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On the Tool Wear in the Intermittent Cutting of the Reinforced Plastics*

(2nd Report, Rheological Study of the Action of the Abrasive Reinforcement on the Tool Wear)

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The object of this paper is to give a rheological explanation for the peculiar behavior of the tool wear which appears in cutting fibrous glass reinforced plastics.

As a result of this theoretical analysis, it became clear that the deformation speed of the work, the contact area between the work and the tool, the flexural rigidity of the fibrous glass and visco-elasticity of plastics have influence on the tool wear.

Using this result, the effects of the conditions such as cutting speed and chip thickness on the tool wear were considered.

1. Introduction

The first report⁽¹⁾ made it clear that in intermittent cutting of fibrous glass reinforced phenolic resin or phenolic resin containing aluminum oxide, a rapid tool wear occurs and that the effect of cutting conditions on the tool wear is extremely different between these two materials. Accordingly, it can be guessed that the properties of the reinforcement in resin affect the tool wear greatly, which was also mentioned in the first report.

In the intermittent cutting, the contact of the tool with the work and the release from the contact are repeated and therefore the heating and cooling of the tool and also the deformation and the delayed elastic recovery of the work are repeated.

As shown in the first report, the heating and cooling of the tool scarcely affects the tool wear due to the low cutting temperature. The delayed elastic recovery affects the true chip thickness, but for the peripheral cutting edge the ratio of this recovery to the true chip thickness is small, so the effect of this recovery on the wear land of peripheral flank is also small.

In this report, to make the effect of the reinforcement on the tool wear more clear, the behavior of the reinforcement combined with plastics which has visco-elastic property is theoretically considered

with respect to the contact speed and area between tool and work.

2. Case of phenolic resin containing aluminum oxide

This material causes much greater tool wear than the common phenolic resin. When the tool wear in cutting the phenolic resin containing aluminum oxide is to be studied, the tool wear in cutting the resin itself may be disregarded, and therefore, only the situation under which the cutting edge is in contact with aluminum oxide grains has to be considered.

As mentioned in the first report, the cutting temperature is not high enough for the adhesion or diffusion between the aluminum oxide grains and the tool to occur. Therefore, the tool wear of this case is thought to be an abrasive wear. In case of the abrasive wear, the wear is given by the following equation⁽²⁾.

$$Q = A \tan \theta WL/H \dots\dots\dots (1)$$

where Q : volume of material removed, A : constant, θ : constant (depends upon the surface roughness), W : normal load, L : sliding distance, H : hardness of material.

Equation (1) can be rewritten in the following form.

$$h = A \tan \theta wL/H \dots\dots\dots (2)$$

where h : width of the wear region, w : normal force intensity.

In case of the same combination of materials,

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the width of the wear region is dependent on the normal force intensity and the sliding distance. The contact pressure between the aluminum oxide grains and the tool in cutting phenolic resin containing aluminum oxide is considered.

The deformation of the work, which occurs when the tool comes in contact with the aluminum oxide grains, is chiefly caused by the deformation of the resin. Accordingly, the aluminum oxide grains supported by the resin is schematically shown as Fig. 1.

The factors such as the grain size, the distribution and the adhering condition of aluminum oxide with resin affect such things as the number of times of tool encounter with the aluminum oxide grains, spring constant and coefficient of viscosity per one grain, the strength of aluminum oxide grain, and adhering strength of aluminum oxide with resin. These effects are considered as follows:

If the aluminum oxide grain fractures in encountering with the tool, the contact pressure between the tool and the aluminum oxide grain is dependent on its strength. If it does not fracture, it is discharged as chip or pushed into resin due to fracture of resin which supports it. Therefore, in this case the contact pressure depends upon the strength of resin. If the strength of the aluminum oxide grain and that of the resin does not change with cutting conditions, the contact pressure becomes constant independently of the cutting conditions and is not affected by the spring constant and the coefficient of viscosity per one grain. Accordingly, if the contact length of the tool with the work is made constant, the tool wear becomes constant as shown in Eq. (2).

As the experimental results of the first report virtually show such tendency, it is considered that the above fact is correct.

3. Case of fibrous glass reinforced phenolic resin

As this material also causes greater tool wear than the common phenolic resin, only the tool wear which occurs while the tool is in contact with the fibrous glass has to be considered.

Fig. 1 Schematically indicated situation of aluminum oxide grain in resin

Fig. 2 Schematically indicated situation of fibrous glass in resin (distributed load)

As explained in the first report, the cutting temperature is low under the cutting speed less than 500 m/min. So, it is taken for granted that the adhesion or diffusion between the tool and the fibrous glass does not occur, and that the tool wear is an abrasive wear. As in the case of phenolic resin containing aluminum oxide, the work is replaced with a model and the contact pressure between the tool and the fibrous glass in cutting fibrous glass reinforced phenolic resin is considered.

