On Float-Area-Type Flow Meters
(Non-Newtonian Fluid)

By Yasuo Mori**, Hiroshi Maki***, and Nobuhiko Nishiwaki****

The float-area-type flow meter, frequently known as a rotor meter, is an important flow rate measuring device for the flow rate is small. The purpose of this report is to introduce relations about the performance of float-area-type flow meters for non-Newtonian fluids following the "power law" model, and to verify the theoretical performance by experiments.

It is shown, as a result, that the procedure for obtaining the flow rate characteristics for this kind of flow meter is theoretically formulated. The theoretical results are compared with those of experiments which are carried out with several-percent solutions of CMC and good agreement is found to exist between them.

1. Introduction

A float-area-type flow meter is widely used when the flow rate is so small that a differential pressure flow meter is inapplicable. Theoretical treatments about the performances, i.e. the flow rate characteristics, the drag coefficient of the float, the sensitivity, etc., of this kind of flow meter have been made with Newtonian fluids as the working fluid, and good agreement has been found to exist between the theoretical and the experimental results.

Non-Newtonian fluids have come to be widely used in industrial processes. However, no theoretical and experimental studies for non-Newtonian fluids have been reported regarding the performance of this kind of flow meter which is most suitable for measuring a small rate of flow.

The purpose of this report is to study the correlations between the performances of a float-area-type flow meter for Newtonian fluids and those for non-Newtonian fluids, and to verify the theoretical results by experiments.

2. Analysis

2-1 Fundamental analysis

From a practical point of view concerning the working fluid of a float-area-type flow meter, we consider in these theoretical treatments that a non-Newtonian fluid follows the "power law" model which contains the "pseudoplastic fluids" and the "dilatant fluids".

In order to analyse the flow in the clearance between the float and the tapered tube, a float having the usual shape, which is in equilibrium at the center of a vertical tapered tube, as shown in Fig. 1, is considered. Here, $x$ is a coordinate running in the direction of the flow and starting from the minimum clearance and $y$ is a coordinate across the clearance, as shown in Fig. 1. Here, $d_f$ denotes the maximum diameter of the float, $l$ the length of the part of the maximum diameter of the float, $D$ the inner diameter of the tapered tube, $c=2m=(D-d_f)/2$ the clearance in the radius between the float and the tapered tube, $U$ the velocity in the

Fig. 1 Flow around the float

---

* Received 7th March, 1968.
** Professor, Tokyo Institute of Technology.
*** Assistant, Tokyo Institute of Technology. Meguro-ku, Tokyo.
**** Graduate Student, Tokyo Institute of Technology.
boundary layer at the point (x, y), U_M the maximum velocity at section x, \( U_m \) the mean velocity at section x, \( p \) the pressure and \( k = \frac{dc}{dx} \) the gradient of the tapered tube. Subscript \( f \) refers the value at the float, \( t \) the value at the tapered tube, \( o \) the value at the minimum clearance, \( I \) the merging point of the boundary layers in the clearance, \( 2 \) the value at the rear of the float, \( u \) the value at the front of the float and \( x \) the value at section x. As the velocity is small in the region upstream of the minimum clearance, that is around the circular-cone portion at the front of the float, and the pressure gradient in the direction of the flow is negative, the boundary layer thickness in this region is very small. It is, therefore, assumed that the boundary layers along the surface of the float and the inner wall of the tapered tube become larger, starting at the location of the minimum clearance. The boundary layer thickness at a point x will be denoted by \( \delta \) and the growing boundary layers along both walls are supposed to merge into each other at the point \( x_f \). In many cases, it is considered that \( c/d_f = 0 \). Now, when \( c \) is usually very small as compared with \( d_f \), and also that \( kl/d_f = 0 \), unless l is extremely large, since \( k \) is generally very small. Hence, one may assume \( \delta_f = \delta = \delta_o \).

In order to examine forces acting on the float, the flow field in the clearance will be analysed. Different flow patterns are formed in the clearance according to whether or not the boundary layer, which grow starting at the minimum clearance along both walls, merge into each other in this region. The two types of flow in the clearance will be studied separately.

