	54.8306 0/8	56.3615 0/8	57.8469 0/8	59.2868 0/8	60.6812 0/8	62.0301 0/8	63.3335 0/8	64.5924 0/8
	16.1400 0/5	18.2761 3/4	20.0288 2/4	21.5752 1/4	22.9931 1/4	24.3239 1/4	25.5914 0/5	26.7983 0/4	27.9511 0/4	29.0544 0/4	30.1120 0/4	31.1275 0/4	32.1043 0/4	33.0356 0/4	33.9244 0/4	34.7737 0/4	35.5868 0/4	36.3568 0/4	37.0857 0/4	37.7756 0/4	38.4285 0/4
	11.4504 -3.96/4	13.9052 0/4	15.6727 2/4	17.1491 1/4	18.4607 1/4	19.6663 1/4	20.7798 0/4	21.8044 0/4	22.7420 0/4	23.5966 0/4	24.3712 0/4	25.0698 0/4	25.6964 0/4	26.2540 0/4	26.7456 0/4	27.1742 0/4	27.5438 0/4	27.8584 0/4	28.1200 0/4	28.3306 0/4	28.4922 0/4
	10.5526 -2/4	12.4663 1/4	13.9366 1/3	15.1933 1/3	16.3213 1/3	17.3634 1/3	18.3278 1/3	19.2198 1/3	20.0444 1/3	20.8066 1/3	21.5112 1/3	22.1644 1/3	22.7712 1/3	23.3376 1/3	23.8688 1/3	24.3696 1/3	24.8352 1/3	25.2704 1/3	25.6800 1/3	26.0584 1/3	26.4096 1/3
	7.6682 -1/3	9.9555 1/3	11.4781 1/3	12.7153 1/3	13.7964 1/3	14.7776 1/3	15.6944 1/3	16.5600 1/3	17.3796 1/3	18.1472 1/3	18.8672 1/3	19.5344 1/3	20.1528 1/3	20.7272 1/3	21.2528 1/3	21.7344 1/3	22.1760 1/3	22.5824 1/3	22.9488 1/3	23.2704 1/3	23.5520 1/3
	7.8679 0/3	9.5176 1/3	10.7632 1/3	11.8184 1/3	12.7532 1/3	13.5936 1/3	14.3520 1/3	15.0336 1/3	15.6432 1/3	16.1856 1/3	16.6640 1/3	17.0832 1/3	17.4472 1/3	17.7600 1/3	18.0256 1/3	18.2480 1/3	18.4224 1/3	18.5536 1/3	18.6464 1/3	18.7040 1/3	18.7312 1/3
	5.9977 0/3	7.8992 1/2	9.1794 1/2	10.2241 0/2	11.0704 1/2	11.7616 1/2	12.3328 1/2	12.8000 1/2	13.1776 1/2	13.4704 1/2	13.6928 1/2	13.8512 1/2	13.9504 1/2	14.0000 1/2	14.0128 1/2	14.0000 1/2	13.9648 1/2	13.9072 1/2	13.8288 1/2	13.7344 1/2	13.6208 1/2
