625. 282. 03

Adhesion and Dynamic Phenomena during Slipping of Electrical Rolling Stock*

(1st Report, Slipping Phenomena and Improvement of Adhesive Characteristics of Rectifier AC Locomotives)

By Tetsuji HIROTSU**

For the purpose of improving the adhesive characteristics of rectifier AC locomotives, the slipping phenomena of the standard four-axle locomotive were observed, and a clear picture was obtained of various phenomena which cannot be explained by simplified theories and of conditions of coefficient of adhesion in a range of small wheel-rail relative speeds. Detailed calculative expressions of slipping phenomena were then formulated and these expressions were used for numerical calculation by means of a digital computer. The results of these calculations were found to coincide approximately with actually measured values. The effects of various factors on adhesion were investigated and the following systems were planned:

- (1) Controlling DC source voltage by feedback of rectifier output DC voltage.
 - (2) Controlling DC source voltage by feedback of maximum difference in armature voltages.
- The characteristics of these controlling systems and their effects on adhesion were made clear.

1. Introduction

Rectifier AC locomotives generally have better adhesive characteristics than other types of locomotives. Seki and his coworkers⁽¹⁾ have theoretically investigated these characteristics. The present author and his coresearchers^{(2) (3)} previously reported the results of their study on how self-excited vibration of the driving axle system in slipping can be prevented and how the adhesive characteristics can be further improved. In this previous study, approximation was made, assuming that the magnetic flux of field of the DC series motor for wheel drive is constant, and that the coefficient of adhesion decreases linearly as slip speed increases. Also investigation was made of cases which may be regarded as one main motor connected to one power source.

In the present study, the slipping phenomena of a typical four-axle AC locomotive were observed, and a clear picture was gained of a few phenomena that cannot be explained by such a simplified theory as referred to above, and of the relation between wheel-rail relative speed and coefficient of adhesion. Moreover, detailed calculative expressions were derived, the calculated values were compared with observed values, and the effects of a few factors that had

not been made clear were examined. On the basis of these investigations, some methods to improve adhesive characteristics were worked out and effects of those methods were appraised.

2. Slipping phenomena of typical four-axle AC locomotive

Slipping phenomena were observed with regard to the Type ED 75 locomotive — a typical four-axle individual drive AC locomotive in Japan. The sample is mounted on two bogies (B-B) which have no weight transfer. The main motor circuit, as shown in Fig. 1, consists of four motors connected in parallel. Observation of slipping phenomena was made during

MT: transformer A_{1-4} : main motor armatures
MR: rectifier MF_{1-4} : main motor fields
 MSL_{1-4} : smoothing reactors
 E_d : rectifier output DC voltage
 I_d : DC current

Fig. 1 Main motor circuit

* Received 6th September, 1967.

** Hitachi Research Laboratory, Hitachi Ltd. Katsuta.

an adhesive characteristics test conducted in a section of 10‰ upgrade between Takahagi Station and Kawajiri Station on the Japanese National Railways' Joban Line.

2.1 Observed slipping phenomena

The test was conducted with the rail surface in three different states, namely, (i) dry, (ii) wet, and (iii) wet and sanded. Observations disclosed that slipping phenomena (changes, during slipping, in the speed of slipping axles and in the motor current) did not show any essential difference depending on rail surface conditions but did vary in accordance with the number of slipping axles and with the locomotive speed at slipping (hereafter the expression "at slipping" will be omitted). For instance, in cases where locomotive speed is zero and the number of slipping axles is two or less, the circumferential speed of the slipping axle is vibratory and often exhibits readhesive characteristics. A typical example is given in Fig. 2. Even if locomotive speed is zero, however, when all axles are involved in slipping, their circumferential speeds are not vibratory but rather tend to converge on a certain value. On the other hand, when locomotive speed is 2~3 km/hr or higher, the axle speeds, regardless of the number of slipping axles, do not vibrate but in most cases converge on a certain value. An example is shown in Fig. 3. During slipping, the notch is held at a fixed position, but if slipping does not stop automatically (as in Fig. 3), it is stopped by setting back the notch manually. In Fig. 3, No. 1 axle and No. 2 axle begin to slipping at approximately the same time; as a result the current in No. 3 axle and No. 4 axle, still in adhesion, increases,

Test No. 506 Dry, no sanding

$I_{m1} \sim I_{m4}$: main motor current in No. 1~4 axles
 $v_1 \sim v_4$: axle speed of No. 1~4

Fig. 2 Some observed results (locomotive speed: zero)

to induce a slipping of No. 3 axle; the current in Axle 4 continues to increase, eventually causing Axle 4 to slip. This phenomenon, in which an axle in adhesion is made to slip as a result of changes in the current due to the slipping of another axle, will be called "induction of slip." The fact that induction of slip and the above-mentioned slip phenomena vary in accordance with the number of slipping axles and locomotive speed cannot be explained by the simplified theories used conventionally.

