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Crystallographic Study of Yield Condition of Polycrystalline Metals*

(3rd Report, Relation between Slip in Crystals and Yield Condition)

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In order to reveal the relation between the mathematical yield condition and the number of slip planes or slip directions of a crystal, the yield stress ratio τ/σ of torsion to tension is calculated for several crystals characterized by its numbers of slip planes and slip directions, adopting Sachs model of polycrystalline aggregates. It is shown that the value τ/σ is most affected by the minor number n_m of slip planes and slip directions, and that the value τ/σ becomes smaller with the increase in n_m , until it reaches 0.50 corresponding to the crystal in which slip occurs on any plane in any direction.

It is also shown that the calculated value τ/σ is equal in fcc and bcc metals. However, in the case of fcc metal, the optional motion of screw dislocation is expected to reduce the value τ/σ slightly.

1. Introduction

In the previous papers^{(1) (2)}, the yield condition of polycrystals was discussed in relation to the slip in crystals, adopting the graphical method of calculation. It was suggested that the yield condition is dependent on the numbers of slip planes and slip directions in the crystal. To make this point clear, more generalized discussions on the analytical basis are given in the present study. That is, several examples are considered at first, then the analysis is applied to the general case.

As representative of the yield condition, the stress ratio τ/σ of torsion to tension will be adopted in the following. Postulating the parallel model of polycrystals proposed by G. Sachs⁽³⁾, τ/σ is given by^{(1) (2)},

$$\frac{\tau}{\sigma} = \frac{\tilde{\mu}_{\text{Ten}}}{\tilde{\mu}_{\text{Tor}}} \quad (1)$$

where μ_{Ten} and μ_{Tor} are the Schmid factors in tension and torsion, respectively, and $\tilde{\mu}_{\text{Ten}}$ and $\tilde{\mu}_{\text{Tor}}$ are their mean values for all orientations of grains. In order to obtain the mean value of the Schmid factor, it is convenient to use the integral method introduced previously⁽²⁾.

2. Yield condition of the crystals having simple slip systems

A few specific examples will serve as clues to elucidation of the problem. The yield condition of some imaginary crystals having mathematically simple slip systems is discussed before commencing the generalized argument.

2.1 Crystal having one slip plane and one slip direction

The Schmid factor for tension is given by

$$\mu_{\text{Ten}} = \sin \theta_0 \cos \theta_0 \cos \alpha \quad (2)$$

where the angles θ_0 and α are defined in Fig. 1. To obtain the value $\tilde{\mu}_{\text{Ten}}$ for all crystal orientations, it is necessary to integrate μ_{Ten} with respect to both α and θ_0 ⁽²⁾, after the multiplication of the weight $\sin \theta_0$ to keep the uniform distribution of orientation. The range of integrals by α and θ_0 is taken from 0 to $\pi/2$,

$$\begin{aligned} \tilde{\mu}_{\text{Ten}} &= \frac{\int_0^{\pi/2} \int_0^{\pi/2} \sin \theta_0 \mu_{\text{Ten}} d\alpha d\theta_0}{\int_0^{\pi/2} \int_0^{\pi/2} \sin \theta_0 d\alpha d\theta_0} \\ &= \frac{2}{3\pi} = 0.2122 \quad (3) \end{aligned}$$

On the other hand, the Schmid factor for torsion is given by⁽²⁾,

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$$\mu_{\text{Tor}} = \cos 2\theta_0 \cos \alpha \sin \lambda + \cos \theta_0 \sin \alpha \cos \lambda \dots (4)$$

where λ is the angular position of a grain in the cylindrical specimen, which also indicates the relative direction of the stress axis and the crystal axis (Fig. 2). The mean value $\tilde{\mu}_{\text{Tor}}$ is obtained by integrating μ_{Tor} with respect to λ , α and θ_0 , after multiplication of $\sin^2 \theta_0 \cos \theta_0$, where $\sin \theta_0 \cos \theta_0$ and $\sin \theta_0$ are the weights in the integrals with λ and α , respectively⁽²⁾.