	6.5185 1.85/2	7.8644 1/2	8.9699 1/2	9.8644 1/2	10.5872 1/2	11.1776 1/2	11.6608 1/2	12.0640 1/2	12.3936 1/2	12.6640 1/2	12.8816 1/2	13.0512 1/2	13.1776 1/2	13.2576 1/2	13.2960 1/2	13.2976 1/2	13.2672 1/2	13.2096 1/2	13.1296 1/2	13.0320 1/2	12.9136 1/2
	5.29572 -2/20	7.80771 16/20	8.68945 16/20	9.37771 16/20	9.89771 16/20	10.28771 16/20	10.58771 16/20	10.81771 16/20	10.99771 16/20	11.13771 16/20	11.24771 16/20	11.33771 16/20	11.41771 16/20	11.48771 16/20	11.54771 16/20	11.59771 16/20	11.63771 16/20	11.66771 16/20	11.68771 16/20	11.69771 16/20	11.70771 16/20
	4.96605 10/14	6.06637 12/14	6.93544 12/14	7.60637 12/14	8.12637 12/14	8.52637 12/14	8.82637 12/14	9.07637 12/14	9.28637 12/14	9.46637 12/14	9.61637 12/14	9.74637 12/14	9.85637 12/14	9.94637 12/14	10.01637 12/14	10.06637 12/14	10.10637 12/14	10.13637 12/14	10.15637 12/14	10.16637 12/14	10.16637 12/14
	4.76988 11.88/12	5.62746 7/13	6.28590 6/14	6.78590 5/15	7.18590 4/16	7.52590 3/17	7.81590 2/18	8.06590 1/19	8.28590 0/20	8.47590 0/20	8.63590 0/20	8.76590 0/20	8.87590 0/20	8.96590 0/20	9.03590 0/20	9.08590 0/20	9.11590 0/20	9.13590 0/20	9.14590 0/20	9.14590 0/20	9.13590 0/20
	3.80418 -9.82/10	4.63460 7/10	5.23460 4/11	5.68460 3/13	6.01460 2/15	6.26460 1/17	6.44460 0/19	6.57460 0/21	6.66460 0/23	6.72460 0/25	6.76460 0/27	6.78460 0/29	6.79460 0/31	6.79460 0/33	6.78460 0/35	6.76460 0/37	6.73460 0/39	6.69460 0/41	6.64460 0/43	6.58460 0/45	6.51460 0/47
	3.94633 -4/8	4.83036 -1/9	5.59348 1/11	6.2709 0/12	6.8846 -1/13	7.4511 -1/14	7.9114 -1/15	8.2857 -1/16	8.5857 -1/17	8.8157 -1/18	8.9857 -1/19	9.1157 -1/20	9.2057 -1/21	9.2557 -1/22	9.2757 -1/23	9.2657 -1/24	9.2257 -1/25	9.1557 -1/26	9.0557 -1/27	8.9257 -1/28	8.7657 -1/29