2.2 Relation between slip speed and coefficient of adhesion during slip

Coefficient of adhesion during slip μ_s between

Test No. 406' Wet, with sanding

$I_{m1} \sim I_{m4}$: main motor current in No. 1~4 axles
 $v_1 \sim v_4$: axle speed of No. 1~4 axles

Fig. 3 Some observed results (locomotive speed: 9 km/hr)

(a) For Test No. 506

(b) For Test No. 406'

(Fig. 2)

(Fig. 3)

Fig. 4 Wheel-rail relative speed and coefficient of adhesion during slip

wheel and rail is obtained by dividing the tangential force f between wheel and rail by axle loading W , as follows:

$$\mu_s = f/W \quad \dots\dots\dots (1)$$

The coefficient can be given also by the following expression in which T_m denotes wheel drive torque, Θ moment of inertia of drive system in terms of axle, $\dot{n}$ angular acceleration of slipping axle, and r wheel radius:

$$\mu_s = (T_m - \Theta \dot{n}) / Wr \quad \dots\dots\dots (2)$$

The author derived the coefficient from Eq. (2). T_m and $\dot{n}$ were found from oscillograms of current and circumferential speed of axle, while W was calculated considering weight transfer.

The relation between wheel-rail relative speed ($V_w - V_l$, where V_l is locomotive speed and V_w is circumferential speed of slipping axle) and μ_s is as shown in Fig. 4. Figure 4 (a) is derived from the oscillogram in Fig. 2, which is a typical case where locomotive speed is zero, and Fig. 4 (b) from the oscillogram in Fig. 3, which is a typical case where locomotive speed is present. In Fig. 4 (b), there are a range wherein μ_s increases with wheel-rail relative speed (slip in this range will be called "creep") and a range wherein μ_s decreases with wheel-rail relative speed (slip in this range will be called "slide"), and almost no discontinuous change is noted in μ_s during transition from creep to slide. On the other hand, in Fig. 4 (a), in which locomotive speed is zero, there is almost no creep, and during transition from creep to slide, μ_s decreases discontinuously. The magnitude of this discontinuous decrease $\Delta\mu$ is as shown in Fig. 5.

The kind of slip that must be considered in discussing adhesive characteristics is the slip within the range of slide. Consequently, it becomes necessary to seek the relation between slide speed (the difference between the circumferential speed of a slipping axle and the circumferential speed of that axle at the boundary of creep and slide) and μ_s . Hereafter in this paper, slide speed will be referred to as slip speed and written as v_s . The relation between v_s and μ_s obtained with regard to oscillograms from all observations indicates obvious differences between cases where locomotive speed is zero (i.e., no change in wheel-rail contact point) and cases where locomotive speed is present (i.e., contact point moves), but because of the small number of observations made, it was not known whether the relation

Fig. 5 Frequency distribution of $\Delta\mu$

changes with locomotive speed. The upper and lower limits of the $v_s - \mu_s$ curve for zero locomotive speed are shown in Fig. 6 and those for a certain locomotive speed in Fig. 7. From Figs. 6 and 7, the following is found about the relation between v_s and μ_s :

- (1) Under the same conditions, the slope of

Fig. 6 Measured coefficient of adhesion during slip (locomotive speed: zero)

Fig. 7 Coefficient of adhesion during slip (locomotive speed: 5 ~ 20 km/hr)

decrease $d\mu_s/dv_s$ becomes steeper as coefficient of static adhesion (μ_s at $v_s=0$) increases.

(2) At a slip speed of 5 km/hr or lower, the value of $d\mu_s/dv_s$ at zero locomotive speed is larger than that where locomotive speed is present.

(3) At zero locomotive speed, $d\mu_s/dv_s$ does not vary much with the rail surface condition.

(4) As v_s increases, the difference between upper and lower limits decreases and, regardless of rail surface conditions and locomotive speed, narrows to almost zero.

That the v_s - μ_s curve varies depending on the presence or absence of locomotive speed is thought to be related to whether the contact point between wheel and rail moves or not.

Reports have been published⁽⁴⁾ on measured μ_s for comparatively large slip speeds, but none of them refers to initial slipping conditions and differences due to the presence or absence of locomotive speed. The author and his collaborators made measurements for various types of locomotives at different places, and obtained the same results as stated above, so that these results are thought to be valid for any type of locomotive observed at any site.

3. Formulation of theoretical expressions for slipping phenomena

3.1 Preconditions

(1) Cases of four-axle individual drive will be considered.

(2) For weight transfer, only static weight transfer will be considered.

(3) It will be assumed that slip is caused by discontinuous decrease of coefficient of static adhesion.

3.2 Explanation of symbols

MKS gravity units will be used.

M : total mass of train including revolutionary mass of adhesive axles.