### 2.1.1 Flow in the inlet length of the clearance

(0 \( \leq x \leq x_o \))

The density of the fluid is denoted by \( \rho \). The equation of continuity is:

\[
\pi d_f \int_0^y \rho U dy + \pi (d_f + 2c_o) \int_y^\infty \rho U dy = \pi c_o (d_f + c_o) \rho U_o \]

where \( c_o \) is the clearance in the radius at section \( x \) and given by

\[
c_o = c + kx
\]

With the assumption that \( x \) is not extremely long, the terms of \( \rho (\delta)^2 \) in Eq. (1) are dropped out. Then Eq. (1) becomes

\[
\pi d_f \int_0^y \rho U dy + \pi (d_f + 2c_o) \int_y^\infty \rho U dy = \pi c_o (d_f + c_o) \rho U_o \]

(3)

With the same assumption as above, the momentum equation is:

\[
\frac{d}{dx} \left[ \pi d_f \int_0^y \rho U^3 dy + \pi (d_f + 2c_o) \int_y^\infty \rho U^3 dy \right] + \pi (d_f + c_o) (c_o - 2\delta) \rho U^2 \rho U_o
\]

\[
= -\pi (d_f + c_o) c_o \frac{dp}{dx} - 2\pi \pi (d_f + c_o)  \]

(4)

Since the working fluid is a non-Newtonian fluid following the "power law" model as mentioned at the beginning, the shearing stress \( \tau \) at the surface of the float is written as

\[
\tau = \left( \mu \frac{\partial U}{\partial y} \right)^n
\]

(5)

where \( \mu \) (sec^\(-1\)cm^\(-1\)g^\(-1\)) is the pseudo viscosity and \( n \) is a rheological constant. The range of the value of the rheological constant \( n \) for working fluids of a float-area-type flow meter is usually considered as \( 0.5 \leq n < 2 \).

Since there are no losses in the main flow, we obtain

\[
\frac{dp}{dx} = -\left( \frac{\rho}{2} \right) \left( \frac{dU}{dx} \right)^2 dy \]

(6)

When the gradient of the tapered tube is very small, i.e. \( k = 0 \), it is considered that \( dp/dx < 0 \). The velocity distribution in the boundary layer may be assumed in the following form of the cubic equation

\[
\frac{U}{U_M} = a \left( \frac{y}{\delta} \right) + a' \left( \frac{y}{\delta} \right)^2 + \frac{a''}{\delta} \left( \frac{y}{\delta} \right)^3
\]

(7)

where \( a, a' \) and \( a'' \) in Eq. (7) are constants and are determined by the limiting conditions

\[
\text{at} \ (y/\delta) = 1 \ : \ \frac{(U/U_M)}{y/\delta} = 1, \ (\partial U/\partial y) = 0
\]

Then Eq. (7) becomes

\[
\frac{U}{U_M} = a \left( \frac{y}{\delta} \right) + (3-2a) \left( \frac{y}{\delta} \right)^2 + (a-2) \left( \frac{y}{\delta} \right)^3
\]

(8)

where \( a \) is a function of the rheological constant \( n \) alone. In order to obtain the relation between \( a \) and \( n \), we consider a portion of a fully developed flow of the unit width between parallel plates having a spacing of 2s. Let \( dp \) be the pressure drop over a distance \( dL \) in the direction of the flow, \( \tau_s \), the surface shear stress and \( \tau \) the shear stress at a distance \( y \) from the center of the spacing. The following relations are then obtained

\[
\frac{y}{s} = \frac{\tau}{\tau_s}
\]

(9)

\[
\tau_s = s \frac{dp}{dL}
\]

(10)

Substitution of \( \tau \) from Eqs. (9) and (10) into Eq. (5) gives

\[
\frac{dU}{dy} = \frac{1}{\mu} \left( \frac{dp}{dL} \right)^{1/n} y
\]

(11)

Integrating Eq. (11) with respect to \( y \) and referring to the boundary condition \( U = 0 \) at \( y = s \), velocity distribution in the clearance between parallel plates becomes.
\[ U(y) = \frac{1}{\mu} \left( \frac{dp}{dz} \right)^{1/n} \left( \frac{y^{(n+1)/n} - y^{(n+1)/n}}{n+1} \right) \]  
\[ Q_s = 2 \int_0^y U(y) dy = \frac{2}{\mu} \left( \frac{dp}{dz} \right)^{1/n} \frac{n}{(n+1)} \left( y^{(2n+1)/n} - y^{(n+1)/n} \right) \]  

The ratio of the mean velocity \( U_{\text{mean}} \) obtained from Eq. (13) to the maximum velocity \( U_{\text{max}} \) obtained from Eq. (12) is:

\[ \frac{U_{\text{mean}}}{U_{\text{max}}} = \frac{n+1}{2n+1} \]  