In the case of the crystal having one slip direction, the range of integration by α is given by $[\alpha_1, \alpha_1 + \pi]$, for μ_{Tor} is the sine function of period 2π (Fig. 4). In order to get the maximum value of $\tilde{\mu}_{\text{Tor}}$ under the condition of $\mu_{\text{Tor}} \geq 0$, the integral range of α has to be chosen as $\mu_{\text{Tor}}(\alpha_1) = \mu_{\text{Tor}}(\alpha_1 + \pi) = 0$. Then,

$$\int_{\alpha_1}^{\alpha_1 + \pi} \mu_{\text{Tor}} d\alpha = 2\sqrt{\cos^2 2\theta_0 \sin^2 \lambda + \cos^2 \theta_0 \cos^2 \lambda} \dots (5)$$

Fig. 1 Angular relationship between crystal axes and stress axes in tension

Fig. 2 Angular relationship between crystal axes and stress axes in tension

$$\tilde{\mu}_{\text{Tor}} = \frac{\int_0^{\pi/2} \int_0^{\pi/2} \int_{\alpha_1}^{\alpha_1 + \pi} \sin^2 \theta_0 \cos \theta_0 \mu_{\text{Tor}} d\alpha d\lambda d\theta_0}{\int_0^{\pi/2} \int_0^{\pi/2} \int_{\alpha_1}^{\alpha_1 + \pi} \sin^2 \theta_0 \cos \theta_0 d\alpha d\lambda d\theta_0} = 12I(\pi/2, \pi/2)/\pi^2 \dots (6)$$

where

$$I(\theta', \lambda') = \int_0^{\theta'} \int_0^{\lambda'} \sin^2 \theta_0 \cos \theta_0 \sqrt{\cos^2 2\theta_0 \sin^2 \lambda + \cos^2 \theta_0 \cos^2 \lambda} d\lambda d\theta_0 \dots (7)$$

After numerical calculation,

$$I(\pi/2, \pi/2) = 0.2957 \dots (8)$$

then,

$$\tilde{\mu}_{\text{Tor}} = 0.3596 \dots (9)$$

Substituting Eqs. (3) and (9) into Eq. (1), the yield stress ratio for the present crystal is given by,

$$\frac{\tau}{\sigma} = \frac{\tilde{\mu}_{\text{Ten}}}{\tilde{\mu}_{\text{Tor}}} = \frac{\pi}{18I(\pi/2, \pi/2)} = 0.5902 \dots (10)$$

It is worth noting that the Miller indices of the slip plane or the slip direction are not necessary in the process of calculation.

2.2 Crystal having one slip plane and arbitrary slip direction

If a slip occurs in arbitrary direction, the value of α in tension has to be so chosen as to make μ_{Ten} maximum, that is, $(\cos \alpha)_{\text{max}} = 1$. Thus,

$$\tilde{\mu}_{\text{Ten}} = \frac{\int_0^{\pi/2} \sin^2 \theta_0 \cos \theta_0 d\theta_0}{\int_0^{\pi/2} \sin \theta_0 d\theta_0} = 1/3 \dots (11)$$

Similarly in torsion, the value α_0 at which μ_{Tor} takes maximum value is obtained from $\partial \mu_{\text{Tor}} / \partial \alpha = 0$. Substituting the resultant value of α_0 into Eq. (4),

$$[\mu_{\text{Tor}}]_{\alpha_0} = \sqrt{\cos^2 2\theta_0 \sin^2 \lambda + \cos^2 \theta_0 \cos^2 \lambda} \dots (12)$$

then,

$$\tilde{\mu}_{\text{Tor}} = \frac{\int_0^{\pi/2} \int_0^{\pi/2} \sin^2 \theta_0 \cos \theta_0 [\mu_{\text{Tor}}]_{\alpha_0} d\lambda d\theta_0}{\int_0^{\pi/2} \int_0^{\pi/2} \sin^2 \theta_0 \cos \theta_0 d\lambda d\theta_0} = \frac{6}{\pi} I(\pi/2, \pi/2) = 0.5648 \dots (13)$$

Hence,

$$\frac{\tau}{\sigma} = \frac{\tilde{\mu}_{\text{Ten}}}{\tilde{\mu}_{\text{Tor}}} = \frac{\pi}{18I(\pi/2, \pi/2)} = 0.5902 \dots (14)$$

The value of $\tilde{\mu}_{\text{Ten}}$ or $\tilde{\mu}_{\text{Tor}}$ is independent of the exchange of the slip plane normal and the slip direction, as described later [Eq. (18)]. Therefore, the result of Eq. (14) is also applicable to the crystal having one slip direction and arbitrary slip plane.