Θ : moment of inertia of drive system in terms of axle

V_t : speed of locomotive

R_t : total mechanical resistance of train

n : angular velocity of wheel

W : axle loading

W_0 : axle loading when tangential force of wheel circumference is zero

H : height from rail surface to coupler

h : height from rail surface to point of tractive force of truck

L : distance between trucks

l : distance between two axles in each truck

μ_0 : coefficient of static adhesion

μ_s : coefficient of adhesion during slip

$\Delta\mu$: discontinuous decrease of μ_0 causing slip

f : tangential force between wheel and rail

τ_m : main motor torque

T_m : drive torque in terms of axle

r : radius of wheel

S : gear ratio

η : efficiency of torque transmission

v_s : slip speed

$K\Phi$: unit revolution flux

I_m : main motor current

E_r : DC power source voltage (no-load rectifier output voltage)

E_m : counter-electromotive force

R_r : equivalent resistance of power source

R_m : resistance of main motor circuit

L_m : inductance of main motor circuit

Subscripts 1 through 4 will be used to indicate the position of axles from front to rear.

3.3 Calculative expressions

For translation of locomotive, the following equation holds:

$$M\dot{V}_t = \sum_{i=1}^4 f_i - R_t \quad \dots\dots\dots (3)$$

Generally, V_t is not a negative value, so it will

be assumed that if $V_t < 0$, then $R_t = \sum_{i=1}^4 f_i$.

The tangential force between wheel and rail is given as follows:

$$\text{At slip: } f_i = \mu_{si} W_i \quad \dots\dots\dots (4)$$

$$\text{At adhesion: } f_i = \frac{T_{mi}}{r} - \frac{\Theta}{r^2} \dot{V}_t \quad \dots\dots\dots (5)$$

Wheel revolution is given by:

$$\text{At slip: } \Theta \dot{n}_i = T_{mi} - \mu_{si} W_i r \quad \dots\dots\dots (6)$$

$$\text{At adhesion: } \dot{n}_i = \dot{V}_t / r \quad \dots\dots\dots (7)$$

It should be noted that when readhesion occurs, coefficient of adhesion at zero slip speed will be discontinuous as shown in Fig. 8. In other words, when readhesion occurs:

(1) Adhesion is maintained when $T_{mi} \leq \mu_0 W_i r$

(2) Reslip occurs when $T_{mi} > \mu_0 W_i r$

Since T_{mi} never assumes a negative value, there is no need to consider negative slip (skid).

Considering only static weight transfer, the axle loadings of the four axles would be given by:

Fig. 8 Relation between slip speed V_s and coefficient of adhesion during slip μ_s

$$\left. \begin{aligned} W_1 &= W_0 - a_1 F - a_2 (f_1 + f_2) \\ W_2 &= W_0 - a_1 F + a_2 (f_1 + f_2) \\ W_3 &= W_0 + a_1 F - a_2 (f_3 + f_4) \\ W_4 &= W_0 + a_1 F + a_2 (f_3 + f_4) \end{aligned} \right\} \dots\dots\dots (8)$$

where $a_1 = (H-h)/2L$, $a_2 = h/l$, and $F = \sum_{i=1}^4 f_i$

Coefficient of adhesion μ_{si} is expressed as follows:

$$\left. \begin{aligned} \mu_{si} &= \mu_0 \quad (v_s = 0) \\ \mu_{si} &= (\mu_0 - C_2) - \Delta\mu + \{C_2 / (1 + C_1 v_{si})\} \quad (v_s > 0) \end{aligned} \right\} \dots\dots\dots (9)$$

where C_1 and C_2 are constants.

Slip speed v_{si} is expressed as:

$$v_{si} = rn_i - V_t \dots\dots\dots (10)$$

Main motor torque τ_{mi} is given by:

$$\tau_{mi} = K\Phi_i I_{mi} / g \dots\dots\dots (11)$$

where g denotes gravitational acceleration.

Drive torque T_{mi} in terms of axle is:

$$T_{mi} = S\eta\tau_{mi} \dots\dots\dots (12)$$

The main motor circuit can be expressed equivalently as in Fig. 9, in which R_r denotes the equivalent resistance of power source including the overhead power line, and R_m and L_m denote, respectively, the winding resistance and inductance of the main motor and smoothing reactor.

- E_d : rectifier output voltage
 E_r : no-load rectifier output voltage
 E_{m1-4} : No. 1 ~ No. 4 main motor counter-electromotive force
 I_{m1-4} : No. 1 ~ No. 4 main motor current
 R_r : equivalent resistance of power source
 L_{m1-4} : inductance of smoothing reactor and main motor winding
 R_m : resistance of smoothing reactor and main motor winding

Fig. 9 Main motor circuit

Fig. 10 Unit revolution flux versus main motor current (Type MT 52 motor for ED 75 locomotive)

For the main circuit, the following voltage equation holds:

$$L_{mi} \dot{I}_{mi} + R_m I_{mi} + R_r \sum_{j=1}^4 I_{mj} = E_r - E_{mi} \dots\dots\dots (13)$$

Since during slip, notch position is fixed,

$$\dot{E}_r = 0 \dots\dots\dots (14)$$

Counter-electromotive force E_{mi} is expressed as follows:

$$E_{mi} = S K\Phi_i n_i \dots\dots\dots (15)$$

Unit revolution flux $K\Phi_i$, as shown in Fig. 10, has saturation characteristics against current, and is expressed by:

$$K\Phi_i = \Phi_e I_{mi} / (I_{mi} + b) \dots\dots\dots (16)$$

where Φ_e and b are constants.