	3.21511 -5.89/6	3.98817 -3/7	4.6584 -2/8	5.2563 -1/9	5.8025 0/10	6.3172 0/11	6.7972 0/12	7.2372 0/13	7.6372 0/14	7.9972 0/15	8.3172 0/16	8.5972 0/17	8.8372 0/18	9.0372 0/19	9.1972 0/20	9.3172 0/21	9.3972 0/22	9.4472 0/23	9.4672 0/24	9.4572 0/25	9.4172 0/26
	2.96412 -1/6	3.64589 -2/7	4.27934 -1/8	4.8387 0/9	5.3361 0/10	5.7861 0/11	6.1861 0/12	6.5361 0/13	6.8361 0/14	7.0861 0/15	7.2861 0/16	7.4361 0/17	7.5361 0/18	7.5961 0/19	7.6261 0/20	7.6361 0/21	7.6261 0/22	7.5961 0/23	7.5461 0/24	7.4761 0/25	7.3861 0/26
	2.78557 5.57/6	3.37921 -1/7	3.96751 -4/8	4.5161 0/9	5.0198 0/10	5.4819 0/11	5.9145 0/12	6.3145 0/13	6.6845 0/14	7.0245 0/15	7.3345 0/16	7.6145 0/17	7.8645 0/18	8.0845 0/19	8.2745 0/20	8.4345 0/21	8.5645 0/22	8.6645 0/23	8.7345 0/24	8.7745 0/25	8.7845 0/26
	3.15781 3/6	3.70137 -5/3	4.2191 0/4	4.7006 0/5	5.1435 0/6	5.5435 0/7	5.9035 0/8	6.2235 0/9	6.5035 0/10	6.7435 0/11	6.9435 0/12	7.1035 0/13	7.2235 0/14	7.3035 0/15	7.3535 0/16	7.3835 0/17	7.3935 0/18	7.3835 0/19	7.3535 0/20	7.3035 0/21	7.2335 0/22
	2.97198 5.98/6	3.47383 -4/7	3.9602 -0.98/1	4.4207 0/1	4.8463 1/2	5.2363 0/3	5.5863 0/4	5.8963 0/5	6.1663 0/6	6.4063 0/7	6.6063 0/8	6.7663 0/9	6.8863 0/10	6.9663 0/11	6.9963 0/12	6.9763 0/13	6.9163 0/14	6.8263 0/15	6.7063 0/16	6.5563 0/17	6.3863 0/18
	2.81156 5.56/6	3.27921 -3/7	3.7335 -0.65/1	4.1738 0/1	4.5837 1/2	4.9537 0/3	5.2837 0/4	5.5737 0/5	5.8237 0/6	6.0337 0/7	6.2037 0/8	6.3337 0/9	6.4237 0/10	6.4737 0/11	6.4937 0/12	6.4837 0/13	6.4437 0/14	6.3737 0/15	6.2737 0/16	6.1437 0/17	5.9837 0/18
	2.66503 -4.97/5	3.11278 2/6	3.5343 0/1	3.9549 0/1	4.3597 1/2	4.7397 0/3	5.0897 0/4	5.4097 0/5	5.6997 0/6	5.9597 0/7	6.1897 0/8	6.3897 0/9	6.5597 0/10	6.6997 0/11	6.8097 0/12	6.8897 0/13	6.9497 0/14	6.9897 0/15	6.9997 0/16	6.9897 0/17	6.9597 0/18