The numbers of slip planes and slip directions and the corresponding values of τ/σ are listed in Table 1. It is interesting that the value of τ/σ is 0.590 in so far as the crystal has one slip plane or

one slip direction. Further discussion on this point will be made later.

2.3 Crystal having arbitrary slip plane and arbitrary slip direction

In this case it is necessary to take such values of α and θ_0 as to make $\tilde{\mu}_{\text{Ten}}$ or $\tilde{\mu}_{\text{Tor}}$ maximum. In tension, substituting

$$(\cos \alpha)_{\max} = 1, \quad (\sin \theta_0 \cos \theta_0)_{\max} = 1/2$$

into Eq. (2), we obtain,

$$\tilde{\mu}_{\text{Ten}} = 1/2 \quad \dots\dots\dots (15)$$

In torsion, the maximum value is given from Eq. (12):

$$[\mu_{\text{Tor}}]_{\alpha_0, \theta_0=0} = 1$$

Hence,

$$\tilde{\mu}_{\text{Tor}} = 1 \quad \dots\dots\dots (16)$$

Consequently,

$$\frac{\tau}{\sigma} = \frac{\tilde{\mu}_{\text{Ten}}}{\tilde{\mu}_{\text{Tor}}} = 0.500 \quad \dots\dots\dots (17)$$

It is natural that the result is in accordance with the value reduced from the Tresca's law or the maximum shear stress theory, for the basic assumptions are identical in both ideas.

3. Generalized relations

It is necessary to systematize the preliminary illustration mentioned above into a more general form. The shear stress τ_s acting on the slip plane in the slip direction under the principal stresses σ_1 , σ_2 and σ_3 is given by⁽⁴⁾:

$$\tau_s = \sigma_1 \cos \phi_1 \cos \phi_1' + \sigma_2 \cos \phi_2 \cos \phi_2' + \sigma_3 \cos \phi_3 \cos \phi_3' \quad \dots\dots\dots (18)$$

where ϕ_i and ϕ_i' ($i=1, 2, 3$) are the angles between the slip plane normal or the slip direction and the stress axes. It follows from Eq. (18) that the value of τ_s remains unchanged by the exchange of ϕ_i and ϕ_i' that is, the slip plane and the slip direction.

For the convenience of the following treatment, it is advantageous to express the one which has the major number between the slip plane and the slip direction by M , while the one having the minor number by m . Let's denote the elements of M and m by M_i ($i=1, \dots, n_M$) and m_i ($i=1, \dots, n_m$)

Table 1 Relation between the numbers of slip planes and slip directions and the yield stress ratio

Number of slip planes	1	1*	1	∞	∞
Number of slip directions	1	3	∞	1	∞
$\tilde{\mu}_{\text{Ten}}$	0.212	0.318	0.333	0.333	0.500
$\tilde{\mu}_{\text{Tor}}$	0.360	0.539	0.565	0.565	1.000
$\frac{\tau}{\sigma} = \frac{\tilde{\mu}_{\text{Ten}}}{\tilde{\mu}_{\text{Tor}}}$	0.590	0.590	0.590	0.590	0.500

* hcp metal

respectively, where n_M and n_m are the numbers of M and m .

$$\left. \begin{aligned} n_M &= \max(n_{sp}, n_{sd}) \\ n_m &= \min(n_{sp}, n_{sd}) \end{aligned} \right\} \quad \dots\dots\dots (19)$$

where n_{sp} and n_{sd} are the numbers of slip planes and slip directions, respectively. If $n_{sp} \neq n_{sd}$, $n_M > n_m$, that is, several components of M correspond to one component of m . It is obvious that, the integral by α of Eqs. (3) or (6) is related with M , while the integral by θ with m .

3.1 Influence of M (integration by α)

It is expected from Eqs. (3) and (5) that, the value τ/σ is affected by the numbers of slip planes and slip directions and the relative angular relation between the components through the range of integrals. Let's denote the integral range by α with $2\Delta\alpha$, which corresponds to the range represented by a single component of M .