Inductance L_{mi} decreases as current increases. Within the range of the current in question, however, this nonlinear characteristic can be approximated by a straight line, as follows:

$$L_{mi} = d_0 + d_1 I_{mi} \dots\dots\dots (17)$$

where d_0 and d_1 are constants.

Equations (3) ~ (17) are calculative expressions for slip phenomena. These can be arranged into nonlinear simultaneous ordinary differential equation with regard to V_t , n_i ($i=1 \sim 4$) and I_{mi} ($i=1 \sim 4$).

4. Numerical calculation and effects of various factors

4.1 Comparison with measured values

Using the theoretical expressions introduced in the preceding chapter, we shall perform numerical calculation of the locomotive slip phenomena described previously and compare the results with measured values.

The numerical calculations were performed by the Runge-Kutta methods using the HITAC 5020 computer, at a time division of 0.02 second. The values were determined after being checked for accuracy against those obtained at smaller time divisions.

The various numerical values for the Type ED 75 locomotive are listed in Table 1. As an example, calculation will be made for the case shown in Fig. 3. In this case, μ_s is as shown in Fig. 4 (b) and the μ_s for each axle can be approximated as follows:

No. 1 axle: $\mu_0 = 0.38$,

$$\mu_s = 0.05 + \{0.30 / (1 + 0.875 v_{s1})\},$$

$$(\Delta\mu = 0.03)$$

No. 2 axle: $\mu_0 = 0.39$,

$$\mu_s = 0.075 + \{0.30 / (1 + 0.875 v_{s2})\},$$

$$(\Delta\mu = 0.015)$$

No. 3 axle: $\mu_0 = 0.36$,

$$\mu_s = 0.03 + \{0.30 / (1 + 0.875 v_{s3})\},$$

$$(\Delta\mu = 0.03)$$

No. 4 axle: $\mu_0 = 0.398$,

$$\mu_s = 0.083 + \{0.30 / (1 + 0.875 v_{s4})\},$$

$$(\Delta\mu = 0.015)$$

In Fig. 3, locomotive speed immediately prior to slip is 9 km/hr and slip occurs first in No. 1 axle. Therefore, initial conditions for the calculation would be: a locomotive speed of 9 km/hr and No. 1 axle at the limit of adhesion. The results of calculation are shown in Fig. 11. Comparing these with the measured results in Fig. 3, one notes that both results agree in change of slip speed and current as well as in the condition of induction of slip.

Next, let us perform numerical calculation for a case where μ_s is taken as the representative characteristics, and see whether the effects of the number of slipping axles and locomotive speed are the same as in the measured results referred to in section 2.1. We shall consider cases where locomotive speed is zero and where it is 10 km/hr. For the

Table 1. Values for type ED 75 locomotive

Symbol	Unit	Value or expression
M	kg sec ² /m	1.5×10^5
θ	kg m sec ²	85
W_0	kg	16.8×10^3
S		4.44
r	m	0.56
η		0.945
H	m	0.88
h	"	0
L	"	7.6
	"	2.5
R_m	Ω	0.085 (at 75°C)
R_t	"	0.035 (average for 9~15 notches)
K	Wb	$11.53 I_m / (I_m + 268.8)$
L_m	H	$0.0253 - 1.43 \times 10^{-5} I_m$ ($400 \text{ A} < I_m < 1000 \text{ A}$)

Fig. 11 Results of calculation for Test No. 406' (Fig. 3)

former case, the v_s - μ_s characteristics will be represented by Characteristics A (heavy line in Fig. 6) and for the latter, they will be represented by Characteristics B (heavy line in Fig. 7). Thus:

Characteristics A: $\mu_0 = 0.35$,

$$\mu_s = 0.05 + \{0.27 / (1 + 1.25 v_s)\},$$

$$(\Delta \mu = 0.03)$$

Characteristics B: $\mu_0 = 0.30$,

$$\mu_s = 0.05 + \{0.22 / (1 + 0.76 v_s)\},$$

$$(\Delta \mu = 0.03)$$

Other values are as listed in Table 1, and the results of the calculation are shown in Fig. 12. It should be noted that in order to make weight transfer zero, h and H were taken as zero; and to investigate the influence of the number of slipping axles, the coefficient of static adhesion μ_0 for the axles in adhesion was taken at sufficiently large values not to induce slip even if the current in the adhesive axles should increase. From Fig. 12 it is found that at zero locomotive speed readhesion occurs

(a) Locomotive speed: zero, μ_s : Characteristics A

(b) Locomotive speed: 10 km/hr, μ_s : Characteristics B
Fig. 12 Results of calculation where μ_s is taken as representative characteristics

when there are two or less slipping axles and does not when there are three or more of them, but that at 10 km/hr speed no readhesion occurs regardless of the number of slipping axles. In either case, the greater the number of slipping axles, the higher the slip speed and the larger the current become in axles in adhesion, so that induction of slipping of other axles becomes the more likely to happen.

