Deformation of Polycrystalline Metal Composed of Anisotropic Crystals Having Linear Stress-Strain Relation*

By Takeji Abe**

It is a basic problem in the deformation of polycrystalline aggregates composed of anisotropic crystals, whether the stress or the strain is constant in the polycrystals. In order to treat the problem analytically, the idea of constraint ratio was introduced as a measure of deformation behaviour of each grain. Basic relations between stress and strain were deduced in terms of the constraint ratio. On the basis of the relations acquired, energetic consideration was made on the deformation characteristics of the polycrystals.

It was found that, if the mutual restriction between the grains is absent, the constant stress condition is energetically stable. In real polycrystals, however, the strain is to be continuous through the grain boundary during deformation. It was concluded that, the mode of deformation changes gradually from constant stress to constant strain in proportion to the increase in strained region around the grain boundary.

1. Introduction

Most metals used in mechanical engineering are polycrystals composed of numerous grains having various orientations. It is important to know the deformation behavior of polycrystals in discussing the strength of metals.

Many investigations have been made on the correlation between the deformations of single crystal and polycrystals. Most of the previous studies intended to associate the slip in each grain with the deformation of the polycrystals directly, as represented by the study of G. I. Taylor[1]. Furthermore, those studies were made chiefly on fcc metals. Usually, two assumptions are included in those analyses, that is, (a) the strain in each grain is constant, and (b) the flow stress $\tau$ on slip plane is dependent only on the shear strain $\gamma$ on the plane, i.e. $\tau = f(\gamma)$. However, the previous observation[2] by X-rays on the deformation behavior of polycrystalline copper suggested that these assumptions are not applicable to the metals having strong anisotropy.

In general, the correlation between the slip in single crystal and the deformation of polycrystals may be composed of the following two stages:

1. Relation between the slips in single crystal of a certain orientation and the stress-strain curve of the crystal.

2. Relation between the stress-strain curves of single crystals which are different from each other according to their orientations and the deformation behavior of the polycrystals composed of them. Of these, (1) is a problem which must be clarified.
by the experimental studies or by the dislocation theory. In the present study, discussion will be
carried out from the standpoint of (2). Although
this treatment is somewhat phenomenalistic when
compared with the one correlating directly the slip
in single crystal and the deformation of the poly-
crystals, it is expected to have a merit of taking
the effect of grain boundary, for instance, into ac-
count.

When a polycrystalline metal composed of grains
having different stress-strain curves is deformed,
certain distributions of stress and strain are pro-
duced. Although many discussions(3) have been
made on whether the stress or the strain becomes con-
stant in polycrystals during deformation, this has
not been clarified as yet.

In the present paper, the idea of the mutual
constraint ratio between grains reported before(4) is
applied to the analysis of the stress and the strain
distribution in a polycrystal and discussions are made
from energetic point of view. The complexity of
the problem, however, forces us to use highly sim-
plified mathematical descriptions, that is, the follow-
ing items are assumed: (a) the deformation pro-
cceeds under uniaxial stress, (b) all grains have a
cubic shape of the same size (Fig. 1), and (c) the
stress-strain curve (work-hardening curve) of each
grain is linear. In the following, fundamental
relations concerning the stress and the strain are
expressed in terms of the constraint ratio. Then,
these relations are applied to the energetic study of
the deformation behavior of the polycrystals. Both
cases when the constraint of strain at the grain
boundary is present and when it is absent are dis-
cussed.

2. Basic relations

It is necessary, at first, to introduce basic rela-
tions for the analysis of deformation of polycrystals.
Let's consider here a row of grains perpendicular to
the stress axis shown in Fig. 2.

2-1 Work-hardening rate $K$

Let's denote the gradient of the stress-strain
relation of a certain grain by $K$.

\[ \sigma = K \varepsilon \]  
\[ K = K + \Delta K \]

where $\sigma$ and $\varepsilon$ are the stress and the strain of
the grain and $K$ is the mean value of $K$ for all grains
(Fig. 3). (The definition of $K$ is given afterwards
in Eq. (18)).

2-2 Constraint ratio $\eta$

To show the condition of mutual interference
between grains analytically, the constraint ratio $\eta$
between grains is defined as follows(3) (Fig. 3).

\[ \eta = \frac{\Delta \varepsilon}{(\Delta \varepsilon)_{\text{max}}} \]

where $0 \leq \eta \leq 1$. It is apparent that $\eta = 0$
corresponds to the constant strain condition and $\eta = 1$
to the constant stress. In the following, it is assumed
that the value of $\eta$ is equal irrespective of the value
$\Delta K$ of each grain.

