Adhesion and Dynamic Phenomena during Slipping of Electrical Rolling Stock*

(1st Report, Slipping Phenomena and Improvement of Adhesive Characteristics of Rectifier AC Locomotives)

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For the purpose of improving the adhesive characteristics of rectifier AC locomotives, the slipping phenomena of the standard four-axle locomotive were observed, and a clear picture was obtained of various phenomena which cannot be explained by simplified theories and of conditions of coefficient of adhesion in a range of small wheel-rail relative speeds. Detailed calculative expressions of slipping phenomena were then formulated and these expressions were used for numerical calculation by means of a digital computer. The results of these calculations were found to coincide approximately with actually measured values. The effects of various factors on adhesion were investigated and the following systems were planned:

(1) Controlling DC source voltage by feedback of rectifier output DC voltage.
(2) Controlling DC source voltage by feedback of maximum difference in armature voltages.

The characteristics of these controlling systems and their effects on adhesion were made clear.

1. Introduction

Rectifier AC locomotives generally have better adhesive characteristics than other types of locomotives. Seki and his coworkers(1) have theoretically investigated these characteristics. The present author and his coresearchers(2-3) previously reported the results of their study on how self-excited vibration of the driving axle system in slipping can be prevented and how the adhesive characteristics can be further improved. In this previous study, approximation was made, assuming that the magnetic flux of field of the DC series motor for wheel drive is constant, and that the coefficient of adhesion decreases linearly as slip speed increases. Also investigation was made of cases which may be regarded as one main motor connected to one power source.

In the present study, the slipping phenomena of a typical four-axle AC locomotive were observed, and a clear picture was gained of a few phenomena that cannot be explained by such a simplified theory as referred to above, and of the relation between wheel-rail relative speed and coefficient of adhesion. Moreover, detailed calculative expressions were derived, the calculated values were compared with observed values, and the effects of a few factors that had not been made clear were examined. On the basis of these investigations, some methods to improve adhesive characteristics were worked out and effects of those methods were appraised.

2. Slipping phenomena of typical four-axle AC locomotive

Slipping phenomena were observed with regard to the Type ED75 locomotive—a typical four-axle individual drive AC locomotive in Japan. The sample is mounted on two bogies (B-B) which have no weight transfer. The main motor circuit, as shown in Fig. 1, consists of four motors connected in parallel. Observation of slipping phenomena was made during

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an adhesive characteristics test conducted in a section of 10% upgrade between Takehagi Station and Kawajiri Station on the Japanese National Railways' Joban Line.

2-1 Observed slipping phenomena

The test was conducted with the rail surface in three different states, namely, (i) dry, (ii) wet, and (iii) wet and sanded. Observations disclosed that slipping phenomena (changes, during slipping, in the speed of slipping axles and in the motor current) did not show any essential difference depending on rail surface conditions but did vary in accordance with the number of slipping axles and with the locomotive speed at slipping (hereafter the expression "at slipping" will be omitted). For instance, in cases where locomotive speed is zero and the number of slipping axles is two or less, the circumferential speed of the slipping axle is vibratory and often exhibits readhesive characteristics. A typical example is given in Fig. 2. Even if locomotive speed is zero, however, when all axles are involved in slipping, their circumferential speeds are not vibratory but rather tend to converge on a certain value. On the other hand, when locomotive speed is 2-3 km/hr or higher, the axle speeds, regardless of the number of slipping axles, do not vibrate but in most cases converge on a certain value. An example is shown in Fig. 3. During slipping, the notch is held at a fixed position, but if slipping does not stop automatically (as in Fig. 3), it is stopped by setting back the notch manually. In Fig. 3, No. 1 axle and No.2 axle begin to slipping at approximately the same time; as a result the current in No. 3 axle and No. 4 axle, still in adhesion, increases, to induce a slipping of No. 3 axle; the current in Axle 4 continues to increase, eventually causing Axle 4 to slip. This phenomenon, in which an axle in adhesion is made to slip as a result of changes in the current due to the slipping of another axle, will be called "induction of slip." The fact that induction of slip and the above-mentioned slip phenomena vary in accordance with the number of slipping axles and locomotive speed cannot be explained by the simplified theories used conventionally.

2-2 Relation between slip speed and coefficient of adhesion during slip

Coefficient of adhesion during slip $\mu_s$ between Test No. 406' Wet, with sanding

\[ I_m \sim I_m' \]: main motor current in No. 1-4 axles
\[ v_1 \sim v_1' \]: axle speed of No. 1-4

Fig. 2 Some observed results (locomotive speed: zero)

\[ I_m \sim I_m' \]: main motor current in No. 1-4 axles
\[ v_1 \sim v_1' \]: axle speed of No. 1-4

Fig. 3 Some observed results (locomotive speed: 9 km/hr)

\[ I_m \sim I_m' \]: main motor current in No. 1-4 axles
\[ v_1 \sim v_1' \]: axle speed of No. 1-4