4.2 Effects of equivalent resistance of power source, number of slipping axles, motor current dependency of flux, and locomotive speed

It has been demonstrated by observations and theoretical calculations that slipping phenomena vary in accordance with locomotive speed and the number of slipping axles. This is thought to be due to the equivalent resistance of power source and the motor current dependency of flux. It is difficult to analyze the effects of these factors on dynamic slipping phenomena, but when the slip speed converges on a certain value, the value can be obtained from equilibrium characteristics. Here, let us investigate the effects of these factors on the convergence value. Equilibrium characteristics refer to the relation between slip speed and wheel circumferential drive force at steady state.

For simplicity's sake, unit revolution flux will be linearly approximated, as shown by the broken line in Fig. 10, in the neighborhood of current I_0 immediately preceding the slip, and expressed as follows:

$$K\Phi = \Phi_0 + ai_m \quad (18)$$

where Φ_0 is unit revolution flux for current I_0 , a denotes a constant, and i_m the change in current due to slip.

Counter-electromotive force E_{ms} at slip is:

$$E_{ms} = S(\Phi_0 + ai_s)(V_t + v_s)/r \quad (19)$$

and counter-electromotive force at adhesion E_{ma} is:

$$E_{ma} = S(\Phi_0 + ai_a)V_t/r \quad (20)$$

where i_s and i_a are changes due to slip in main motor current, respectively, in slipping axles and adhesive axles.

When the number of slipping axles N_s is 3 or less, the following voltage equations hold for a steady state:

$$R_m i_s - R_m i_a = E_{ma} - E_{ms} \quad (21)$$

$$R_r \{4I_0 + N_s i_s + (4 - N_s) i_a\} + R_m (I_0 + i_a) = E_r - E_{ma} \quad (22)$$

Substituting Eqs. (19) and (20) in Eqs. (21) and (22), and considering that the following relation holds:

$$4R_r I_0 + R_m I_0 = E_r - S\Phi_0 V_t/r$$

we obtain the following voltage equations for changes:

$$(R_m + SaV_t/r + Sav_s/r) i_s - (R_m + SaV_t/r) i_a = -S\Phi_0 v_s/r \quad (23)$$

$$N_s R_r i_s + (R_m + SaV_t/r + (4 - N_s) R_r) i_a = 0 \quad (24)$$

If $v_s \ll V_t$, then we can neglect the coefficient

Sav_s/r for i_s on the left side of Eq. (23). Neglecting this, we find that $R_m + SaV_t/r$ can be called the equivalent resistance of motor circuit in cases where unit revolution flux $K\Phi$ is regarded as constant at Φ_0 . Therefore, we shall put:

$$R_{me} = R_m + SaV_t/r \quad (25)$$

From Eqs. (23) and (24), i_s and i_a are given by the following:

$$i_s = -S\Phi_0 v_s / r R_s \quad (26)$$

$$i_a = S\Phi_0 v_s / r R_a \quad (27)$$

where

$$R_s = R_{me} + Sav_s/r + N_s / \{1/R_r + (4 - N_s)/R_{me}\} \quad (28)$$

$$R_a = \{(R_{me} + Sav_s/r) R_{me} / R_r + 4R_{me} + (4 - N_s) Sav_s/r\} / N_s \quad (29)$$

When $N_s = 4$, then the voltage equation at steady state would be:

$$(4R_r + R_m)(I_0 + i_s) = E_r - E_{ms} \quad (30)$$

and for the change:

$$(4R_r + R_m + SaV_t/r + Sav_s/r) i_s = -S\Phi_0 v_s / r \quad (31)$$

Thus,

$$i_s = -S\Phi_0 v_s / r \{4R_r + R_m + SaV_t/r + Sav_s/r\} \quad (32)$$

Wheel circumferential drive forces F_s and F_a respectively for slipping axles and adhesive axles are given by:

$$F_s = S(\Phi_0 + ai_s)(I_0 + i_s)/gr \quad (33)$$

$$\approx S\{\Phi_0 I_0 - S\Phi_0(\Phi_0 + aI_0)v_s/R_r r\}/gr \quad (34)$$

$$F_a = S(\Phi_0 + ai_a)(I_0 + i_a)/gr \quad (35)$$

$$\approx S\{\Phi_0 I_0 + S\Phi_0(\Phi_0 + aI_0)v_s/R_r r\}/gr \quad (36)$$

Equations (33) and (35) show the equilibrium characteristics of F_s and F_a against slip speed.

As an example, the equilibrium characteristics for a case where slip occurs in the Type ED 75 locomotive while traveling at 10 km/hr, as calculated by Eqs. (26) ~ (32), (33) and (35) are given in Fig. 13. It is noted that within the range of a small v_s , linearly approximated Eqs. (34) and (36) hold.