2-3 Stress and strain

The stress and the strain in a grain are repre-
sented by,

\[ \sigma = \tilde{\sigma} + \Delta \sigma \]  
\[ \varepsilon = \tilde{\varepsilon} + \Delta \varepsilon \]

where $\tilde{\sigma}$ and $\tilde{\varepsilon}$ are the mean values of $\sigma$ and $\varepsilon$
for all grains, respectively. As reported before(3),
the following two relations have been found to hold,

\[ \Delta \sigma = -\eta \Delta K \tilde{\varepsilon} \]  
\[ \Delta \varepsilon = (1 - \eta) \frac{\Delta K \tilde{\varepsilon}}{(\Delta \varepsilon)_{\text{max}}} \]

Fig. 2 Model of polycrystals

Fig. 3 Relation between $K$, $\eta$, $\Delta \sigma$, $\Delta \varepsilon$, and $K'$ on stress-
strain diagram
2.4 Physical meaning of $\eta$

When $\eta$ is constant for all grains, the deformed grains are located along the curve in $\sigma$-$\varepsilon$ diagram, as shown in Fig. 3. The gradient $K'$ of the curve at the cross point with the line of $\bar{K}$ is given by

$$K' = \frac{\partial (\sigma)}{\partial (\varepsilon)} \bigg|_{\Delta K = 0}$$

Substituting Eqs. (3) and (4), into Eq. (7),

$$K' = \left(1 - \eta \right) \frac{\bar{K}}{\bar{\eta}}$$

or

$$\eta = \frac{\bar{K}}{\bar{K} - K'}$$

It follows from Eq. (8) that $\eta$ corresponds to the gradient $K'$ on the $\sigma$-$\varepsilon$ diagram.

2.5 Mean value of $K$: $\bar{K}$

The mean value $\bar{K}$ of $K$ for all grains is dependent on $\eta$ as shown in Fig. 4. Therefore, it is necessary to express $\bar{K}$ as a function of $\eta$. From Eq. (1),

$$\int (\sigma - K \varepsilon) d\Omega = 0$$

where $\int d\Omega$ expresses the integration with respect to all grain orientations and $\int d\Omega = 1$ (normalized).

Let’s denote the mean values $\bar{K}$ in the conditions of the constant stress and the constant strain as $\bar{K}_s$ and $\bar{K}_e$ respectively. Then, from Eq. (9),

$$\bar{K}_s = \frac{1}{\int \frac{1}{K} d\Omega}$$

$$\bar{K}_e = \int K d\Omega$$

In a more general case, substituting Eqs. (3), (4) and (6) into Eq. (9),

$$\int [\sigma + (1 - \eta) (\Delta \sigma)_{\text{max}} - K (\varepsilon + (1 - \eta) (\Delta \varepsilon)_{\text{max}})] d\Omega = 0$$

Substituting $\int (\Delta \sigma)_{\text{max}} d\Omega = 0$ and Eq. (11),

$$\bar{K}_s = \bar{K} + \frac{\bar{K}}{\bar{K} + K'}$$

In the case of constant stress ($\eta = 1$),

$$\sigma = \bar{K}_s \varepsilon + \int K (\Delta \varepsilon)_{\text{max}} d\Omega$$

Substituting $K = \bar{K}_s + \Delta K$, and $\int (\Delta \varepsilon)_{\text{max}} d\Omega = 0$,

$$\bar{K}_s = \bar{K}_s - \frac{1}{\varepsilon} \int \Delta K \varepsilon d\Omega$$

where $\Delta K$ is the value of $\Delta K$ under the condition of constant strain. On the other hand, from Eqs. (3) and (5), we have

$$\frac{\Delta K}{\varepsilon} = \frac{(\Delta \varepsilon)_{\text{max}}}{\varepsilon}$$

Substituting Eq. (16) into Eq. (15),

$$\bar{K}_e = \frac{1}{\bar{K}_e} \int \Delta K d\Omega$$

Now, let’s define the mean value $\bar{K}$ corresponding to an arbitrary value of $\eta$ by the following relation:

$$\bar{K} = \bar{K}_e$$

Denoting,

$$\int \Delta K d\Omega = \int \Delta K$$

then, from Eqs. (13), (14), (17), (18) and (19), the mean value $\bar{K}$ is finally obtained:

$$\bar{K} = \bar{K}_s + (1 - \eta) \frac{\Delta K}{\bar{K}_s}$$

(The symbol $\sim$ expresses the mean value for all grains, just as same as the previous symbol $\sim$)

It follows from Eq. (20) that $\bar{K}$ changes in proportion to $\eta$. These relations are shown schematically in Fig. 4.