Fig. 4 Wheel-rail relative speed and coefficient of adhesion during slip
wheel and rail is obtained by dividing the tangential force $f$ between wheel and rail by axle loading $W$, as follows:

$$
\mu_s = \frac{f}{W} \tag{1}
$$

The coefficient can be given also by the following expression in which $T_m$ denotes wheel drive torque, $I$ moment of inertia of drive system in terms of axle, $\dot{\theta}$ angular acceleration of slipping axle, and $r$ wheel radius:

$$
\mu_s = \frac{(T_m - Gh)}{Wr} \tag{2}
$$

The author derived the coefficient from Eq. (2). $T_m$ and $\dot{\theta}$ were found from oscillograms of current and circumferential speed of axle, while $W$ was calculated considering weight transfer.

The relation between wheel-rail relative speed ($V_w - V_r$, where $V_w$ is locomotive speed and $V_r$ is circumferential speed of slipping axle) and $\mu_s$ is as shown in Fig. 4. Figure 4 (a) is derived from the oscillogram in Fig. 2, which is a typical case where locomotive speed is zero, and Fig. 4 (b) from the oscillogram in Fig. 3, which is a typical case where locomotive speed is present. In Fig. 4 (b), there are a range wherein $\mu_s$ increases with wheel-rail relative speed (slip in this range will be called "creep") and a range wherein $\mu_s$ decreases with wheel-rail relative speed (slip in this range will be called "slide"). and almost no discontinuous change is noted in $\mu_s$ during transition from creep to slide. On the other hand, in Fig. 4 (a), in which locomotive speed is zero, there is almost no creep, and during transition from creep to slide, $\mu_s$ decreases discontinuously. The magnitude of this discontinuous decrease $\Delta \mu_s$ is as shown in Fig. 5.

The kind of slip that must be considered in discussing adhesive characteristics is the slip within the range of slide. Consequently, it becomes necessary to seek the relation between slide speed (the difference between the circumferential speed of a slipping axle and the circumferential speed of that axle at the boundary of creep and slide) and $\mu_s$. Hereafter in this paper, slide speed will be referred to as speed and written as $v_s$. The relation between $v_s$ and $\mu_s$ obtained with regard to oscillograms from all observations indicates obvious differences between cases where locomotive speed is zero (i.e., no change in wheel-rail contact point) and cases where locomotive speed is present (i.e., contact point moves), but because of the small number of observations made, it was not known whether the relation

changes with locomotive speed. The upper and lower limits of the $v_s$-$\mu_s$ curve for zero locomotive speed are shown in Fig. 6 and those for a certain locomotive speed in Fig. 7. From Figs. 6 and 7, the following is found about the relation between $v_s$ and $\mu_s$:

(1) Under the same conditions, the slope of

![Graph showing the relationship between $v_s$, $\mu_s$, and characteristics A and B.]

Fig. 6 Measured coefficient of adhesion during slip (locomotive speed: zero)

![Graph showing the frequency distribution of $\Delta \mu_s$.]

Fig. 5 Frequency distribution of $\Delta \mu_s$

![Graph showing the coefficient of adhesion during slip with locomotive speed (5 to 20 km/hr).]

Fig. 7 Coefficient of adhesion during slip (locomotive speed: 5 to 20 km/hr)
decrease $d\mu_s/dv_i$ becomes steeper as coefficient of static adhesion ($\mu_s$ at $v_i=0$) increases.

2. At a slip speed of 5 km/hr or lower, the value of $d\mu_s/dv_i$ at zero locomotive speed is larger than that where locomotive speed is present.

3. At zero locomotive speed, $d\mu_s/dv_i$ does not vary much with the rail surface condition.

4. As $v_i$ increases, the difference between upper and lower limits decreases and, regardless of rail surface conditions and locomotive speed, narrows to almost zero.

That the $v_i-\mu_s$ curve varies depending on the presence or absence of locomotive speed is thought to be related to whether the contact point between wheel and rail moves or not.

Reports have been published on measured $\mu_s$ for comparatively large slip speeds, but none of them refers to initial slipping conditions and differences due to the presence or absence of locomotive speed. The author and his collaborators made measurements for various types of locomotives at different places, and obtained the same results as stated above, so that these results are thought to be valid for any type of locomotive observed at any site.

3. Formulation of theoretical expressions for slipping phenomena

3.1 Preconditions

1. Cases of four-axle individual drive will be considered.

2. For weight transfer, only static weight transfer will be considered.

3. It will be assumed that slip is caused by discontinuous decrease of coefficient of static adhesion.

3.2 Explanation of symbols

MKS gravity units will be used.