In Fig. 13, letting v_{s0} represent the slip speed at the intersection of curve $v_s - F_s/W$ and curve $v_s - \mu_s$, we find that when v_s is smaller than v_{s0} ,

Type ED 75 locomotive, speed 10 km/hr
 μ_s : Characteristics B
 Fig. 13 Equilibrium characteristics of driving force versus slip speed

wheel revolution is accelerated because the drive force is larger than the friction force, and when v_s is larger than v_{s0} the revolution is decelerated because the drive force is smaller than the frictional force. Therefore, it is found that slip speed converges on v_{s0} and F_a/W converges on the β_0 value for v_{s0} .

For adhesive characteristics, it is desirable that v_{s0} be small, and to prevent slip from being induced, β_0 do not increase beyond the value prior to slip (μ_0). From Eq. (34), we note that the smaller the value of R_s , the steeper the fall of curve $v_s - F_s/W$, so that v_{s0} becomes small. From Eq. (36) it is found that the larger the value of R_a , the gentler the rise of curve $v_s - F_a/W$, so that if v_{s0} is small β_0 becomes small. According to Eqs. (28) and (29), when R_r is not 0, the smaller the values R_r and N_s , the smaller the value of R_s and the larger the value of R_a , so that both v_{s0} and β_0 are reduced and satisfactory adhesive characteristics are obtained. When R_r is zero, from Eq. (28) we get $R_s = R_{me} + SaV_s/r$, so that curve $v_s - F_s/W$ is not affected by N_s and consequently v_{s0} is not affected by N_s . When R_r is zero, from Eq. (29) R_a becomes infinitely large, so that from Eq. (27) we get $i_a = 0$ and wheel circumferential drive force is held constant at the value before slip ($\beta_0 = \mu_0$ constant).

By varying equivalent resistance of power source R_r and number of slipping axles N_s , we obtained v_{s0} and β_0 as shown in Fig. 14. The v_{s0} and β_0 calculated from the theoretical expressions in section 3.3 are plotted in the graph and these values, as may be expected, agree approximately with the values obtained schematically, as described above, from the equilibrium characteristics.

Locomotive speed V_t acts as an equivalent of resistance SaV_t/r inserted in series in the main motor circuit [equivalent resistance of main motor

Except for R_r , values are for Type ED 75 locomotive
Locomotive speed: 10 km/hr μ_s : characteristics B
Fig. 14 Influence of R_r and N_s on V_{s0} and β_0

is given by Eq. (25)], and when main motor flux is constant, i. e. when $a=0$, equivalent resistance of main motor R_{me} is constant at R_m . When there is motor current dependency of flux, i. e. $a>0$, as V_t increases R_{me} increases with it and unless the $v_s - \mu_s$ characteristics change with locomotive speed, v_{s0} also increases. For example, the SaV_t/r for locomotive speed 10 km/hr for the Type ED 75 locomotive is calculated as follows:

$$SaV_t/r = 4.44 \times 3.2 \times 10^{-3} \times 10 / 3.6 \times 0.56 \\ = 0.0705 (\Omega)$$

This value is close to $R_m = 0.085 (\Omega)$.

In the foregoing we have quantitatively explained the cause of changes in slip phenomena due to the number of slipping axles and locomotive speed, and the effects of various factors on converging values.

5. Method of improving adhesive characteristics

To improve adhesive characteristics, v_{s0} should be small and from the viewpoint of preventing induction of slip, β_0 should be equal to or smaller than μ_0 . Here, a few methods of improving adhesive characteristics will be investigated on the basis of the discussion in section 4.2.

5.1 Controlling DC source voltage by feedback of rectifier output DC voltage

In this method, as shown in Fig. 15, rectifier output voltage E_d (Fig. 1) is negatively fed back and compared with reference voltage E_p , and power source voltage E_r is controlled in accordance with the difference. In this way, equivalent resistance of power source R_r can be equivalently made zero. Let us first seek the equilibrium characteristics for such a case.

The voltage equations for a steady state are given by Eqs. (21) and (22) for $N_s \leq 3$, and by Eq. (30) for $N_s = 4$, but the power source voltage E_r to be controlled in this way is not constant but is expressed as follows:

$$E_r = GE_p - G_f E_d = GE_p - G_f \{E_{ms} + R_m(I_0 + i_s)\} \quad \dots\dots\dots (37)$$

E_p : reference voltage, E_d : rectifier output voltage
 E_r : no-load rectifier output voltage
 k_f : gain of E_d detector, G : gain
 T : time constant

s : operator of Laplace transformation

Fig. 15 Block diagram of the system controlling DC source voltage by feedback of rectifier output DC voltage

where $G_f = K_f G$ and E_{ms} is given by Eq. (19)

When $N_s \leq 3$, the voltage equations for current changes i_s and i_a are given as follows, by rearranging Eqs. (19), (21), (22) and (37):

$$(R_{me} + Sav_s/r)i_s - R_{me}i_a = -S\Phi_0 v_s/r \quad (38)$$

$$\{N_s R_r + G_f(R_{me} + Sav_s/r)\}i_s + \{R_{me} + (4 - N_s)R_r\}i_a = -G_f S\Phi_0 v_s/r \quad (39)$$