Now, there are some macroscopic models about FRP.

(i) The model composed of filaments and resin^{(3) (4)}: Both the filaments which are parallel or normal to an applied force and the resin support the applied force corresponding to the ratio of these elements.

(ii) The net model⁽⁵⁾: This model is used for analysis of the pressure vessel. The resin does not support the load at all but only the filaments do.

These models are effective for analysis of the construction made of FRP but can not be used for analysis of the local behavior of FRP, for example, analysis of a cutting phenomenon.

Fibrous glass is microscopically a beam having flexural rigidity. The fibrous glass in resin does not exist at enough intervals nor are they straight. But if the load is applied to the fibrous glass vertically to its axis, the flexural rigidity of the fibrous glass, and elasticity and viscosity of resin which supports the fibrous glass, ought to affect the deflection and fracture of the fibrous glass. One fibrous glass is schematically shown in Fig. 2. It is supported by spring and dash-pot continuously along its axis.

In cutting fibrous glass reinforced plastics, the cutting edge encounters the fibrous glass ceaselessly, and some of these are discharged as chips and others are pushed under the clearance face. In the former case, both fibrous glass and the resin are fractured.

The factors such as the thickness, the distribution of fibrous glass, and its adhering condition with resin affect the following: the number of times of the tool encounter with fibrous glass, the schematically considered spring constant and coefficient of viscosity, the adhesive strength, and also the strength or the flexural rigidity of the fibrous glass. These effects are considered below:

In cutting the fibrous glass reinforced phenolic resin in parallel with the laminate, it is apparent from the great tool wear that the contact pressure between the corner of the cutting edge and the fibrous glass which is parallel to the cutting edge is higher than in other portions. However, in order to make the study easy, the contact pressure is made constant along the contact length (Fig. 2), and if the tool is in contact with the fibrous glass over

a certain length, it is treated in a similar way even in case of the fibrous glass existing in parallel with the cutting direction. Also, though the action of the spring and the dash-pot on the fibrous glass is really three-dimensional, here provisionally it is treated one-dimensionally as shown in Fig. 2.

3.1 Study based on one-dimensional model

The notations are as follows:

- k : spring constant per unit length of fibrous glass kg/m/m
- η : coefficient of viscosity per unit length of fibrous glass kg sec/m/m
- v : the deflection speed of fibrous glass at the point on which the load acts m/sec
- E : modulus of longitudinal elasticity of fibrous glass kg/m²
- I : geometrical moment of inertia of fibrous glass m⁴
- t : time sec
- p : applied load per unit length of fibrous glass kg/m
- l : width of the applied load m
- δ : deflection of fibrous glass m

The fibrous glass can be regarded as a perfectly elastic body which does not deform plastically until it fractures⁽⁶⁾.

The coordinate is established as shown in Fig. 2. Using "Beam Theory", the equilibrium of the distributed load which acts along the fibrous glass is considered.

At first the case is studied assuming that the applied load is a concentrated one (Fig. 3). Supposing that the load " P " is applied along the y -axis at a certain speed " v ", and that the inertia force of fibrous glass is neglected, the point on the fibrous glass bends at a certain speed, and the resistant force which is proportional to this speed acts at this point. The deflection speed at $x=0$ is v and if the fibrous glass bends smoothly, the deflection speed at $x=x$ is $v\delta/\delta_{x=0}$. Accordingly the equilibrium of the distributed load is as follows:

$$EI \frac{d^4 \delta}{dx^4} = -k\delta - \eta v \frac{\delta}{\delta_{x=0}} = -\left(k + \frac{\eta v}{\delta_0}\right) \delta \quad \dots\dots\dots (3)$$

where

$$\delta_0 \equiv \int_0^t v dt \quad \dots\dots\dots (4)$$

This equation takes the following solution.

$$\delta = e^{ax}(c_1 \cos ax + c_2 \sin ax) + e^{-ax}(c_3 \cos ax + c_4 \sin ax) \quad \dots\dots\dots (5)$$

where " a " is defined as the following equation, and $c_1 \sim c_4$ are the constants of integration.

$$4a^4 \equiv \frac{k + (\eta v / \delta_0)}{EI} \quad \dots\dots\dots (6)$$

$$= \frac{k + (\eta / t)}{EI} \quad \dots\dots\dots (6)'$$

Using the following boundary conditions, Eq. (5) becomes Eq. (7).