In tension, it is sufficient to choose the integral range as $(-\Delta\alpha, \Delta\alpha)$ to make $\tilde{\mu}_{\text{Ten}}$ maximum, for μ_{Ten} is a function of $\cos \alpha$ [Eq. (2)]. In torsion, Eq. (4) is rewritten as follows:

$$\mu_{\text{Tor}} = A(\theta, \lambda) \sin(\alpha + \omega) \quad \dots\dots\dots (20)$$

$$A(\theta, \lambda) = \sqrt{\cos^2 2\theta \sin^2 \lambda + \cos^2 \theta \cos^2 \lambda} \quad \dots\dots\dots (21)$$

where ω is a function of θ and λ . In order to make $\tilde{\mu}_{\text{Tor}}$ maximum, it is necessary to choose the integral range of $(\alpha + \omega)$ as $(\pi/2 - \Delta\alpha, \pi/2 + \Delta\alpha)$.

It is shown that the results of integral with respect to α are equal in $\tilde{\mu}_{\text{Ten}}$ and $\tilde{\mu}_{\text{Tor}}$, for both μ_{Ten} and μ_{Tor} are the sine-function of the first order and the range of integration is equal. Then, it is concluded from Eq. (1) that the value of n_M , that is, the major number of slip planes and slip directions, does not affect the yield stress ratio τ/σ .

3.2 Influence of m (integration by θ)

It is possible, from the above discussion, to postulate $n_m = \infty$, without losing the generality of the following analysis. The functional form of θ is different between Eq. (2) and Eq. (21), so that it is expected that the number n_m and the relative angular relations of the components of m have influence on the yield condition of polycrystals.

When the elements m_i and m_j are close to each other, the integral range $\Delta\theta$ of θ is given as the rotation angle from the position that m_i directs to the axis ($\theta=0$) to the position that m_i and m_j are

Fig. 3 Range of integral with respect to α

symmetrical with respect to the axial direction ($\theta = \Delta\theta$). Hereafter, the direction of the slip m_1 and its angle θ_1 are chosen as representative of the crystal orientation. Let's define the slip m_i by the angles $\Delta\theta_i$ and ζ_i , when m_1 is directed to $\theta_1 = \Delta\theta$, (Fig. 4). The angles $\Delta\theta_i'$ and ψ_i' shown in Fig. 4 are given by,

$$\left. \begin{aligned} \cos \Delta\theta_i' &= \cos \Delta\theta_i / \cos \zeta_i \\ \cos \psi_i' &= \cot \Delta\theta_i / \cot \Delta\theta_i' \end{aligned} \right\} \dots\dots\dots (22)$$

Similarly when m_1 takes an arbitrary value θ_1 ($0 \leq \theta_1 \leq \Delta\theta$), the angles θ_i and λ_i of m_i are given by

$$\cos \theta_i = \cos(\Delta\theta_i' - \Delta\theta + \theta_1) \cos \zeta_i \dots\dots\dots (23)$$

$$\left. \begin{aligned} \lambda_i &= \lambda_1 - \psi_i \\ \cos \psi_i &= \cot \theta_i \tan(\Delta\theta_i' - \Delta\theta + \theta_1) \end{aligned} \right\} \dots\dots\dots (24)$$

It is now possible to calculate the value $\mu_{\text{Ten}}^{(i)}$ or $\mu_{\text{Tor}}^{(i)}$ for an arbitrary slip m_i .

3.2.1 Tension

The Schmid factor for slip m_i is given from Eq. (2) :

$$\mu_{\text{Ten}}^{(i)} = \sin \theta_i \cos \theta_i = \frac{1}{2} \sin 2\theta_i \dots\dots\dots (25)$$

From this it follows that $\mu_{\text{Ten}}^{(i)}$ takes maximum value at $\theta_i = \pi/4$. Therefore, the slip occurs in such direction m_i that has the nearest angle of θ_i to $\pi/4$. Substituting the value θ_i obtained from Eq. (23) into Eq. (25), the maximum value of μ_{Ten} is given by

$$\mu_{\text{Ten}} = \max(\mu_{\text{Ten}}^{(1)}, \dots, \mu_{\text{Ten}}^{(i)}, \dots) \dots\dots\dots (26)$$

The weight in the integration is $\sin \theta_1$ corresponding to the representative direction m_1 . Consequently, the mean value $\tilde{\mu}_{\text{Ten}}$ for all grains is

$$\tilde{\mu}_{\text{Ten}} = \frac{\int_S \sin \theta_1 \mu_{\text{Ten}} d\theta_1}{\int_S \sin \theta_1 d\theta_1} \dots\dots\dots (27)$$

where S is the area on the unit sphere occupied by the representative direction m_1 , and is determined from the symmetry of the crystal (cf. Fig. 5).