In the case of elastic deformation, the gradient $K$ in Eq. (20) is replaced by Young’s modulus $E$:

$$\bar{E} = \bar{E}_s - \eta \varepsilon$$

For example, the well known relations\(^{(2)}\) on Young’s modulus are illustrated in Table 1.

2.6 $\Delta K$

The value of $\Delta K$ for an arbitrary value of $\eta$ is also deduced from the above relations as follows.

$$K = \bar{K} + \Delta K = \bar{K}_s + \Delta K$$

From Eqs. (17) and (19),

$$K = \bar{K}_s + \Delta K$$

Table 1 Young’s modulus of polycrystals

<table>
<thead>
<tr>
<th>Constant strain Voigt(^{(3)})</th>
<th>Constant stress Reuss(^{(3)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{E}_s = \int E d\Omega$</td>
<td>$\frac{1}{\bar{E}_s} = \int \frac{1}{E} d\Omega$</td>
</tr>
<tr>
<td>$E_s = (e_{01} + 2e_{02})(e_{01} - e_{02} + 2e_{04})$</td>
<td>$E_s = 3(3 - 6\mu + 2\mu + \mu)$</td>
</tr>
<tr>
<td>$2e_{01} + 3e_{02} + 4e_{04}$</td>
<td>$\sigma_{01} :$ elastic modulus, $\varepsilon_{01} :$ elastic compliance</td>
</tr>
</tbody>
</table>
\[ |\Delta K_+| = |\Delta K_-| = |\Delta K_0| = \frac{\varepsilon_2}{K}\]  \hspace{1cm} (22)

where the symbols + and - correspond to the grains of \( \Delta K > 0 \) and \( \Delta K < 0 \), respectively. It is apparent from Fig. 4 that,

\[ |\Delta K_+| = |\Delta K_-| = |\Delta K_0| \]

then from Eq. (20), we obtain

\[ |\Delta K_+| = K_+ - K = |\Delta K_0| + \eta \frac{\varepsilon_2}{K} \]

\[ |\Delta K_-| = K - K_- = |\Delta K_0| - \eta \frac{\varepsilon_2}{K} \]  \hspace{1cm} (23)

3. Deformation of polycrystals without constraint at grain boundary

It is thought that the value of the constraint ratio \( \eta \) in actual metals is dependent on the mode of the mutual restriction between the grains at the grain boundaries. However, the microscopic mechanism of the mutual restriction may be affected by the orientation of respective grains or the relative orientation of the grain boundary to the stress axis, and is considered to behave in a complicated manner. In the following, the problem is discussed from a phenomenological or energetic point of view to avoid this complication. In this chapter, the imaginary case when the constraining force does not act between the neighboring grains, that is, when the grains can slip freely at the grain boundary, is considered. Following discussion will be made based on the polycrystal model shown in Fig. 2. The deformation behavior of a system composed of two grains is considered at first, then the result is applied to the polycrystals composed of many grains.

Let’s denote here the gradients of the two grains in the stress-strain diagram by \( K_+ (K_+ > K) \) and \( K_- (K_- < K) \), respectively, and the energies of the respective grains by \( W^+ (\Delta K > 0) \) and \( W^- (\Delta K < 0) \). Assuming the symmetric distribution of \( \Delta K \), the energy \( W' \) of the system (Fig. 5) is given by

\[ W' = \frac{1}{2} (W^+ + W^-) \]

\[ = \frac{1}{4} \left( (\bar{\varepsilon} + |\Delta \varepsilon|)(\bar{\varepsilon} - |\Delta \varepsilon|) + (\bar{\sigma} - |\Delta \sigma|)(\bar{\sigma} + |\Delta \sigma|) \right) \]

\[ = \frac{1}{2} \left( \bar{\varepsilon}^2 - |\Delta \varepsilon|^2 \right) \]  \hspace{1cm} (24)

(The energy per unit volume is considered, which also means the energy per unit area on the plane perpendicular to the stress axis.) The second term of the right side of Eq. (24) is calculated from Eqs. (5) and (6):

\[ |\Delta \sigma| |\Delta \varepsilon| \approx \eta (1 - \eta) \frac{(\Delta K)^2}{K} \]

\[ \approx \eta (1 - \eta) \frac{(\Delta K)^2}{K} \]  \hspace{1cm} (25)