- $M$: total mass of train including revolutionary mass of adhesive axles.
- $\Theta$: moment of inertia of drive system in terms of axle
- $V_i$: speed of locomotive
- $R_i$: total mechanical resistance of train
- $n$: angular velocity of wheel
- $W$: axle loading
- $W_o$: axle loading when tangential force of wheel circumference is zero
- $H$: height from rail surface to coupler
- $h$: height from rail surface to point of tractive force of truck
- $L$: distance between trucks
- $l$: distance between two axles in each truck
- $\mu_s$: coefficient of static adhesion
- $\mu_i$: coefficient of adhesion during slip
- $\Delta\mu$: discontinuous decrease of $\mu_s$ causing slip

- $f$: tangential force between wheel and rail
- $\tau_m$: main motor torque
- $T_m$: drive torque in terms of axle
- $r$: radius of wheel
- $S$: gear ratio
- $\eta$: efficiency of torque transmission
- $v_i$: slip speed
- $K\Phi$: unit revolution flux
- $I_m$: main motor current
- $E_p$: DC power source voltage (no-load rectifier output voltage)
- $E_m$: counter-electromotive force
- $R_e$: equivalent resistance of power source
- $R_m$: resistance of main motor circuit
- $L_m$: inductance of main motor circuit

Subscripts 1 through 4 will be used to indicate the position of axles from front to rear.

3.3 Calculative expressions

For translation of locomotive, the following equation holds:

$$MV_i = \sum_{i=1}^{4} f_i - R_i$$

Generally, $V_i$ is not a negative value, so it will be assumed that if $V_i < 0$, then $R_i = \sum_{i=1}^{4} f_i$.

The tangential force between wheel and rail is given as follows:

At slip: $f_i = \mu_i W_i$ \hspace{1cm} (4)

At adhesion: $f_i = \frac{T_m}{r} - \frac{\Theta}{r^2} V_i$ \hspace{1cm} (5)

Wheel revolution is given by:

At slip: $\Theta h_i = T_m - \mu_i W_i r$ \hspace{1cm} (6)

At adhesion: $n_i = \frac{V_i}{r}$ \hspace{1cm} (7)

It should be noted that when readhesion occurs, coefficient of adhesion at zero slip speed will be discontinuous as shown in Fig. 8. In other words, when readhesion occurs:

1. Adhesion is maintained when $T_m \leq \mu_o W_i$

2. Reslip occurs when $T_m > \mu_o W_i$

Since $T_m$ never assumes a negative value, there is no need to consider negative slip (skid).

Considering only static weight transfer, the axle loadings of the four axles would be given by:

![Fig. 8 Relation between slip speed $v_i$ and coefficient of adhesion during slip $\mu_s$](image-url)
For the main circuit, the following voltage equation holds:
\[ L_{m1}i_{m1} + R_{m}i_{m1} + R_{s} \sum_{j=1}^{4} i_{j} = E_{r} - E_{m1} \]  
(13)

Since during slip, notch position is fixed, 
\[ \dot{E}_{r} = 0 \]  
(14)

Counter-electromotive force \( E_{m1} \) is expressed as follows:
\[ E_{m1} = S K \Phi_{b} n_{i} \]  
(15)

Unit revolution flux \( K \Phi_{b} \), as shown in Fig. 10, has saturation characteristics against current, and is expressed by:
\[ K \Phi_{b} = \frac{\Phi_{b} I_{m1}}{I_{m1} + b} \]  
(16)

where \( \Phi_{b} \) and \( b \) are constants.

Inductance \( L_{m1} \) decreases as current increases. Within the range of the current in question, however, this nonlinear characteristic can be approximated by a straight line, as follows:
\[ L_{m1} = d_{b} + d_{1} I_{m1} \]  
(17)

where \( d_{b} \) and \( d_{1} \) are constants.

Equations (3) \sim (17) are calculative expressions for slip phenomena. These can be arranged into nonlinear simultaneous ordinary differential equation with regard to \( V_{i}, n_{i} \) (\( i = 1 \sim 4 \)) and \( I_{m1} \) (\( i = 1 \sim 4 \)).

4. Numerical calculation and effects of various factors

4-1 Comparison with measured values

Using the theoretical expressions introduced in the preceding chapter, we shall perform numerical calculation of the locomotive slip phenomena described previously and compare the results with measured values.

The numerical calculations were performed by the Runge-Kutta methods using the HITAC 5020 computer, at a time division of 0.02 second. The values were determined after being checked for accuracy against those obtained at smaller time divisions.