3.2.2 Torsion

Substituting θ_i and λ_i obtained from Eqs. (23) and (24) into Eq. (21), and putting $\sin(\alpha + \omega) = 1$ in Eq. (20), we obtain

Fig. 4 Angular relationship between m_1 and m_i

$$\begin{aligned} \mu_{\text{Tor}}^{(i)} &= A(\theta_i, \lambda_i) \\ &= \sqrt{\cos^2 2\theta_i \sin^2 \lambda_i + \cos^2 \theta_i \cos^2 \lambda_i} \dots\dots\dots (28) \end{aligned}$$

The maximum value is given by

$$\mu_{\text{Tor}} = \max(\mu_{\text{Tor}}^{(1)}, \dots, \mu_{\text{Tor}}^{(i)}, \dots) \dots\dots\dots (29)$$

Hence, the mean value is

$$\tilde{\mu}_{\text{Tor}} = \frac{\int_S \int_0^\pi \sin^2 \theta_1 \cos \theta_1 \mu_{\text{Tor}} d\lambda_1 d\theta_1}{\int_S \int_0^\pi \sin^2 \theta_1 \cos \theta_1 d\lambda_1 d\theta_1} \dots\dots\dots (30)$$

It is concluded that both the number n_m of m and the relative angular relation between the components m_i have influence on the area S or the values given from Eqs. (22)~(24), and consequently on the yield stress ratio τ/σ .

3.3 Example of calculation on $m = \{111\}$

Based on the general discussion mentioned above, the value τ/σ is computed numerically on the particular case of $m = \{111\}$, which corresponds to a slip in fcc or bcc metal [For the convenience, m and its components are expressed with the Miller indices $\{ \}$ and $()$, respectively, which originally represent crystal planes]. Considering from the symmetry of the components of $m = \{111\}$, the integral area S of Eqs. (27) or (30) is given by the triangle (111)-(011)-(001) on the unit sphere, as shown in Fig. 5. The range $\Delta\theta$ of the integral with respect to θ_1 is given by,

$$\cot \Delta\theta = \sqrt{2} \cos \eta, \quad 0^\circ \leq \eta \leq 60^\circ \dots\dots\dots (31)$$

where η is the angle between the plane on which m_1 rotates and the plane which contains (111) and (011).

3.3.1 $\tilde{\mu}_{\text{Ten}}$

When m_1 moves in the triangular area (Fig. 5), either the slip $m_1 = (111)$ or the slip $m_2 = (\bar{1}11)$, which makes an angle $70^\circ 32'$ with the former, is operative. It is apparent from the geometrical configuration that the angle $\Delta\theta_2$ of m_2 is equal to $\Delta\theta$ of m_1 . Furthermore, the following relation holds in general:

$$\cos^2 \zeta_i = \cos^2 \gamma_i + \cos^2 \eta_i \sin^2 \gamma_i \dots\dots\dots (32)$$

Fig. 5 Surface area S represented by $m_1 = \{111\}$ on unit sphere

where γ_i is the angle between m_1 and m_i , and η_i is the angle between the plane of rotation of θ_1 and the plane containing m_1 and m_i . Substituting the value $\gamma_2=70^\circ32'$ and $\eta_2=\eta$ into Eq. (32), ζ_2 is obtained. Then, $\Delta\theta_2'$ and θ_2 are calculated from Eq. (22) and Eq. (23), respectively. In the present case, Eq. (27) is rewritten as,

$$\tilde{\mu}_{Ten} = \frac{\int_0^{60^\circ} \Delta\theta \int_0^{\Delta\theta} \sin \theta_1 \mu_{Ten} d\theta_1 d\eta}{\int_0^{60^\circ} \Delta\theta \int_0^{\Delta\theta} \sin \theta_1 d\theta_1 d\eta} \quad \dots\dots\dots(33)$$

Substituting Eqs. (25) and (26) into Eq. (33), we obtain after the numerical calculation,

$$\tilde{\mu}_{Ten} = 0.475 \quad \dots\dots\dots(34)$$

This value is slightly larger than the value 0.472 obtained for bcc metal having 48 slip planes ($n_M=48$), which is thought to result from the assumption of $n_M=\infty$ in the present calculation.