In each column:

Upper line: Calculated value

Lower line: (Calculated value) - (Standard value)

(Tolerance)

region ② in Fig. 2), while the Skeleton Tables (1963) include the water region and the critical region. On the saturation line, i.e. the boundary with the water region, the difference of the specific free enthalpy $g_1 - g_2$ is less than 0.2 J/g as shown in Fig. 8. Boundary with the equation for the critical region⁽⁶⁾ is expressed by the following equation,

$$\beta = M_0 + M_1 \theta + M_2 \theta^2 \dots (21)$$

where M_i is constant shown in Table 8. On this boundary, the specific free enthalpy, the specific

Fig. 5 Calculated results of the specific heat at constant volume

Table 3. Calculated Results and Comparisons of Enthalpy																			h J/g		$t^{\circ}\text{C}$	P bar
0	50	100	150	200	250	300	350	375	400	425	450	475	500	550	600	650	700	750	800			
2501.80 0/2	2594.74 0/2	2688.67 0/2	2783.73 0/2	2880.04 0/2	2977.71 0/2	3076.83 0/2	3177.49 -1/2	3228.41 -1/2	3279.74 0/2	3331.48 -1/2	3383.65 0/2	3436.23 0/2	3489.24 0/2	3596.55 0/3	3705.59 0/3	3816.34 0/4	3928.81 0/4	4042.95 0/4	4158.72 0/4	4275.28 0/4	0	
		2675.15 -1/2	2776.55 0/2	2875.53 0/2	2974.53 0/2	3074.41 0/3	3175.56 1/3	3226.67 0/3	3278.16 0/3	3330.04 0/3	3382.33 -1/3	3435.02 0/3	3488.13 0/3	3595.61 0/3	3704.78 0/3	3815.65 0/4	3928.21 0/4	4042.44 -1/4	4156.50 -1/4	4274.58 -1/4	1	
				2855.92 -1/3	2961.27 0/3	3064.52 1/4	3167.76 0/4	3219.65 0/4	3271.81 0/4	3324.26 0/4	3377.05 0/4	3430.18 0/4	3483.69 0/4	3591.84 0/4	3701.56 0/4	3812.89 0/4	3925.83 0/4	4040.38 0/4	4156.50 -1/4	4274.58 -1/4	5	
				2827.69 -2/4	2943.52 1/3	3051.75 1/4	3157.85 0/4	3210.77 0/4	3263.79 0/4	3316.98 0/4	3370.41 -1/4	3424.11 0/4	3478.11 0/4	3587.11 0/5	3697.52 0/5	3809.43 -1/5	3922.85 0/5	4037.80 0/5	4154.26 0/5	4274.58 -1/5	10	
					2881.28 0/5	3010.43 1/5	3126.94 1/4	3183.36 -1/4	3239.22 -1/4	3294.78 0/4	3350.23 0/4	3405.69 0/4	3461.24 -1/4	3572.83 -1/5	3685.36 0/5	3799.00 0/5	3913.88 0/6	4030.06 0/6	4147.56 0/6	4274.58 -1/6	25	
						3070.64 3/5	3134.61 1/4	3196.22 -1/4	3256.38 1/4	3315.61 -1/4	3374.28 -1/4	3432.57 -1/4	3491.11 -1/4	3549.73 -1/5	3664.88 1/6	3781.50 1/6	3898.84 1/6	4017.09 1/6	4136.34 0/6	4274.58 -1/6	50	
							3005.78 3/5	3080.68 2/4	3149.93 1/4	3215.77 0/4	3279.46 -1/4	3341.75 -1/4	3403.11 -1/4	3524.17 -2/5	3644.14 1/6	3763.93 1/6	3883.70 1/6	4004.06 1/6	4125.08 1/6	4274.58 -1/6	75	
							2927.99 4/5	3019.47 3/4	3099.22 1/4	3172.34 0/4	3241.43 -1/4	3307.94 -1/4	3372.73 -1/4	3489.11 -2/5	3623.11 -2/5	3745.98 -1/8	3868.45 1/8	3990.96 1/8	4113.79 1/8	4274.58 -1/8	100	
							2829.99 4/6	2948.21 2/6	3042.79 0/4	3125.42 0/4	3201.15 0/4	3272.61 -2/5	3341.29 -2/5	3478.51 -2/8	3601.78 -2/10	3727.97 1/10	3853.11 1/10	3977.81 2/10	4102.47 2/10	4274.58 -2/10	125	