The various numerical values for the Type ED 75 locomotive are listed in Table 1. As an example, calculation will be made for the case shown in Fig. 3. In this case, \( \mu_{s} \) is as shown in Fig. 4 (b) and the \( \mu_{s} \) for each axle can be approximated as follows:

No. 1 axle: \( \mu_{s} = 0.38 \), 
\[ \mu_{s} = 0.05 + (0.30/(1+0.875v_{s})) \]  
(\( \Delta \mu = 0.03 \))

No. 2 axle: \( \mu_{s} = 0.39 \), 
\[ \mu_{s} = 0.075 + (0.30/(1+0.875v_{s})) \]  
(\( \Delta \mu = 0.015 \))

No. 3 axle: \( \mu_{s} = 0.36 \), 
\[ \mu_{s} = 0.03 + (0.30/(1+0.875v_{s})) \]  
(\( \Delta \mu = 0.03 \))

No. 4 axle: \( \mu_{s} = 0.386 \), 
\[ \mu_{s} = 0.083 + (0.30/(1+0.875v_{s})) \]  
(\( \Delta \mu = 0.015 \))
In Fig. 3, locomotive speed immediately prior to slip is 9 km/hr and slip occurs first in No. 1 axle. Therefore, initial conditions for the calculation would be: a locomotive speed of 9 km/hr and No. 1 axle at the limit of adhesion. The results of calculation are shown in Fig. 11. Comparing these with the measured results in Fig. 3, one notes that both results agree in change of slip speed and current as well as in the condition of induction of slip.

Next, let us perform numerical calculation for a case where \( \mu_s \) is taken as the representative characteristic, and see whether the effects of the number of slipping axles and locomotive speed are the same as in the measured results referred to in section 2-1. We shall consider cases where locomotive speed is zero and where it is 10 km/hr. For the former case, the \( v_s-\mu_s \) characteristics will be represented by Characteristics A (heavy line in Fig. 6) and for the latter, they will be represented by Characteristics B (heavy line in Fig. 7). Thus:

Characteristics A: \( \mu_s=0.35 \)
\[
\mu_s = 0.05 + (0.27/(1 + 1.25v_s)), \quad (\Delta \mu = 0.03)
\]

Characteristics B: \( \mu_s=0.30 \)
\[
\mu_s = 0.05 + (0.22/(1 + 0.76v_s)), \quad (\Delta \mu = 0.03)
\]

Other values are as listed in Table 1, and the results of the calculation are shown in Fig. 12. It should be noted that in order to make weight transfer zero, \( h \) and \( H \) were taken as zero; and to investigate the influence of the number of slipping axles, the coefficient of static adhesion \( \mu_0 \) for the axles in adhesion was taken at sufficiently large values not to induce slip even if the current in the adhesive axles should increase. From Fig. 12 it is found that at zero locomotive speed readhesion occurs

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Value or expression</th>
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<tr>
<td>( M )</td>
<td>kg sec(^3)/m</td>
<td>1.5 \times 10^4</td>
</tr>
<tr>
<td>( \theta )</td>
<td>kN m sec(^2)</td>
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<tr>
<td>( W_s )</td>
<td>kg</td>
<td>16.8 \times 10^4</td>
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<td>( h )</td>
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<tr>
<td>( L )</td>
<td></td>
<td>7.6</td>
</tr>
</tbody>
</table>

(a) Locomotive speed: zero, \( \mu_s \): Characteristics A

(b) Locomotive speed: 10 km/hr, \( \mu_s \): Characteristics B

Fig. 11 Results of calculation for Test No. 406 (Fig. 3)

Fig. 12 Results of calculation where \( \mu_s \) is taken as representative characteristics
when there are two or less slipping axles and does not when there are three or more of them, but that at 10 km/hr speed no readhesion occurs regardless of the number of slipping axles. In either case, the greater the number of slipping axles, the higher the slip speed and the larger the current become in axles in adhesion, so that induction of slipping of other axles becomes the more likely to happen.

4-2 Effects of equivalent resistance of power source, number of slipping axles, motor current dependency of flux, and locomotive speed

It has been demonstrated by observations and theoretical calculations that slipping phenomena vary in accordance with locomotive speed and the number of slipping axles. This is thought to be due to the equivalent resistance of power source and the motor current dependency of flux. It is difficult to analyze the effects of these factors on dynamic slipping phenomena, but when the slip speed converges on a certain value, the value can be obtained from equilibrium characteristics. Here, let us investigate the effects of these factors on the convergence value. Equilibrium characteristics refer to the relation between slip speed and wheel circumferential drive force at steady state.

For simplicity’s sake, unit revolution flux will be linearly approximated, as shown by the broken line in Fig. 10, in the neighborhood of current \( I_o \), immediately preceding the slip, and expressed as follows:

\[ K \Phi = \Phi_0 + ai_n \]  \hspace{1cm} (18)

where \( \Phi_0 \) is unit revolution flux for current \( I_o \) a denotes a constant, and \( i_n \) the change in current due to slip.

Counter-electromotive force \( E_{\text{em}} \) at slip is:

\[ E_{\text{em}} = S(\Phi_0 + ai_n)(V_o + v_o)/r \]  \hspace{1cm} (19)

and counter-electromotive force at adhesion \( E_{\text{en}} \) is:

\[ E_{\text{en}} = S(\Phi_0 + ai_n)V_o/r \]  \hspace{1cm} (20)

where \( i_o \) and \( i_n \) are changes due to slip in main motor current, respectively, in slipping axles and adhesive axles.