Substituting Eqs. (20) and (25) into Eq. (24), we obtain,

\[ W' = \frac{1}{2} \left( \bar{K} \varepsilon^2 + (1 - \eta^2) \frac{(\Delta K)^2}{K} \right) \]  \hspace{1cm} (26)

Next, it is necessary to apply the above discussion on the two grains to the polycrystals having numerous grains. Namely, the term \( (\Delta K)^2 \) in Eq. (26) has to be replaced by the mean values \( \langle (\Delta K)^2 \rangle \) for many grains defined by Eq. (19). Then, the deformation energy \( W \) per unit volume of the polycrystals is given by,

\[ W = \int W' d\Omega = \frac{1}{2} \left( \bar{K} \varepsilon^2 + (1 - \eta^2) \frac{\langle (\Delta K)^2 \rangle}{K} \right) \]  \hspace{1cm} (27)

As may be seen from Eq. (27), \( W \) takes its minimum value \( W = W_s = \frac{1}{2} \bar{K} \varepsilon^2 \) at \( \eta = 1 (K' = 0) \). It is worth noting that if we neglect the mutual restriction between grains the state of constant stress is energetically stable. \( W \) takes its maximum value \( W = W_s = \frac{1}{2} \bar{K} \varepsilon^2 \) at \( \eta = 0 (K' = -\infty) \), that is, at constant strain. This result agrees with the inclination of the experimental data on the mode of deformation of polycrystals deviates from the condition of constant strain to that of constant stress.

It follows from Eq. (27) that if the values of \( \langle (\Delta K)^2 \rangle \) are the same the difference \( W_s - W_s = \frac{\langle (\Delta K)^2 \rangle 2}{2 \bar{K}} \), becomes smaller for larger value of the averaged gradient \( \bar{K} \). Although we have assumed in the above discussion that \( \eta \) is independent of the value of \( \Delta K \), it is considered that \( \eta \) represents the mean value \( \eta \) in general.

The former assumption of the cubic shape of grains (Fig. 1) is not always necessary so long as the deformation without constraint at the grain boundary is considered. It is necessary, however, that the distribution of the volume of grain is uniform with respect to \( \Delta K \).
4. Deformation of polycrystals with constraint at grain boundary

As mentioned above, if the value of \( K \) (or \( \Delta K \)) of a grain differs from that of the neighboring grains, a certain discrepancy of strain is produced at the boundary during deformation \( \text{cf. Eq. (5)} \). In real polycrystalline metals, however, there is the continuity of strain between the neighboring grains, which exerts force on each other. In the following, the work (or the increase in the deformation energy) due to constraint in the neighborhood of the grain boundary (which is called grain boundary region hereafter) is estimated.

Let's consider a polycrystal model piled up in a cubic form as shown in Fig. 1. An arbitrary grain A is surrounded by other grains such as B, C and D. Now, if we consider the case of uniaxial stress, the mode of deformation of crystal grain A is thought to be most strongly affected by the neighboring grains B. Therefore, in the first approximation, only the influence of the constraint from the neighboring grains (B) on the normal axial strain is considered in the following analysis. Namely, the polycrystal model of Fig. 2 is assumed.

As may be seen from the crystal configuration viewed from the stress axial direction (Fig. 6), the grain A is constrained by the four surrounding grains B. Then, the grain boundary of an arbitrary one of them is taken as representative. For instance, let us consider the boundary between the grains I and J as shown in Fig. 7, and denote the values of \( K \) of the two grains by \( K_i \) and \( K_j \), respectively, and their difference by \( 2\Delta K_{ij} \).

\[
2\Delta K_{ij} = K_i - K_j \tag{28}
\]

The analysis of the preceding chapter on the deformation energy \( W \) of two grains could be applied directly to the behavior of the grain boundary region. It is convenient to discuss the problem on the stress-strain diagram. Let \( \bar{K} \) be the mean value of \( K \) for all grains under the constraint ratio \( \eta \), as shown in Fig. 8 (a) (i.e., all the grains are assumed to be located on a curve \( \eta = \text{constant} \), or approximately, on a straight line with gradient \( K' \) as given by Eq. (8)). The two grains with hardening coefficients \( K_i \) and \( K_j \) are located at points I and J, respectively. The difference of the strain at the grain boundary is given from Eqs. (4), (5) and (28):

\[
2\Delta \varepsilon_{ij} = \varepsilon_i - \varepsilon_j = \Delta \varepsilon_i - \Delta \varepsilon_j \\
\approx -\eta \left[ \frac{(\Delta K)_i}{K_i} - \frac{(\Delta K)_j}{K_j} \right] \varepsilon \\
\approx -2\eta \frac{\Delta K_{ij}}{K_i} \varepsilon \tag{29}
\]