							2693.72 2/8	2862.81 1/6	2979.11 1/5	3074.26 1/5	3158.23 1/5	3235.56 -1/5	3308.68 -3/8	3447.33 -2/10	3580.15 -1/10	3709.80 -2/10	3837.67 3/10	3964.61 3/10	4091.13 2/10	4274.58 -2/10	150	
								2754.88 0/8	2906.17 1/6	3018.09 1/6	3112.29 1/6	3196.59 1/6	3274.80 -2/6	3420.53 -2/8	3558.20 -2/10	3691.45 -1/10	3822.15 1/11	3951.37 2/11	4079.78 3/11	4274.58 -3/11	175	
								2604.03 -1/8	2820.69 2/8	2956.03 1/6	3062.98 1/6	3155.53 1/6	3239.56 -1/6	3393.10 -3/8	3535.93 -2/10	3672.95 0/10	3806.58 2/11	3938.10 3/11	4068.41 3/11	4274.58 -3/11	200	
									2716.24 1/8	2886.96 1/6	3009.95 1/6	3112.23 1/6	3202.89 -2/6	3365.02 -3/8	3513.34 -2/10	3654.29 0/10	3790.89 2/11	3924.80 3/11	4057.05 4/12	4274.58 -4/12	225	
									2579.37 -1/8	2809.16 2/6	2932.87 1/6	3066.58 1/6	3164.75 -2/6	3336.29 -3/8	3490.44 -2/10	3635.48 0/10	3775.15 2/12	3911.49 3/12	4045.68 5/13	4274.58 -5/13	250	
										2719.61 1/6	2891.37 1/6	3018.51 0/6	3125.13 0/6	3306.93 0/10	3467.24 2/10	3616.53 2/12	3756.36 4/13	3898.16 4/13	4034.33 4/13	4274.58 -4/13	275	
										2612.84 -1/6	2824.99 3/6	2968.00 1/6	3084.04 0/6	3276.94 0/10	3443.76 1/10	3597.45 1/10	3743.52 4/13	3884.82 5/13	4022.99 5/13	4274.58 -5/13	300	
											2674.43 2/6	2869.95 2/6	2997.77 0/6	3215.28 -1/8	3386.02 2/10	3538.95 2/13	3711.74 5/13	3858.16 5/13	4000.38 5/13	4274.58 -5/13	350	
												2907.08 3/6	3151.78 -1/8	3347.45 0/10	3520.11 2/10	3679.96 4/13	3831.55 4/13	3977.99 7/14	4274.58 -7/14	400		
												2622.04 -2/6	2814.33 0/6	3027.16 0/10	3188.41 3/10	3347.99 5/13	3505.06 7/13	3655.59 8/13	3813.48 8/13	4274.58 -8/13	450	
												2723.72 1/6	2923.72 -1/8	3129.19 0/10	3249.19 0/10	3441.99 5/13	3616.19 8/13	3778.73 8/13	3933.48 8/14	4274.58 -8/14	500	
												2642.45 1/8	2958.79 -1/8	3200.42 0/10	3403.12 3/10	3584.58 6/13	3752.62 9/13	3911.60 10/14	4274.58 -10/14	550		
													2897.81 -2/8	3152.62 0/10	3364.68 3/10	3553.24 6/13	3726.76 9/13	3889.98 11/13	4274.58 -11/13	600		
														2841.21 -3/8	3106.44 3/10	3286.96 6/13	3522.28 9/13	3701.22 12/15	3868.64 12/15	4274.58 -12/15	650	
														2790.82 -2/8	3062.49 0/10	3290.26 2/10	3491.83 6/13	3676.02 10/14	3847.60 12/16	4274.58 -12/16	700	
														2748.43 0/8	3021.33 0/10	3254.91 2/10	3462.00 6/13	3651.20 10/15	3826.86 13/17	4274.58 -13/17	750	
														2715.55 7/8	2983.31 0/10	3221.25 2/10	3432.92 5/13	3626.81 10/15	3806.43 13/18	4274.58 -13/18	800	
															2948.43 0/10	3189.62 3/10	3404.73 5/13	3602.86 10/16	3786.30 13/19	4274.58 -13/19	850	
																2916.11 0/10	3160.32 5/13	3379.40 9/16	3579.40 13/20	3766.46 13/20	900	
																2884.80 -2/10	3133.53 4/13	3351.61 8/16	3556.44 13/20	3746.88 13/20	950	
																2851.60 -9/10	3109.71 7/10	3326.99 9/13	3534.01 7/16	3727.55 13/20	1000	