When the number of slipping axles \( N_s \) is 3 or less, the following voltage equations hold for a steady state:

\[ R_{m_i}i - R_{n_i}i = E_{\text{en}} - E_{\text{em}} \]  \hspace{1cm} (21)

\[ R_i(4I_o + N_s i_o + (4 - N_s) i_n) + R_{m_i}(I_o + i) = E_{\text{en}} - E_{\text{em}} \]  \hspace{1cm} (22)

Substituting Eqs. (19) and (20) in Eqs. (21) and (22), and considering that the following relation holds:

\[ 4R_iI_o + R_{m_i} = E_{\text{en}} - S\phi_0 V_o/r \]

we obtain the following voltage equations for changes:

\[ (R_m + 4S\phi_0 V_o/r) i_o + (R_{m_i} + 4S\phi_0 V_o/r) i_n = 0 \]  \hspace{1cm} (23)

\[ N_s R_i + (R_m + 4S\phi_0 V_o/r + (4 - N_s) R_{m_i}) i_o = 0 \]  \hspace{1cm} (24)

If \( v_o < V_o \), then we can neglect the coefficient \( S\phi_0/r \) for \( i_o \) on the left side of Eq. (23). Neglecting this, we find that \( R_m + 4S\phi_0 V_o/r \) can be called the equivalent resistance of motor circuit in cases where unit revolution flux \( K \Phi \) is regarded as constant at \( \Phi_0 \). Therefore, we shall put:

\[ R_{m_i} = R_m + 4S\phi_0 V_o/r \]  \hspace{1cm} (25)

From Eqs. (23) and (24), \( i_o \) and \( i_n \) are given by the following:

\[ i_o = -S\phi_0 V_o/r R_{m_i} \]  \hspace{1cm} (26)

\[ i_n = S\phi_0 V_o/r R_{m_i} \]  \hspace{1cm} (27)

where

\[ R_i = R_{m_i} + 4S\phi_0 V_o/r + (4 - N_s) R_{m_i} \]  \hspace{1cm} (28)

\[ R_o = (R_{m_i} + 4S\phi_0 V_o/r) R_{m_i} / R_i + 4R_{m_i} + 4S\phi_0 V_o/r + (4 - N_s) S\phi_0 V_o/r / N_s \]  \hspace{1cm} (29)

When \( N_s = 4 \), then the voltage equation at steady state would be:

\[ (4R_i + R_m)(I_o + i) = E_{\text{en}} - E_{\text{em}} \]  \hspace{1cm} (30)

and for the change:

\[ (4R_i + R_m + 4S\phi_0 V_o/r + S\phi_0 V_o/r) i_o = -S\phi_0 V_o/r \]  \hspace{1cm} (31)

Thus:

\[ i_o = -S\phi_0 V_o/r (4R_i + R_m + 4S\phi_0 V_o/r) \]  \hspace{1cm} (32)

Wheel circumferential drive forces \( F_i \) and \( F_o \) respectively for slipping axles and adhesive axles are given by:

\[ F_i = S(\phi_0 + ai_o)(I_o + i_o)/gr \]  \hspace{1cm} (33)

\[ F_o = S(\phi_0 + ai_n)(I_o + i_n)/gr \]  \hspace{1cm} (34)

Equations (33) and (35) show the equilibrium characteristics of \( F_i \) and \( F_o \) against slip speed.

As an example, the equilibrium characteristics for a case where slip occurs in the Type ED 75 locomotive while traveling at 10 km/hr, as calculated by Eqs. (26) to (32), (33) and (35) are given in Fig. 13. It is noted that within the range of a small \( v_o \), linearly approximated Eqs. (34) and (36) hold.

In Fig. 13, letting \( v_o \) represent the slip speed at the intersection of curve \( v_o - F_i/W \) and curve \( v_o - \mu_o \), we find that when \( v_o \) is smaller than \( v_o \),
wheel revolution is accelerated because the drive force is larger than the friction force, and when \( v_0 \) is larger than \( v_{\text{so}} \) the revolution is decelerated because the drive force is smaller than the frictional force. Therefore, it is found that slip speed converges on \( v_{\text{so}} \) and \( F_0/W \) converges on the \( \beta_0 \) value for \( v_{\text{so}} \).

For adhesive characteristics, it is desirable that \( v_0 \) be small, and to prevent slip from being induced, \( \beta_0 \) do not increase beyond the value prior to slip \( (\beta_0) \). From Eq. (34), we note that the smaller the value of \( R_s \), the steeper the fall of curve \( v_0 - F_0/W \), so that \( v_{\text{so}} \) becomes small. From Eq. (36) it is found that the larger the value of \( R_s \), the gentler the rise of curve \( v_0 - F_0/W \), so that if \( v_{\text{so}} \) is small \( \beta_0 \) becomes small. According to Eqs. (28) and (29), when \( R_s \) is not 0, the smaller the values of \( R_s \) and \( N_s \), the smaller the value of \( R_s \) and the larger the value of \( R_s \), so that both \( v_{\text{so}} \) and \( \beta_0 \) are reduced and satisfactory adhesive characteristics are obtained. When \( R_s \) is zero, from Eq. (28) we get \( R_s = R_{\text{so}} + Sv_0/\rho \), so that curve \( v_0 - F_0/W \) is not affected by \( N_s \) and consequently \( v_{\text{so}} \) is not affected by \( N_s \). When \( R_s \) is zero, from Eq. (29) \( R_s \) becomes infinitely large. So that from Eq. (27) we get \( i_s = 0 \) and wheel centrifugal drive force is held constant at the value before slip \( (\beta_0 = \mu_0 \text{ constant}) \).