In order to satisfy the continuity of strain at the grain boundary, the strains of the two crystals must be equal at the boundary, that is, it is necessary that \( \eta = 0 \) at the boundary. The situation is shown in Fig. 8 (b) which is enlarged from the encircled part in Fig. 8 (a). Since the state of the polycrystal as a whole is unchanged, the mean value of the grains I and J remains on the curve \( \eta = \text{constant} \) of the polycrystal (Fig. 8 (a)), and each grain is so located as to have \( \eta = 0 \) at the grain boundary (Fig. 7). That is, the respective sides of the boundary of the I-grain and the J-grain are located at the points \( I_0 \) and \( J_0 \) in Fig. 8. Accordingly, the position of the mean moves along the curve \( \eta = \text{constant} \) from point R to point Q. It is apparent that the relation \( |\Delta \sigma_{ij}| = |\Delta \sigma_{ij}| \) holds on the both sides of the boundary, that is, the condition of the equilibrium of the normal axial stress at the grain boundary is satisfied.
Next, let us consider the energy required to keep the strain continuous in the grain boundary region. For this purpose, it is necessary to apply the energy $W'$ of Eq. (26), which was obtained for an arbitrary pair of crystals in polycrystals to the system consisting of two neighboring grains. Let's denote the mean values of $\varepsilon$ and $K$ for the two grains $I$ and $J$ by $\bar{\varepsilon}$ and $\bar{K}$, respectively. ($\bar{K}$ implies the mean value under the condition of constant strain.) Then, according to Eq. (26), the energy $\tilde{W}$ for the two grains is given by

$$\tilde{W}(\bar{\varepsilon}, \bar{K}) = \frac{1}{2} \left[ \bar{K} \bar{\varepsilon}^2 + (1 - \bar{\varepsilon})^2 \left( \frac{dK_{ij}}{K} \right)^2 \right]$$

(30)

The energy increase $\Delta W$ of the system corresponding to the change in constraint ratio from $\eta = \eta'$ to $\eta = \eta''$ ($\eta' < \eta''$) is given by

$$\Delta W = W(\eta'') - W(\eta)$$

$$= \frac{1}{2} \left[ (1 - \eta'')^2 - (1 - \eta')^2 \right] \left( \frac{dK_{ij}}{K} \right)^2 \bar{\varepsilon}^2$$

(31)

It is assumed here that the strain or the constraint ratio $\eta$ changes linearly across the grain boundary region as shown in Fig. 7. Then,

$$\eta'' = \eta + \frac{x}{\Delta l/2}$$

(32)

where $\Delta l/2$ and $x$ are the breadth of the boundary region in each grain and the distance from the boundary, respectively. The energy $W''$ for one grain boundary is

$$W'' = \frac{2}{l} \int_{0}^{\Delta l/2} \Delta W d\eta = \frac{2}{\eta} \int_{0}^{\eta} \Delta W d\eta$$

(33)

Each grain boundary in one grain has the energy $W''/2$ and each grain has four grain boundaries as shown in Fig. 6. Then, the energy increase $W_s'$ per unit volume of the grain boundary region is given by

$$W_s' = \frac{2}{l} \int_{0}^{\Delta l/2} \Delta W d\eta = \frac{2}{\eta} \int_{0}^{\eta} \Delta W d\eta$$

(34)

where $l$ is the length of the edges of a cubic grain. Now, let's define the ratio of the boundary region by $P$.

$$P = \Delta l / l$$

(35)

$P$ is thought to be dependent on $\eta$. Then, let $P_1$ be the value of $P$ at $\eta = 1$.

$$P = \eta P_1$$

(36)

(which means that the grain boundary region in Fig. 7 changes in proportion to $\eta$.) Substituting Eqs. (31), (35) and (36) into Eq. (34), we obtain,

$$W_s' = 2P_1 \int_{0}^{\eta} \Delta W d\eta' = \frac{2}{\eta} \int_{0}^{\eta} \left( (1 - \eta'')^2 - (1 - \eta')^2 \right) d\eta'$$

$$W_s' = P_1 \eta^2 \left( 1 - \frac{2}{3} \eta \right) \left( \frac{dK_{ij}}{K} \right)^2 \bar{\varepsilon}^2$$