In each column:

Upper line: Calculated value

Lower line: $\frac{\text{Calculated value} - \text{Standard value}}{\text{Tolerance}}$

In each column :
Upper line : Calculated value
Lower line : $\frac{(\text{Calculated value}) - (\text{Standard value})}{(\text{Tolerance})}$

																				$s \text{ J/g}^\circ\text{K}$	
0	50	100	150	200	250	300	350	375	400	425	450	475	500	550	600	650	700	750	800	$t^\circ\text{C}$	
6.905 74	7.118 14	7.388 37	7.627 41	7.842 51	8.038 72	8.219 65	8.383 01	8.468 13	8.545 83	8.621 31	8.694 71	8.766 20	8.835 89	8.970 37	9.098 96	9.222 30	9.340 93	9.455 30	9.565 77	0	
		7.359 09	7.614 24	7.835 35	8.034 23	8.216 56	8.383 74	8.466 16	8.544 11	8.619 78	8.693 36	8.765 00	8.834 82	8.969 51	9.098 26	9.221 73	9.340 46	9.454 91	9.565 45	1	
				7.060 87	7.272 58	7.461 09	7.633 78	7.715 43	7.794 38	7.870 89	7.945 17	8.017 41	8.087 75	8.223 29	8.332 68	8.476 66	8.595 80	8.710 58	8.821 38	5	
				6.694 11	6.927 03	7.124 67	7.302 16	7.385 43	7.465 69	7.543 28	7.618 47	7.691 47	7.762 47	7.899 06	8.029 28	8.153 90	8.273 54	8.388 73	8.499 85	10	
					6.411 36	6.647 42	6.842 39	6.931 17	7.015 63	7.096 78	7.174 82	7.250 21	7.323 24	7.463 10	7.595 81	7.722 36	7.843 55	7.959 96	8.072 07	25	
						6.215 14	6.454 01	6.554 67	6.647 96	6.735 70	6.819 07	6.898 81	6.975 47	7.121 05	7.238 04	7.387 91	7.511 70	7.630 18	7.743 97	50	
						5.867 54	6.185 74	6.303 63	6.408 47	6.504 52	6.594 16	6.678 85	6.759 53	6.911 26	7.052 76	7.186 05	7.312 50	7.433 11	7.548 59	75	
							5.952 54	6.096 54	6.217 32	6.323 99	6.421 24	6.511 66	6.596 85	6.755 27	6.901 52	7.038 37	7.167 56	7.290 32	7.407 53	100	
							5.718 63	5.904 82	6.048 08	6.168 63	6.275 23	6.372 40	6.462 70	6.628 45	6.779 75	6.920 29	7.052 30	7.177 26	7.296 22	125	
							5.444 94	5.701 14	5.887 64	6.026 49	6.144 69	6.249 84	6.345 99	6.519 81	6.676 48	6.820 88	6.955 79	7.082 99	7.203 72	150	
								5.497 98	5.727 30	5.890 65	6.023 28	6.137 91	6.240 76	6.423 47	6.585 86	6.734 29	6.872 18	7.001 67	7.124 20	175	
								5.230 17	5.558 85	5.756 44	5.907 03	6.032 89	6.143 39	6.335 92	6.504 41	6.657 03	6.797 99	6.929 82	7.054 17	200	
									5.370 68	5.620 04	5.793 24	5.932 34	6.051 57	6.254 89	6.429 88	6.586 87	6.730 99	6.865 20	6.991 39	225	
									5.141 64	5.477 50	5.679 92	5.834 58	5.963 70	6.178 86	6.360 72	6.522 28	6.669 65	6.806 28	6.934 34	250	
										5.323 48	5.565 48	5.738 42	5.878 66	6.106 71	6.295 87	6.462 17	6.612 87	6.751 97	6.881 92	275	
										5.149 42	5.448 48	5.643 04	5.795 67	6.037 68	6.234 52	6.405 74	6.559 86	6.701 47	6.833 33	300	
											5.200 05	5.452 55	5.633 85	5.906 82	6.120 11	6.301 64	6.462 85	6.609 60	6.745 33	350	
												5.261 16	5.476 13	5.783 30	6.014 24	6.206 62	6.375 20	6.527 22	6.666 89	400	
													5.323 13	5.665 67	5.914 99	6.118 56	6.294 71	6.452 14	6.595 80	450	
													5.178 51	5.553 53	5.821 27	6.036 11	6.219 95	6.382 86	6.530 55	500	
													5.050 11	5.447 22	5.732 51	5.958 40	6.149 90	6.318 33	6.470 07	550	
														5.347 67	5.648 52	5.884 84	6.063 84	6.257 78	6.413 56	600	
														5.256 23	5.569 36	5.815 12	6.021 27	6.200 62	6.360 43	650	
														5.174 55	5.495 23	5.749 07	5.961 80	6.146 43	6.310 20	700	
														5.104 32	5.426 31	5.686 63	5.905 19	6.094 85	6.262 51	750	
														5.046 99	5.362 68	5.627 86	5.851 25	6.045 60	6.217 05	800	
															5.304 04	5.572 84	5.799 86	5.998 46	6.173 56	850	
															5.249 48	5.521 69	5.750 96	5.953 26	6.131 81	900	
															5.197 07	5.474 50	5.704 52	5.909 83	6.091 61	950	
															5.143 38	5.431 29	5.660 56	5.868 05	6.052 79	1 000	