By varying equivalent resistance of power source \( R_s \) and number of slipping axles \( N_s \), we obtained \( v_{\text{so}} \) and \( \beta_0 \) as shown in Fig. 14. The \( v_{\text{so}} \) and \( \beta_0 \) calculated from the theoretical expressions in section 3.3 are plotted in the graph and these values, as may be expected, agree approximately with the values obtained schematically, as described above, from the equilibrium characteristics.

Locomotive speed \( V_l \) acts as an equivalent of resistance \( Sv_l/\rho \) inserted in series in the main motor circuit [equivalent resistance of main motor

is given by Eq. (25)], and when main motor flux is constant, i.e. when \( a_0 = 0 \), equivalent resistance of main motor \( R_m \) is constant at \( R_m \). When there is motor current dependency of flux, i.e. \( a_0 \neq 0 \), as \( V_l \) increases \( R_m \) increases with it and unless the \( v_0 - \mu_0 \) characteristics change with locomotive speed, \( v_{\text{so}} \) also increases. For example, the \( SaV_{l/\rho} \) for locomotive speed 10 km/hr for the Type ED75 locomotive is calculated as follows:

\[
SaV_{l/\rho} = 4.44 \times 3.2 \times 10^{-4} \times 10/3.6 \times 0.56 = 0.0705 \text{ (\Omega)}
\]

This value is close to \( R_{\text{so}} = 0.085 \text{ (\Omega)} \).

In the foregoing we have quantitatively explained the cause of changes in slip phenomena due to the number of slipping axles and locomotive speed, and the effects of various factors on converging values.

5. Method of improving adhesive characteristics

To improve adhesive characteristics, \( v_{\text{so}} \) should be small and from the viewpoint of preventing induction of slip, \( \beta_0 \) should be equal to or smaller than \( \mu_0 \). Here, a few methods of improving adhesive characteristics will be investigated on the basis of the discussion in section 4.2.

5.1 Controlling DC source voltage by feedback of rectifier output DC voltage

In this method, as shown in Fig. 15, rectifier output voltage \( E_d \) (Fig. 1) is negatively fed back and compared with reference voltage \( E_r \), and power source voltage \( E_r \), is controlled in accordance with the difference. In this way, equivalent resistance of power source \( R_s \) can be equivalently made zero. Let us first seek the equilibrium characteristics for such a case.

The voltage equations for a steady state are given by Eqs. (21) and (22) for \( N_s = 3 \), and by Eq. (30) for \( N_s = 4 \), but the power source voltage \( E_r \) to be controlled in this way is not constant but is expressed as follows:

\[
E_r = G_E p - G_j E_d = G_E p - G_j (E_m + R_m (i_0 + i_s)) \quad \cdots \cdots (37)
\]

\( E_p \): reference voltage, \( E_d \): rectifier output voltage, \( E_r \): no-load rectifier output voltage, \( G_j \): gain of \( E_d \) detector, \( G \): gain, \( T \): time constant, \( s \): operator of Laplace transformation.

Fig. 15 Block diagram of the system controlling DC source voltage by feedback of rectifier output DC voltage.
where \( G_F = K_F G \) and \( E_{ma} \) is given by Eq. (19)

When \( N_s \leq 3 \), the voltage equations for current changes \( i_e \) and \( i_a \) are given as follows, by rearranging Eqs. (19), (21), (22) and (37):

\[
\begin{align*}
(R_{me} + S a v_i / r) i_e - R_{me} i_a &= -S \Phi_0 v_i / r \quad (38) \\
(N_s R_e + G_e (R_{me} + S a v_i / r)) i_e + (R_{me} + 4N_i R_e) i_a &= -G_e S \Phi_0 v_i / r \quad (39)
\end{align*}
\]

From Eqs. (38) and (39), we get:

\[
\begin{align*}
i_e &= -S \Phi_0 v_i / R_e \quad (40) \\
i_a &= S \Phi_0 v_i / R_r \quad (41)
\end{align*}
\]

where \( R_e \) and \( R_r \) are as follows:

\[
R_e = R_{me} + S a v_i / r + N_s / (1 + G_F) / R_e \\
+ (4 - N_s) / R_{me}
\]

\[
R_r = (R_{me} + S a v_i / r) R_{me} / (1 + G_F) / R_e + 4R_{me}
\]

\[
+ (4 - N_s) S a v_i / r / N_s
\]