$$x = \infty; \quad \delta = 0$$

$$x = 0; \quad d\delta/dx = 0, \quad \delta = \delta_0(t)$$

$$\delta = \delta_0(t)e^{-ax}(\cos ax + \sin ax) \quad \dots\dots\dots (7)$$

The shearing force which occurs in the beam is given by the following equation.

$$V = -EI(d^3\delta/dx^3) \quad \dots\dots\dots (8)$$

$$= -4EI\delta_0(t)a^3e^{-ax}\cos ax \quad \dots\dots\dots (9)$$

Since the shearing force for the right part of the beam is equal to $-P/2$, the concentrated load at time $t=t$ is given by the following equation.

$$P(t) = -8EI\delta_0(t)a^3 \quad \dots\dots\dots (10)$$

So, Eq. (7) becomes (7)'.

$$\delta = -\frac{P(t)}{8EIa^3}e^{-ax}(\cos ax + \sin ax) \quad \dots\dots\dots (7)'$$

In case of a uniform load (Fig. 2), the deflection at $x=x$ is obtained by integrating Eq. (7)', whose $P(t)$ is substituted by $p(t)dx$, from zero to both ends of distributed load. The extent of $x > 0$ is all that has to be considered because of the symmetry of the distributed load. It is obtained by the following equation for the point within the distributed load.

$$\begin{aligned} \delta_p &= -\frac{p(t)}{8EIa^3} \left[\int_0^{(l/2)+x} e^{-ax}(\cos ax + \sin ax) dx \right. \\ &\quad \left. + \int_0^{(l/2)-x} e^{-ax}(\cos ax + \sin ax) dx \right] \\ &= -\frac{p(t)}{8EIa^4} \left[2 - e^{-a[(l/2)+x]} \cos a\left(\frac{l}{2}+x\right) \right. \\ &\quad \left. + e^{-a[(l/2)-x]} \cos a\left(\frac{l}{2}-x\right) \right] \quad \dots\dots\dots (11) \end{aligned}$$

The shearing force is given by the following equation.

$$\begin{aligned} V_{p[|x| \leq (l/2)]} &= \frac{p(t)}{4a} \left[e^{-a[(l/2)+x]} \left\{ \cos a\left(\frac{l}{2}+x\right) \right. \right. \\ &\quad \left. \left. - \sin a\left(\frac{l}{2}+x\right) \right\} - e^{-a[(l/2)-x]} \left\{ \cos a\left(\frac{l}{2}-x\right) \right. \right. \\ &\quad \left. \left. - \sin a\left(\frac{l}{2}-x\right) \right\} \right] \quad \dots\dots\dots (12) \end{aligned}$$

The shearing force at the point without the distributed load is obtained by substituting $x = (l/2)$ for $(l/2) - x$ in Eq. (12). That is,

$$\begin{aligned} V_{p[x > (l/2)]} &= \frac{p(t)}{4a} \left[e^{-a[x+(l/2)]} \right. \\ &\quad \times \left\{ \cos a\left(x+\frac{l}{2}\right) - \sin a\left(x+\frac{l}{2}\right) \right\} \\ &\quad \left. - e^{-a[x-(l/2)]} \left\{ \cos a\left(x-\frac{l}{2}\right) - \sin a\left(x-\frac{l}{2}\right) \right\} \right] \quad \dots\dots\dots (12)' \end{aligned}$$

Fig. 3 Schematically indicated situation of fibrous glass in resin (concentrated load)

Equations (12) and (12)' become maximum at about $x=l/2$. And if (la) becomes a little large, the maximum shearing force is approximately as follows:

$$|(V_p)_{\max}| = \frac{p(t)}{4a}$$

It is supposed that the fibrous glass fractures when this maximum shearing force reaches the shearing strength of fibrous glass K . That is,

$$K = |(V_p)_{\max}| = \frac{p(t)}{4a} \dots\dots\dots (13)$$

Now, the fibrous glass used as the reinforcement is $5 \sim 13 \mu$ thick. According to the bending test⁽⁷⁾ it does not fracture until the radius of curvature becomes $50 \sim 200 \mu$.