Table 7 Calculated results of the specific heat at constant pressure

c_p J/g °K																			t °C	P bar	
0	50	100	150	200	250	300	350	375	400	425	450	475	500	550	600	650	700	750			800
1.850	1.868	1.889	1.913	1.939	1.968	1.998	2.029	2.045	2.061	2.078	2.095	2.112	2.129	2.163	2.198	2.232	2.266	2.299	2.331	0	0
		2.088	1.991	1.975	1.987	2.009	2.037	2.052	2.067	2.083	2.100	2.116	2.133	2.166	2.200	2.234	2.268	2.301	2.333	1	1
				2.156	2.076	2.061	2.071	2.081	2.092	2.105	2.118	2.133	2.148	2.179	2.210	2.243	2.275	2.307	2.338	5	5
				2.473	2.212	2.133	2.116	2.118	2.124	2.132	2.142	2.154	2.166	2.194	2.223	2.253	2.284	2.314	2.344	10	10
					2.838	2.411	2.273	2.243	2.227	2.219	2.217	2.220	2.225	2.240	2.261	2.285	2.310	2.337	2.363	25	25
						3.189	2.623	2.504	2.431	2.385	2.356	2.338	2.328	2.321	2.327	2.339	2.355	2.375	2.395	50	50
						4.866	3.156	2.863	2.691	2.584	2.516	2.471	2.440	2.407	2.395	2.394	2.402	2.413	2.428	75	75
							4.010	3.378	3.034	2.832	2.705	2.622	2.565	2.498	2.466	2.432	2.449	2.453	2.461	100	100
							5.528	4.138	3.498	3.144	2.931	2.795	2.705	2.597	2.540	2.511	2.497	2.493	2.495	125	125
							8.966	5.342	4.130	3.541	3.205	2.997	2.862	2.702	2.619	2.572	2.546	2.533	2.529	150	150
								7.559	5.025	4.048	3.536	3.232	3.038	2.817	2.701	2.635	2.596	2.574	2.563	175	175
								12.411	6.393	4.701	3.936	3.505	3.238	2.940	2.788	2.701	2.648	2.616	2.598	200	200
									8.738	5.567	4.418	3.820	3.461	3.073	2.879	2.768	2.701	2.659	2.633	225	225
									13.207	6.777	4.999	4.182	3.711	3.216	2.975	2.838	2.755	2.702	2.668	250	250
										8.599	5.708	4.594	3.988	3.369	3.075	2.911	2.810	2.746	2.704	275	275
										11.549	6.593	5.061	4.293	3.532	3.179	2.985	2.867	2.790	2.740	300	300
											9.297	6.178	4.982	3.885	3.400	3.140	2.983	2.881	2.813	350	350
												7.592	5.754	4.269	3.635	3.303	3.103	2.973	2.886	400	400
												9.460	6.546	4.671	3.877	3.470	3.226	3.068	2.960	450	450
													7.224	5.070	4.120	3.639	3.351	3.163	3.035	500	500
													7.528	5.436	4.356	3.806	3.477	3.259	3.109	550	550
														5.724	4.576	3.967	3.601	3.355	3.184	600	600
														5.872	4.770	4.117	3.723	3.450	3.257	650	650
														5.789	4.932	4.250	3.838	3.544	3.330	700	700
														5.347	5.061	4.361	3.946	3.636	3.402	750	750
														4.364	5.164	4.446	4.042	3.724	3.472	800	800
															5.267	4.500	4.124	3.807	3.540	850	850
															5.421	4.523	4.188	3.885	3.605	900	900
															5.715	4.519	4.231	3.954	3.667	950	950
															6.294	4.498	4.248	4.015	3.726	1000	1000

Fig. 6 Calculated results of the isentropic exponent

Fig. 7 Calculated results of the isothermal exponent

Fig. 8 Difference of the free enthalpy on the boundary with the equation for the compressed water region