When \( N_s = 4 \), current change \( i_e \) is expressed as follows, by rearranging Eqs. (19), (30) and (37):

\[
i_e = -S \Phi_0 v_i / R_r (R_{me} + S a v_i / r + 4N_i / (1 + G_F)) \quad (44)
\]

Comparing Eqs. (42), (43) and (44) with Eqs. (28), (29) and (32) where no control is made, we note that when control is made the \( R_e \) in Eqs. (28), (29) and (32) is \( R_e / (1 + G_F) \). Therefore, by making gain \( G_F \) sufficiently large, \( R_e / (1 + G_F) \) approaches zero, so that equivalent resistance of power source becomes equivalently zero.

Since from Eq. (43) we note that \( \lim_{G_F \to \infty} R_e \to 0 \), from Eq. (41) we get \( \lim_{G_F \to \infty} i_a = 0 \). Thus, with a sufficiently large gain \( G_F \), the wheel circumference drive force of an adhesive axle is held constant at the value prior to slip. However, from Eq. (42) we get:

\[
\lim_{G_F \to \infty} R_e = R_{me} + S a v_i / r \quad (45)
\]

Therefore, with a large gain \( G_F \), the \( v_e - F_e / W \) curve given by Eq. (33) converges on the \( v_e - F_e / W \) curve for \( R_e = R_{me} + S a v_i / r \) and the converging value of slip speed then is at the lower limit and does not fall below this. However, since the \( R_{me} \) on the right side of Eq. (45), as in Eq. (25), has a term that increases in proportion to locomotive speed \( V_i \), the lower limit of \( R_e \), given by Eq. (45) increases with locomotive speed. Consequently, the lower limit value of slip speed convergence \( v_{e1} \) also increases.

Next, let us attempt numerical calculations of slip phenomena by the method described in section 3-3 for slip phenomena where such control is performed, taking the time lag in the system into consideration. By approximating the transfer function of the control system with a time lag of first order (Fig. 15) and letting time constant be denoted by \( T_e \) power source voltage \( E_e \) is given by:

\[
\begin{align*}
T_e E_e + E_e &= G e p - G_F E_e \quad (46) \\
E_a &= L_m l_i + R_m i_n + E_m \quad (47)
\end{align*}
\]

Other theoretical equations are as shown in section 3-3. Figure 16 gives the results of calculation for a case where such control is applied to the Type ED75 locomotive. Comparing these results with Fig. 12 (no control), we find that the increment in drive force of axles in adhesion is notably reduced, but that adhesion does not occur at 10 km/hr locomotive speed. Therefore, it cannot be said that adhesive characteristics where a locomotive speed is present have been sufficiently improved.

5-2 Controlling DC source voltage by feedback of maximum difference in armature voltages

By controlling power source voltage by negative feedback of maximum difference in voltage — i.e., the difference between maximum and minimum values of armature voltages — and comparing it with the reference voltage, we can compensate the voltage drop due to equivalent resistance \( S a v_i / r \) resulting from the current dependency of main motor flux.

In this way, good adhesive characteristics can be obtained even where a locomotive speed exists.

Let us consider the equilibrium characteristics. Maximum armature voltage \( E_{max} \) is the armature voltage of the slipping main motor, and is given by the following equation, where the armature's winding resistance is represented by \( R_e \):

\[
E_{max} = E_{ma} + R_e (I_v + i_e) \quad (48)
\]

\[
\begin{array}{c}
\text{Drive force} \\
\text{Axle loading}
\end{array}
\]

\[
\begin{array}{c}
\text{Slip speed} \\
\text{km/hr}
\end{array}
\]

(a) Locomotive speed: zero, \( \mu_i \): Characteristics A

\[
\begin{array}{c}
\text{Drive force} \\
\text{Axle loading}
\end{array}
\]

\[
\begin{array}{c}
\text{Slip speed} \\
\text{km/hr}
\end{array}
\]

(b) Locomotive speed: 10 km/hr, \( \mu_i \): Characteristics B

Fig. 16 Results of calculation for controlling DC source voltage by feedback of rectifier output DC voltage (Gain \( G_F = 15 \). Time constant \( T_e = 1 \) sec)
On the other hand, minimum armature voltage \( (E_{a})_{\text{min}} \) is the armature voltage of a main motor in adhesion and can be expressed as follows:

\[
(E_{a})_{\text{min}} = E_{ao} + R_{o}(I_{a0} + i_{a0})
\]

where \( E_{ao} \) and \( E_{a} \) are given by Eqs. (19) and (20). From Fig. 17, the power source voltage \( E_{r} \) in a steady state is given as follows:

\[
E_{r} = G_{e} E_{a} - G_{f} ((E_{a})_{\text{max}} - (E_{a})_{\text{min}})
\]