3.3.2 $\tilde{\mu}_{Tor}$

In torsion, one of the four slips $m_1=(111)$, $m_2=(\bar{1}11)$, $m_3=(1\bar{1}1)$ and $m_4=(\bar{1}\bar{1}1)$ is operative in accordance with the variation in θ_1 or α_1 . The angles θ_i related to m_i ($i=3, 4$) are given by,

$$\cos \Delta\theta_i = \cos \gamma_i \cos \Delta\theta + \sin \gamma_i \sin \Delta\theta \cos \eta_i \quad \dots\dots\dots(35)$$

Substituting Eqs. (32) and (35) into Eqs. (22)~(24), the angle ϕ_i of m_i is expressed in terms of η . Integrating μ_{Tor} with respect to λ_1 and θ_1 [Eqs. (28)~(30)], and also with respect to η after the multiplication of $\Delta\theta$, the mean value $\tilde{\mu}_{Tor}$ is obtained. After the numerical calculation, we obtain

$$\tilde{\mu}_{Tor} = 0.821 \quad \dots\dots\dots(36)$$

3.3.3 τ/σ

Substituting Eqs. (34) and (36) into Eq. (1), the yield stress ratio is obtained:

$$\tau/\sigma = 0.579 \quad \dots\dots\dots(37)$$

which is very close to the value 0.577 obtained from the von Mises' law of yield condition in the theory of plasticity.

4. Discussion

4.1 Effect of preferred orientation on the yield condition of fcc metal

As mentioned above, fcc and bcc metals are identical with respect to the yield stress ratio τ/σ , for in both metals the slip is expressed by $m=\{111\}$. Therefore, the influence of preferred orientation on the yield stress ratio may be similar. In the case of bcc metal reported previously⁽¹⁾, it was found that the value τ/σ becomes greater in accordance with the increase in the ratio $[100]/[111]$ of the grain orientations. Examples of the wire texture of fcc metals are listed in Table 2⁽⁵⁾. Comparing the cases of bcc and fcc metals, it is suggested that the value τ/σ of fcc metal having cold-drawn texture is smaller in Al or Cu, while it is

larger in Ag, than that of uniform distribution of orientations.

4.2 Effects of slip other than basal slip and twin formation in hcp metal

When a slip occurs on the plane other than the basal (0001) plane, the number of slip directions becomes smaller than that of slip planes. Therefore, the slip direction becomes a dominant factor affecting the yield condition. In the case of prismatic slip, the slip direction is $\langle 1120 \rangle$, which is equivalent to that of the basal slip and makes the angle $\pi/3$ to each other. Then, it is expected that the value τ/σ becomes 0.586 (cf. Fig. 8), which is slightly smaller than the value 0.591 for the basal slip⁽²⁾. Twin formation, which occurs in one direction and is not reversible, is also expected to reduce the value of τ/σ slightly.

4.3 Effect of the motion of screw dislocation

Although the slip direction of screw dislocation is defined, its motion is not restricted on any particular crystal plane⁽⁶⁾. Therefore, it is expected that the effect of screw dislocation on the yield condition is similar to that of the increase in the number of available slip systems in so far as the number of slip planes is smaller than the number of slip directions ($n_{sp} \leq n_{sd}$). It does not affect, on the contrary, in the case of $n_{sp} \geq n_{sd}$. The experimental results^{(7) (8)} shown in Fig. 6 that the value of τ/σ is smaller in fcc metal than in bcc, seem to support the above conclusion, for $n_{sp} \leq n_{sd}$ in fcc metal. Similar trends are also reported on the case of fatigue strength under combined stresses⁽⁹⁾.

Table . Preferred orientation of fcc metals after drawing⁽⁵⁾

Metals	[100]	[111]
Al	0%	100%
Cu	40	60
Au	50	50
Ag	75	25

Fig. 6 Yield conditions of fcc and bcc metals^{(7) (8)}

4.4 Comparison with the previous studies

U. Dehlinger⁽⁴⁾ discussed the correlation between the yield condition of polycrystals and the critical shear stress for slip [Eq. (18)] from the stand point of symmetric nature of mathematical formulae. He concluded that the von Mises' law can be directly deduced from the shear stress for slip in crystals. In his analysis, however, the following points were not taken into consideration: (1) the slip system of the highest shear stress has to be chosen among the possible ones; (2) ϕ_i and ϕ_i' in Eq. (18) are not independent, for the slip plane normal makes a right angle with the slip direction; (3) the Schmid factor of a grain is different depending not only on its orientation, but also on the angular position in the cylindrical specimen, for example, in the case of torsion [Eq. (4)].