On the other hand, according to the velocity distribution given by Eq. (8), the flow rate \( Q_s \) of the unit width is:

\[ Q_s = 2 \int_0^y U(y) dy = \frac{6+a}{12} 2sU_M \]  

From Eq. (15), the ratio of the mean velocity \( U_n \) to the maximum velocity \( U_M \) is:

\[ \frac{U_n}{U_M} = \frac{6+a}{12} \]  

On the other hand, substituting Eqs. (6), (8) and (20) into Eq. (4), we obtain:

\[ \frac{\rho d}{dz} \left[ \frac{(2a^2 + 13a - 32)\ddot{x} + 12}{21} \right] U_M = -\left( \left( \frac{\mu dU_M}{\dot{z}} \right)^n - \frac{\rho k U_M^2}{4} \right) \]  

Using Eqs. (19) and (21), Eq. (22) becomes:

\[ \left[ \frac{(6a-a^2)^v}{12} \right] \left[ \frac{2}{kR_{\text{nu}}^*} \right] dc_s = \frac{A}{2} \left( f^2 + f \right) \frac{12}{7(a-a^2)} \left[ (f+1)^{2n} \left( \frac{c_s}{c_0} \right)^{2n} \right] df \]  

where \( A \) denotes:

\[ A = \frac{2(2a^2 + 13a - 32)}{35(6-a)} \]  

and is a function of the rheological constant \( n \) alone, and \( R_{\text{nu}}^* \) is the generalized Reynolds number defined by:

\[ R_{\text{nu}}^* = \frac{\rho m_n U_{\text{nu}}^{2-n}}{\mu^n} \]  

which is widely used for a non-Newtonian fluid.

It is seen from substituting \( \ddot{x} = \dot{c}_s/2 \) into Eq. (21) that:

\[ f_1 = \frac{(6-a)}{(6+a)} \]  

where \( f_1 \) denotes the value of \( f \) at the point where the boundary layers merge into each other.

Here, we introduce a function defined by:

\[ \eta_s = \frac{x}{m_s R_{\text{nu}}^*} \]  

and consequently we have \( \eta = \eta_s \) at \( x = x_0 \) i.e.

\[ \eta_t = \frac{x_1}{m_s R_{\text{nu}}^*} \]  

Integration of the left hand side of Eq. (23) with respect to \( x \) from 0 to \( x_1 \) yields \( \eta_t \). When integrating the right hand side with respect to \( f \) from 0 to \( f_1 \), the term \( (c_s/c_0)^{2n} \) in the denominator reduces to:

\[ \left( \frac{c_s}{c_0} \right)^{2n} = \left[ 1 + (1-n)kR_{\text{nu}}^* \eta_s \right] \left( \frac{1}{2} \right)^{(1-n)(1-2n)} \left( kR_{\text{nu}}^* \eta_s \right)^{(1-n)(2n)} \]  

Then, combining Eqs. (14) and (16), \( a \) is found to be:

\[ a = \frac{6}{2n+1} \]  

The value of \( a \) in Eq. (17) becomes 2 when the working fluid is a Newtonian fluid, i.e. for \( n=1 \), and Eq. (8) agrees with the velocity distribution assumed in the previous report [1].

From the condition of continuity, the mean velocity at section \( x \) is obtained as follows:

\[ U_{\text{mean}} = \frac{c_s U_{\text{nu}}}{c_0 + kx} \]  

The velocity of the main flow or the maximum velocity \( U_M \) at section \( x \) may be assumed in the following form:

\[ U_M/U_{\text{mean}} = f_1 + \frac{1}{f} \]  

On the other hand, from Eqs. (5) and (8), the shearing stress \( \tau \) at the surface of the float is written as:

\[ \tau = \left( \frac{dU}{dz} \right)^n = \left( \frac{\mu dU}{\dot{z}} \right)^n \]  

Substituting Eqs. (8), (18) and (19) into Eq. (3) gives the following result:

\[ \frac{\dot{z}}{c_s} = \frac{6}{6-a} f_1 + \frac{1}{f} \]  