From Eqs. (38) and (39), we get:

$$i_s = -S\Phi_0 v_s / R_s r \quad (40)$$

$$i_a = S\Phi_0 v_s / R_a r \quad (41)$$

where R_s and R_a are as follows:

$$R_s = R_{me} + Sav_s/r + N_s / \{(1 + G_f)/R_r + (4 - N_s)/R_{me}\} \quad (42)$$

$$R_a = \{(R_{me} + Sav_s/r)R_{me}(1 + G_f)/R_r + 4R_{me} + (4 - N_s)Sav_s/r\} / N_s \quad (43)$$

When $N_s = 4$, current change i_s is expressed as follows, by rearranging Eqs. (19), (30) and (37):

$$i_s = -S\Phi_0 v_s / r \{R_{me} + Sav_s/r + 4R_r/(1 + G_f)\} \quad (44)$$

Comparing Eqs. (42), (43) and (44) with Eqs. (28), (29) and (32) where no control is made, we note that when control is made the R_r in Eqs. (28), (29) and (32) is $R_r/(1 + G_f)$. Therefore, by making gain G_f sufficiently large, $R_r/(1 + G_f)$ approaches zero, so that equivalent resistance of power source becomes equivalently zero.

Since from Eq. (43) we note that $\lim_{G_f \rightarrow \infty} R_a \rightarrow \infty$, from Eq. (41) we get $\lim_{G_f \rightarrow \infty} i_a = 0$. Thus, with a sufficiently large gain G_f , the wheel circumference drive force of an adhesive axle is held constant at the value prior to slip. However, from Eq. (42) we get:

$$\lim_{G_f \rightarrow \infty} R_s = R_{me} + Sav_s/r \quad (45)$$

Therefore, with a large gain G_f , the $v_s - F_s/W$ curve given by Eq. (33) converges on the $v_s - F_s/W$ curve for $R_s = R_{me} + Sav_s/r$ and the converging value of slip speed then is at the lower limit and does not fall below this. However, since the R_{me} on the right side of Eq. (45), as in Eq. (25), has a term that increases in proportion to locomotive speed V , the lower limit of R_s given by Eq. (45) increases with locomotive speed. Consequently, the lower limit value of slip speed convergence v_{s0} also increases.

Next, let us attempt numerical calculations of slip phenomena by the method described in section 3.3 for slip phenomena where such control is performed, taking the time lag in the system into consideration. By approximating the transfer function of the control system with a time lag of first order (Fig. 15) and letting time constant be denoted by T , power source voltage E_r is given by:

$$T \dot{E}_r + E_r = GE_p - G_f E_d \quad (46)$$

$$E_d = L_{m1} \dot{I}_{m1} + R_{m1} I_{m1} + E_{m1} \quad (47)$$

Other theoretical equations are as shown in section 3.3. Figure 16 gives the results of calculation for a case where such control is applied to the Type ED 75 locomotive. Comparing these results with Fig. 12 (no control), we find that the increment in drive force of axles in adhesion is notably reduced, but that readhesion does not occur at 10 km/hr locomotive speed. Therefore, it cannot be said that adhesive characteristics where a locomotive speed is present have been sufficiently improved.

5.2 Controlling DC source voltage by feedback of maximum difference in armature voltages

By controlling power source voltage by negative feedback of maximum difference voltage — i. e. the difference between maximum and minimum values of armature voltages — and comparing it with the reference voltage, we can compensate the voltage drop due to equivalent resistance SaV_i/r resulting from the current dependency of main motor flux. In this way, good readhesive characteristics can be obtained even where a locomotive speed exists.

Let us consider the equilibrium characteristics. Maximum armature voltage $(E_a)_{\max}$ is the armature voltage of the slipping main motor, and is given by the following equation, where the armature's winding resistance is represented by R_0 :

$$(E_a)_{\max} = E_{ms} + R_0(I_0 + i_s) \quad (48)$$

(a) Locomotive speed: zero, μ_s : Characteristics A

(b) Locomotive speed: 10 km/hr, μ_s : Characteristics B
Fig. 16 Results of calculation for controlling DC source voltage by feedback of rectifier output DC voltage (Gain $G_f = 15$, Time constant $T = 1$ sec)

On the other hand, minimum armature voltage $(E_a)_{\min}$ is the armature voltage of a main motor in adhesion and can be expressed as follows:

$$(E_a)_{\min} = E_{ma} + R_0(I_0 + i_a) \quad (49)$$

where E_{ma} and E_{ms} are given by Eqs. (19) and (20). From Fig. 17, the power source voltage E_r in a steady state is given as follows:

$$E_r = GE_p - G_f\{(E_a)_{\max} - (E_a)_{\min}\} \quad (50)$$

When $N_s \leq 3$, the voltage equations for current changes i_s and i_a are derived as follows, by rearranging Eqs. (19), (20), (21), (22), (48), (49) and (50):

$$(R_{me} + Sav_s/r)i_s - R_{me}i_a = -S\Phi_0 v_s/r \quad (51)$$

$$[N_s R_r + G_f\{R_0 + Sa(V_t + v_s)/r\}]i_s + \{(4 - N_s)R_r + R_{me} - G_f(R_0 + SaV_t/r)\}i_a = -G_f S\Phi_0 v_s/r \quad (52)$$