Suppose the deflection speed of the point which has the maximum shearing force is v and also v is constant, from Eqs. (4) and (11) the following equation is obtained.

$$vt = p(t)/8EIa^4 \dots\dots\dots (14)$$

From Eqs. (6)', (13) and (14), the following equation is made, and its root shows the amount of time required for the fracture of fibrous glass.

$$v^4 k^3 t_0^4 + 3v^4 k^2 \eta t_0^3 + 3v^4 k \eta^2 t_0^2 + v^4 \eta^3 t_0 = \frac{K^4}{2EI} \dots\dots\dots (15)$$

If v is very small, the 2nd $\sim$ 4th terms of the left hand side can be ignored after comparing them with the first term, and " t_0 " has the following approximation.

$$t_0 = \frac{K}{(2EI)^{1/4} v k^{3/4}} \dots\dots\dots (16)$$

If v becomes pretty large and the only one root of Eq. (15), which is real and positive, is far smaller than 1, the 1st and 2nd term of left hand side can be ignored in comparison with 3rd and 4th term. Then t_0 is given by Eq. (17).

$$t_0 = \frac{-\eta^3 v^4 + \sqrt{\eta^6 v^8 + 6K^4 k \eta^2 v^4 / EI}}{6k \eta^2 v^4} \dots\dots\dots (17)$$

By substituting Eq. (16) or (17) for " t " which is in " a " of Eq. (13), the following equations are obtained.

For small v

$$p(t_0) = 4K \sqrt[4]{\frac{k + \eta(2EI)^{1/4} v k^{3/4} / K}{4EI}} \dots\dots\dots (18)$$

For large v

$$p(t_0) = 2 \sqrt[4]{\frac{4kK^4}{EI} + 4v^4 \eta^4 + 4 \left(v^8 \eta^8 + \frac{6v^4 k \eta^4 K^4}{EI} \right)^{1/2}} \dots\dots\dots (19)$$

k , η , I and v in these equations are variables.

Even if the shearing strength K is constant, the intensity of the distributed load which is applied at the time of the fracture of fibrous glass changes depending upon these variables.

The effect of " v " on $p(t_0)$ is very different for these two equations. For small " v ", the power of

" v " in equation $p(t_0)$ is one-fourth, and for large v , $p(t_0)$ is approximately proportional to " v ".

3.2 Qualitative correction of one-dimensional model

In section 3.1 the study was based on the one-dimensional model. Since the action of the spring and the dash-pot is actually three-dimensional, the real phenomenon is slightly different from that of one-dimensional model. In this section the difference between one- and two-dimensional models is considered, using the theory of two-dimensional elasticity, and then from this the actual situation is analogized.

Following is the study of a two-dimensional semi-infinite body. The coordinate is defined as Fig. 4.

According to the theory of elasticity, in case of a uniform load " q " distributed over a width Δz , the deflection (v_0) at a certain point ($z=z$) within the distributed load is obtained from the following equation⁽⁸⁾.

$$v_0 = \frac{2q}{\pi E} \left[\left(\frac{\Delta z}{2} + z \right) \log_e \frac{C}{\Delta z/2 + z} + \left(\frac{\Delta z}{2} - z \right) \log_e \frac{C}{\Delta z/2 - z} \right] + \frac{1-\nu}{\pi E} q \Delta z \dots\dots\dots (20)$$

where, E : modulus of longitudinal elasticity of semi-infinite body, C : constant, ν : Poisson's ratio.

Suppose the fibrous glass is supported on the horizontal boundary of semi-infinite body, and also it bends after the deflection curve which Eq. (20) gives. Then the shearing force distribution on the fibrous glass is obtained by differentiating Eq. (20) three times with z . That is,

$$\frac{\partial^3 v_0}{\partial z^3} = \frac{2q}{\pi E} \left[\frac{1}{(\Delta z/2 + z)^2} - \frac{1}{(\Delta z/2 - z)^2} \right] \dots\dots\dots (21)$$

The shearing force becomes infinite at $z = \pm \Delta z/2$, but it is not zero on other portions and it varies depending upon the width Δz . It is believed that the shearing force at $z = \pm \Delta z/2$ is actually finite and that this force is also affected by the width Δz as well as the force on other portions. Now, the effect of the width Δz on the shearing force is examined at the arbitrary point $z = k \Delta z$ (where k is a parameter and $-1/2 < k < 1/2$) within the distributed load.

Equation (21) can be rewritten as follows:

Fig. 4 Two-dimensional model of distributed load on semi-infinite body

$$\left(\frac{\partial^3 v_0}{\partial z^3}\right)_{z=\pm \Delta z} = \frac{2q}{\pi E} \left(\frac{1}{\Delta z}\right)^2 \left[\frac{1}{(1/2+k)^2} - \frac{1}{(1/2-k)^2} \right] \dots\dots\dots(21)'$$

$$\frac{\partial}{\partial(\Delta z)} \left(\frac{\partial^3 v_0}{\partial z^3} \right)_{z=\pm \Delta z} = \frac{4q}{\pi E} \left(-\frac{1}{\Delta z} \right)^3 \left[\frac{1}{(1/2+k)^2} - \frac{1}{(1/2-k)^2} \right] \dots\dots\dots(22)$$

Equations (21)' and (22) show that the absolute value of shearing force of fibrous glass decreases with the increase of the width Δz .