It is thought that such numerical calculations as made in the present study are necessary to elucidate the yield condition of polycrystals, owing to the complexity of the problem in question.

J. F. W. Bishop et al.⁽¹⁰⁾ and J. W. Hutchinson⁽¹¹⁾ calculated the yield condition of fcc and bcc metals based on the Taylor's model of polycrystalline aggregates⁽¹²⁾. The values of τ/σ of their results⁽¹⁾ are somewhat smaller than that of the present study based on the Sachs model. This may be due to the difference in the number of slip systems considered in the respective models. That is, in Taylor's model, several slip systems are considered in order to keep the continuity of the strain during deformation⁽¹³⁾.

5. Conclusions

The correlation between the yield condition of polycrystalline metals and the slip in crystal was studied based on the Sachs model of polycrystalline aggregates, and discussions were made on the effects of crystal structure or preferred orientation. The results are summarized as follows:

(1) The minor number n_m of slip planes and slip directions of the crystal and the relative orientation between the components are the most important factor governing the yield stress ratio τ/σ of torsion to tension.

(2) The value of τ/σ is equal to 0.590 at $n_m=1$, which decreases with an increasing number of n_m , and finally reaches 0.500 corresponding to $n_m=\infty$.

(3) The value τ/σ of bcc or fcc metal is in good agreement with that of von Mises law, which is thought to result from the characteristic configuration of the slip systems in respective metals.

(4) The cold-drawn structures of fcc metals are expected to make the value τ/σ smaller for Al

or Cu, and greater for Ag, than that of uniform distribution of orientation.

(5) The value τ/σ of hcp metal is thought to become slightly smaller when the slip occurs on the other plane than the basal plane or when the twin formation takes place.

(6) The motion of screw dislocations on optional planes may decrease the value τ/σ for fcc metal, but not for bcc metal.

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Appendix Relation between τ/σ and n_m when the components of m lie on one plane

In order to illustrate the correlation between the value τ/σ and the number n_m discussed in Chapter 3, an imaginary case when the components of m lie on one plane is considered. The integral area S (Fig. 5) reduces in the present case to a great circle on the unit sphere. Therefore, Eqs. (27) and (30) are represented by the integral with respect to θ_1 . Let's assume here that the angles between the neighboring components of m are equal, and denote the angle by θ^* .

$$\theta^* = 360^\circ / 2n_m \quad \dots\dots\dots (38)$$

The integral range $\Delta\theta$ of θ_1 is

$$\Delta\theta = \frac{\theta^*}{2} = \frac{360^\circ}{4n_m} \quad \dots\dots\dots (39)$$

$$(1) \quad \tilde{\mu}_{\text{Ten}}$$

μ_{Ten} is expressed as the function of θ_1 from Eqs. (25) and (26), taking the geometrical configuration

Fig. 7 Changes in $\tilde{\mu}_{\text{Ten}}$ and $\tilde{\mu}_{\text{Tor}}$ with n_m when the components of m lie on one plane

of the components m_i into account. The mean value $\tilde{\mu}_{Ten}$ for each value of n_m is calculated from Eq. (27). The result is shown in Fig. 7 (a).

(2) $\tilde{\mu}_{Tor}$

In the case of torsion, only the slip m_1 is operative, because in general μ_{Tor} takes the maximum value at $\theta_i=0$ [Eq. (28)]. Substituting Eqs. (28) and (39) into Eq. (30),

$$\tilde{\mu}_{Tor} = \frac{6}{\pi \sin^3 \Delta\theta} I(\Delta\theta, \pi/2) \dots\dots\dots (40)$$

The result of the numerical calculation of Eq. (40) is shown in Fig. 7 (b).

Fig. 8 Yield stress ratio τ/σ versus n_m when the components of m lie on one plane

(3) τ/σ

Substituting the values $\tilde{\mu}_{Ten}$ and $\tilde{\mu}_{Tor}$ shown in Fig. 7 into Eq. (1), the yield stress ratio τ/σ is obtained for each number n_m of slips, as shown in Fig. 8. It is concluded that the value τ/σ decreases with the increase in n_m , in general (These values of τ/σ in Fig. 8 which are less than 0.50, are not important, for they possibly result from the assumption that the components of m lie on one plane).

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