From Eqs. (51) and (52), we get:

$$i_s = -S\Phi_0 v_s / R_s r, \quad i_a = S\Phi_0 v_s / R_a r$$

where R_s and R_a are as follows:

$$R_s = \frac{R_{me}(4R_r + R_{me} + G_f Sav_s/r) + Sav_s\{(4 - N_s)R_r + R_{me} - G_f(R_0 + SaV_t/r)\}/r}{R_r(4 - N_s) + R_{me}(1 + G_f) - G_f(R_0 + SaV_t/r)} \quad (53)$$

$$R_a = \frac{R_{me}(4R_r + R_{me} + G_f Sav_s/r) + Sav_s\{(4 - N_s)R_r + R_{me} - G_f(R_0 + SaV_t/r)\}/r}{R_r(4 - N_s) + G_f(R_0 - R_m)} \quad (54)$$

From Eq. (53), we get: $\lim_{G_f \rightarrow \infty} R_s = Sav_s/r$

and from Eq. (54): $\lim_{G_f \rightarrow \infty} R_a = -Sav_s/r$

Therefore, by making gain G_f sufficiently large, R_s would not be affected by locomotive speed V_t and would assume a sufficiently small value. Similarly, with a large gain, R_a would be equal to $-R_s$, so that the current in adhesive axles would be the same as that in slipping axles, both currents decreasing with slip. Therefore, even when a locomotive speed exists, slip speed can be held down to a small value and induction of slip can be prevented.

Next, let us make calculations for slip phenomena where such control is effected, taking the time lag of the system into consideration. By approximating the control system's transfer function with the time lag of first order, power source voltage E_r is expressed as follows:

$$T\dot{E}_r + E_r = GE_p - G_f\{(E_a)_{\max} - (E_a)_{\min}\} \quad (55)$$

Now, letting the slipping main motor's armature voltage be represented by No. 1 axle's armature voltage E_{a1} , letting the adhesive main motor's armature voltage be represented by No. 4 axle's armature voltage E_{a4} , and denoting the armature winding's inductance by L_0 , we get:

$$(E_a)_{\max} = E_{a1} = L_0 \dot{I}_{m1} + R_0 I_{m1} + E_{m1} \quad (56)$$

$(E_a)_{\max}$: maximum armature voltage
 $(E_a)_{\min}$: minimum armature voltage

Fig. 17 Block diagram of system controlling DC source voltage by feedback of maximum difference in armature voltages

$$(E_a)_{\min} = E_{a4} = L_0 \dot{I}_{m4} + R_0 I_{m4} + E_{m4} \quad (57)$$

Other theoretical expressions are the same as in section 3.3. Figure 18 shows the results of calculations for a case where such control is applied to the Type ED 75 locomotive. It is noted that readhesion can be caused even at a locomotive speed of 30 km/hr.

6. Conclusion

In the foregoing, the adhesive characteristics of rectifier AC locomotives has been discussed. Slip phenomena for a standard four-axle AC locomotive were observed and a few phenomena that could not be explained by conventional simplified theories were clarified, and the relation between wheel-rail relative speed and coefficient of adhesion during slip was established. The author went on to derive detailed theoretical expressions for slip phenomena and compared the calculated results with the measured results, finding that both results agreed. Furthermore, the effects of such factors as equivalent resistance of power source, number of slipping axles, motor current dependency of flux, and locomotive speed, which had not been taken up in previous studies, were investigated, and the following methods

Fig. 18 Results of calculation for controlling DC source voltage by feedback of maximum difference in armature voltages
 (Gain $G_f = 15$, Time constant $T = 1$ sec)

of improving adhesive characteristics were worked out and their validity was examined:

(1) Controlling DC source voltage by feedback of rectifier output DC voltage

(2) Controlling DC source voltage by feedback of maximum difference in armature voltages

In closing, the author wishes to express his deep gratitude to Mr. Irie, formerly an engineer with the Japanese National Railways Design Office, Mr. Kawazoe, engineer with the same office; Mr. Kimura, engineer with the Railway Technology Research

Institute; and staff members of the Mito Works, Hitachi, Ltd. and of the Mito branch of Hitachi Research Laboratory, for their valuable guidance and generous assistance.

References

- (1) Seki et al.: *Jour. Inst. Elect. Engrs. Japan*, Vol. 80, No. 5 (1960-5), p. 36.
- (2) Onoda et al.: *Bulletin of JSME*, Vol. 8, No. 32 (1965-1), p. 584.
- (3) Hirotsu and Onoda: *Proc. 13th Japan Nat. Congr. Appl. Mech.* 1963, (1965). p. 285 ~ 289.
- (4) J. Metzkow: *El. Bahnen*, Bd. 16, Nr. 12 (1940).