When \( N_{s} \leq 3 \), the voltage equations for current changes \( i_{a} \) and \( i_{r} \) are derived as follows, by rearranging Eqs. (19), (20), (21), (22), (48), (49) and (50):

\[
(R_{s} + G_{s} V_{r}) i_{a} - R_{s} i_{r} = - S \Phi_{r} v_{r}/r
\]

\[
[N_{s} R_{r} + G_{r} (R_{r} + S a V_{i} + v_{r})] i_{a} + (4 - N_{s}) R_{r} + R_{s} - G_{r} (R_{r} + S a V_{i} + v_{r}) i_{r} = - G_{r} S \Phi_{r} v_{r}/r
\]

From Eqs. (51) and (52), we get:

\[
i_{a} = - S \Phi_{r} v_{r}/R_{s} \quad i_{r} = S \Phi_{r} v_{r}/R_{r}
\]

where \( R_{s} \) and \( R_{r} \) are as follows:

\[
R_{s} = \frac{R_{s} (3 R_{s} + R_{s} + G_{s} S a V_{i} + v_{r}) + S a V_{i} (4 - N_{s}) R_{s} + R_{s} - G_{r} (R_{s} + S a V_{i} + v_{r})}{R_{s} (4 - N_{s}) + R_{s} (1 + G_{r}) - G_{r} (R_{s} + S a V_{i} + v_{r})}
\]

\[
R_{r} = \frac{(4 - N_{s}) R_{r} + R_{s} (4 - N_{s}) + G_{r} (R_{r} + S a V_{i} + v_{r}) + S a V_{i} (4 - N_{s}) R_{r} + R_{s} - G_{r} (R_{s} + S a V_{i} + v_{r})}{R_{r} (4 - N_{s}) + G_{r} (R_{r} - R_{r})}
\]

From Eq. (56), we get:

\[
\lim_{G_{r} \to \infty} R_{r} = S a V_{i}/r
\]

and from Eq. (54):

\[
\lim_{G_{r} \to \infty} R_{s} = - S a V_{i}/r
\]

Therefore, by making gain \( G_{r} \) sufficiently large, \( R_{r} \) would not be affected by locomotive speed \( V_{i} \) and would assume a sufficiently small value. Similarly, with a large gain, \( R_{s} \) would be equal to \( - R_{s} \), so that the current in adhesive axles would be the same as that in slipping axles, both currents decreasing with slip. Therefore, even when a locomotive speed exists, slip speed can be held down to a small value and induction of slip can be prevented.

Next, let us make calculations for slip phenomena where such control is effected, taking the time lag of the system into consideration. By approximating the control system’s transfer function with the time lag of first order, power source voltage \( E_{r} \) is expressed as follows:

\[
T \dot{E}_{r} + E_{r} = G_{e} E_{a} - G_{f} ((E_{a})_{\text{max}} - (E_{a})_{\text{min}})
\]

Now, letting the sliding main motor’s armature voltage be represented by No. 1 axle’s armature voltage \( E_{a1} \), letting the adhesive main motor’s armature voltage be represented by No. 4 axle’s armature voltage \( E_{a4} \), and denoting the armature winding’s inductance by \( L_{o} \), we get:

\[
(E_{a})_{\text{max}} = E_{a1} = L_{o} \dot{I}_{a1} + R_{o} I_{a1} + E_{a1}
\]

\[
(E_{a})_{\text{min}} = E_{a4} = L_{o} \dot{I}_{a4} + R_{o} I_{a4} + E_{a4}
\]

Fig. 17 Block diagram of system controlling DC source voltage by feedback of maximum difference in armature voltages.

\[
(E_{a})_{\text{max}} : \text{maximum armature voltage} \quad (E_{a})_{\text{min}} : \text{minimum armature voltage}
\]

\[
\text{Fig. 18 Results of calculation for controlling DC source voltage by feedback of maximum difference in armature voltages (Gain } G_{r} = 15, \text{ Time constant } T = 1 \text{ sec)}
\]

Other theoretical expressions are the same as in section 3-3. Figure 18 shows the results of calculations for a case where such control is applied to the Type ED 75 locomotive. It is noted that readhesion can be caused even at a locomotive speed of 30 km/hr.

6. Conclusion

In the foregoing, the adhesive characteristics of rectifier AC locomotives has been discussed. Slip phenomena for a standard four-axle AC locomotive were observed and a few phenomena that could not be explained by conventional simplified theories were clarified, and the relation between wheel-rail relative speed and coefficient of adhesion during slip was established. The author went on to derive detailed theoretical expressions for slip phenomena and compared the calculated results with the measured results, finding that both results agreed. Furthermore, the effects of such factors as equivalent resistance of power source, number of slipping axles, motor current dependency of flux, and locomotive speed, which had not been taken up in previous studies, were investigated, and the following methods...
of improving adhesive characteristics were worked out and their validity was examined:

(1) Controlling DC source voltage by feedback of rectifier output DC voltage

(2) Controlling DC source voltage by feedback of maximum difference in armature voltages

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References