(Subscripts 2 and 3 show the calculated values by this investigation and by the equation for the critical region, respectively)

Fig. 9 Comparisons on the boundary

Table 8 Constants M_i in Eq.(21)

M_0	$+1.303\ 440\ 24 \times 10^4$
M_1	$-2.884\ 429\ 19 \times 10^4$
M_2	$+1.670\ 445\ 79 \times 10^4$

volume, the specific enthalpy and the specific entropy have been calculated by the equations for both regions at every 5 °C from 350 °C to 600 °C and compared as shown in Fig. 9. Although the difference of the specific free enthalpy is within the limit recommended by the IFC Meeting at Prague, the differences in the specific volume, the specific enthalpy, the specific entropy and the specific heat at constant pressure exceed the Prague values. Similar differences exist also among formulations of other investigations, such as German formulation⁽¹¹⁾, and in order to reduce these differences to within the Prague values, it is necessary to add some correction terms which are effective only in the very vicinity of the boundary.

9. Conclusion

This investigation has been intended to formulate the equation of state for the extensive superheated steam region of the Skeleton Tables (1963). After enormous number of calculations, the new equation of state having a comparatively simple form has been developed, which is valid for temperatures up to 800 °C and for pressures up to 1 000 bar. The new equations reproduce the standard values of the specific volume and the specific enthalpy in the Skeleton Tables (1963) within the respective tolerances. Equations for various other derived properties, such as the specific entropy, the specific heat at constant pressure, the specific heat at con-

stant volume, the isentropic exponent and the isothermal exponent, have also been obtained. The values calculated by them show reasonable trends and agree with experimental investigations.

10. Acknowledgment

Co-operations of Mr. Okazaki at Computer Center of Keio University and of Mr. Kawai of Tokyo Electric Power Company in the computational works are sincerely acknowledged.

References

- (1) *Revised Steam Tables and Diagrams of J.S.M.E.*, 2nd Ed. (1955), Japan Soc. Mech. Engrs.
- (2) *Proceedings of the Sixth Int. Conf. Properties of Steam*, New York, (1963), A.S.M.E.
- (3) R. Mollier: *The Mollier Steam Tables and Diagrams*, Translation from the 3rd German edition of 1925, (1927), p. 12, Sir Isaac & Sons.
- (4) H. N. Davis: *Mech. Engng.*, Vol. 47 (1925), p. 107.
- (5) F. G. Keyes et al.: *Proc. Amer. Acad. Arts Sci.*, Vol. 70 (1936), p. 319.
- (6) *VDI-Wasserdampf tafeln*, Vierte Auflage (1956), S. 11, Springer.
- (7) I. Tanishita: *Trans. Japan Soc. Mech. Engrs.*, Vol. 10, No. 38 (1944), p. II-24.
- (8) I. Tanishita et al.: *Trans. Japan Soc. Mech. Engrs.*, Vol. 34, No. 259 (1968), p. 501.
- (9) I. Tanishita et al.: *Trans. Japan Soc. Mech. Engrs.*, Vol. 34, No. 263 (1968), p. 1255.
- (10) D. D. Wagman et al.: *Jour. Res. Nat. Bur. Standards*, Vol. 34 (1945), p. 143.
- (11) *Statement of the 1965 Formulation of the Thermodynamic Properties of Ordinary Water Substance, First Meeting of IFC of Int. Conf. Prop. Steam, Prague*, (1965).
- (12) I. Tanishita and A. Nagashima: *Trans. Japan Soc. Mech. Engrs.*, Vol. 34, No. 259 (1968), p. 517.
- (13) A. M. Сирота и др.: *Теплоэнергетика*, Год. 10, Но. 5 (1963), стр. 64.
- (14) L. S. Dzung: *BWK*, Bd. 16, Nr. 7 (1964), S. 331.
- (15) W. Endres and E. Somm: *BWK*, Vol. 15, No. 9 (1963), p. 439.