In case of one-dimensional model as shown in Fig. 5, the shearing force of fibrous glass at $z = \pm \Delta z/2$ should become infinite and on other portions zero according to the assumption. However, owing to the existence of fibrous glass itself, the shearing force at $z = \pm \Delta z/2$ becomes finite but it does not depend upon the width Δz .

These are the differences between one- and two-dimensional models.

From above facts, it may be concluded that in practice as the width of the distributed load increases, there is a decrease of the shearing force in the fibrous glass due to the deformation of the body which supports the fibrous glass. And it may also be concluded that on the contrary, in order to break the fibrous glass it is necessary to increase the intensity of the distributed load with the increase of the extent of the distributed load.

Above study is based on the theory of elasticity but as for the deformation based on viscosity, the same conclusion may be obtained by the principle of viscosity-elasticity analogy⁽⁹⁾. In case of the deformation based on viscosity, however, the increase of the width of the distributed load requires also the increase of its intensity in order to break the fibrous glass within the same period.

Therefore, the intensity of the distributed load to break the fibrous glass which is supported by a visco-elastic body within a certain period is dependent upon the width of the distributed load.

4. Application of the theory to the cutting process of fibrous glass reinforced plastics

(i) Effect of cutting speed on the tool wear
The deforming speed of the work depends upon

Fig. 5 One-dimensional deformation model of semi-infinite body caused by distributed load

the cutting speed. For the great deforming speed such as cutting process, Eq. (19) is applicable.

As the deflection speed of the fibrous glass increases with cutting speed, the contact pressure between the cutting edge and the fibrous glass increases. So, a high cutting speed causes greater tool wear than a low cutting speed. As to the abrasive wear, according to Eq. (2) the wear land is proportional to the contact pressure and does not depend on the sliding speed. If the deflection speed is proportional to the cutting speed, the curve expressed by Eq. (19) does not only show the relation between the contact pressure and the deflection speed, but also the relation between the wear land and the cutting speed. The experimental curve (Fig. 6) shown in the 1st report indicates the same tendency as the theoretical curve (Fig. 7) which corresponds to Eq. (19).

(ii) Effect of chip thickness on the tool wear

As the chip thickness increases, the contact area of the tool with the work increases. According to

Feed rate: 0.38 mm/rev, Radial rake angle: 10°, Peripheral clearance angle: 13°, Tool: K-10 carbide tool, Dry cutting, Work material: fibrous glass reinforced phenolic resin.

Fig. 6 Relation between the tool wear and the cutting speed in milling

Fig. 7 Theoretical relation between the contact pressure and the deflection speed of fibrous glass

the study in section 3.2, the increase of chip thickness causes the increase of contact pressure between the cutting edge and the fibrous glass. And therefore it causes great tool wear.

5. Conclusions

The main results obtained from this research are as follows:

In cutting fibrous glass reinforced plastics, peculiar phenomena occur because of the flexural rigidity of fibrous glass and the rheological property of plastics.

(1) Under such cutting condition that yields higher deflection speed of fibrous glass, the contact pressure between the tool and the fibrous glass increases and therefore the tool wear becomes great.

(2) Under the cutting condition which yields

a larger contact area of the tool with the work, the contact pressure increases and therefore the tool wear becomes great.

References

- (1) M. Tsueda et al.: This Bulletin, p. 601.
- (2) R. T. Spurr and T. P. Newcomb: *Inst. Mech. Engr. Proc. Conf. on Lubrication and Wear*, (1957), p. 269.
- (3) B. W. Shaffer: *AIAA Jour.*, Vol. 2, No. 2 (1964), p. 348.
- (4) N. Fried: *Mod. Plast.*, Vol. 41, No. 1 (1963), p. 155.
- (5) W. S. Read: *Trans. ASME*, Ser. B, Vol. 85, No. 1 (1963), p. 115.
- (6) K. Hisano and M. Munakata: *Garasusen'i to Sonoohyoo (Fibrous Glass and Its Application)*, (1955), p. 25, Ohm. (in Japanese)
- (7) K. Hisano and M. Munakata: *Garasusen'i to Sonoohyoo (Fibrous Glass and Its Application)*, (1955), p. 29, Ohm. (in Japanese)
- (8) Timoshenko and Goodier: *Theory of Elasticity*, (1951), p. 96, McGraw-Hill.
- (9) T. Nakagawa and H. Kanbe: *Reorogih (Rheology)*, (1959), p. 305, Misuzu Shobo. (in Japanese)