(37)

Let's denote here the mean value of the grain boundary region $P_1$ for many grains by $P_1$. In order to obtain the energy $W_s$ of the grain boundary region averaged over the whole polycrystals, the values of $\bar{K}$, $\bar{\varepsilon}$ in Eq. (37) have to be replaced by $\bar{K}$, and $\bar{\varepsilon}$, respectively:

$$W_s = \int W_s' d\Omega$$

$$= P_1 \eta^2 \left( 1 - \frac{2}{3} \eta \right) \left( \frac{dK_{ij}}{K} \right)^2 + \sigma \left( \frac{dK_{ij}}{K} \right)^2 \bar{\varepsilon}^2$$

(38)

where

$$\left( \frac{dK_{ij}}{K} \right)^2 = \int \left( \frac{dK_{ij}}{K} \right)^2 d\Omega$$

(39)

Consequently, the total deformation energy $W_t$ of polycrystals is obtained by summing the two energies given from Eqs. (27) and (38):

$$W_t = W + W_s$$

$$= \frac{1}{2} \bar{K} \bar{\varepsilon}^2 + \frac{1}{2} \left( 1 - \eta \right) \left( \frac{dK_{ij}}{K} \right)^2 \bar{\varepsilon}^2$$

$$+ P_1 \eta^2 \left( 1 - \frac{2}{3} \eta \right) \left( \frac{dK_{ij}}{K} \right)^2 \bar{\varepsilon}^2$$

(40)

The value of $\eta$ at which $W_t$ takes its minimum value is calculated from $\delta W_t / \delta \eta = 0$:

$$(1 - \eta) \left( 2P_1 \eta \left( \frac{dK_{ij}}{K} \right)^2 - \left( \frac{dK_{ij}}{K} \right)^2 \right) = 0$$

(41)

Noting that $0 \leq \eta \leq 1$, we finally obtain

$$\eta = \min \left( 1, \frac{\left( \frac{dK_{ij}}{K} \right)^2}{2P_1 \left( \frac{dK_{ij}}{K} \right)^2} \right)$$

(42)

Although the model employed in the above analysis is somewhat idealized, the results may contain the essential feature of the deformation behavior of polycrystalline metal. It follows from Eq. (42) that $\eta$ decreases as the ratio of the grain boundary region $P_1$ increases. $P_1 = \infty$ corresponds to $\eta = 0$, that is, to the condition of constant strain, while $P_1 = 0$ corresponds to the case of unconstraint described in the previous chapter.

In the above discussion, $P_1$ was assumed to be constant. The behavior of the grain boundary, however, is thought to be much more complicated and subjected to influences of various factors. Therefore, it seems necessary to clarify the physical meaning of $P_1$ in more detail. In the case of uniform and random distribution of $dK$, it is possible to show that $\left( \frac{dK_{ij}}{K} \right)^2 = 2$ in Eq. (42).

The effects of the upper and the lower grains C in Fig. 1 are not taken into account in the above discussion. It is thought that the strain-difference $\Delta \varepsilon_{ab}$ between the grains A and B increases under the influence of the grain C. That is, the mutual restriction between grains may slightly increase due to the effect of the grains C.

It seems also necessary to clarify the effect of the shape of the grains on the deformation behavior of the polycrystals. Further discussions on these points, however, will be reported elsewhere.
5. Conclusions

The following conclusions may be drawn from the above consideration:

(1) As a parameter reflecting the deformation behavior of a polycrystal consisting of anisotropic crystals, the constraint ratio \( \eta \) between grains is introduced. The basic relations of stress and strain are expressed in terms of \( \eta \), and then applied to the analysis of uniaxial deformation of the polycrystals consisting of grains having linear stress-strain relation.

(2) It becomes evident that, if the mutual restriction between grains at the grain boundary is absent, the condition of constant stress is energetically stable. Furthermore, mutual restriction at the grain boundary region is discussed and it is concluded that the stability condition in the deformation of polycrystal deviates from constant stress to constant strain in proportion to the increase in the grain boundary region.

The author is indebted to Prof. Shuji Taira for his valuable advice and encouragement, and also to Prof. Moriya Oyane for making this study possible.

References

(3) E. Macherauch: Materialprüfung, Bd. 5 (1963), S. 14.