Fig. 2 Relation between \( n \) and \( \eta_t \).
Thus, if \( n \) and \( kR_m^* \) are specified, Eq. (23) can be numerically integrated to give \( \eta_v \). It is, however, difficult to perform the integration for arbitrary values of \( kR_m^* \). Therefore, the relations between \( n \) and \( \eta_v \) have been numerically calculated from Eq. (23) for \( kR_m^* \) equal to 0.1 and 1.0, and are shown in Fig. 2. Since Eq. (23) cannot be used for \( kR_m^* \approx 0 \), an alternative equation has been derived, by putting \( c_v = c_c \) in Eqs. (3), (4), (21) and (22) and \( h \approx 0 \) in Eqs. (2) and (22), as follows:

\[
\left( \frac{6a - \alpha^2}{12} \right) \frac{dx}{m_n R_m^*} = - \frac{1}{2} \left( A \left( 2f + 1 \right) + A + \frac{1}{1} \right) \left( \frac{1}{f + 1} \right) \frac{df}{f^{\eta_v}}
\]

By integrating Eq. (30) numerically by the same procedure as Eq. (23), the relations between \( n \) and \( \eta_v \) for \( kR_m^* \approx 0 \) have been obtained and are also included in Fig. 2. Since \( kR_m^* \) is usually smaller than 0.1 when the working fluid of the float-area type flow meter is a non-Newtonian fluid, it is evident, from Fig. 2, that the use of Eq. (30) in place of Eq. (23) in the range \( kR_m^* \approx 0 \) is justified.

The approximate expressions of \( \eta_v \) as a function of \( n \) for \( kR_m^* \approx 0 \) are:

\[
\eta_v = 0.244(n - 0.75)^3 - 0.1603\eta_v = 0.244(n - 0.75)^3 + 0.1104(0.5 < n \leq 1)
\]

\[
\eta_v = -0.0457n + 0.1494(1 < n < 2)
\]

The pressure drop \( p_1 - p_0 \) across a portion from \( x = 0 \) to \( x = x_i \) is obtained by integrating Eq. (6), and we have:

\[
p_1 - p_0 = -\frac{\theta}{2} \int_{u_0}^{u_f} dB, dB = \frac{\theta}{2} \left[ c_v \right] \left( \frac{f + 1}{2} \right)^{2 - 1}
\]

\[
= \left( \frac{\theta}{2} \right) \left[ \gamma \frac{f + 1}{2} \left( 2 - 3 \gamma \phi + 8 \gamma \phi \gamma \phi \right) \right] - \left( f_i + 1 + 2 f_i \right)
\]

where:

\[
\alpha = k l / d_f
\]

\[
\beta = m_n / d_f
\]

\[
\tau = \alpha \beta
\]

\[
\eta = l / m_n R_m^*
\]

\[
\phi = \eta / \eta
\]

The shearing stress \( \tau \) at the surface of the float is obtained from Eqs. (20) and (21) as:

\[
\tau = \left[ \frac{m_n \dot{u}_n}{\delta} \right] = \left[ \frac{\mu (6a - \alpha^2)}{6} \right] \left( f + 1 \right)^{2} \frac{U_m}{m_n}
\]

Consequently from Eqs. (30) and (39), the shearing force \( T_r \) at the surface of the float for the distance from \( x = 0 \) to \( x = x_i \) is:

\[
T_r = \int_{0}^{l} \tau \pi d_f dx = 16 \pi \left( \frac{\eta}{2} \right) U_m \left( \frac{1}{4} \right) d_f \int_{0}^{l} \tau
\]

where \( t_i \) denotes:

\[
t_i = \int_{0}^{l} \left[ \frac{1}{2} (A + 1) f + (A + 1) f_i \right] \frac{df}{f^{\eta_v}}
\]

\[
\times \left[ 1 - n k R_m^* \eta \right] f_i
\]

Since the value of \( \eta_v \) in Eq. (41) is equal to or smaller than 0.11, as shown in Fig. 2, and the value of \( k R_m^* \) is smaller than 0.1 as discussed above, the term \( n k R_m^* \eta \) in Eq. (41) may be neglected when integrating, so that, \( t_i \) is expressed as:

\[
t_i = -((A + 0.5) f_i + (A + 1) f_i) / 2
\]

The pressure difference and the frictional force acting on the float for the region between \( x = 0 \) and \( x = x_i \) can be calculated by Eqs. (33) and (40), respectively.

2-1-2 Flow after the inlet length of the clearance \( c_f \leq x \leq l \)

It may be considered that the \( dp / dx < 0 \) in the limiting case of very small values of \( k \). For simplicity, by putting \( y = m_n x \leq x \) and \( y = m_n \) in the velocity distribution shown in Eq. (8), we have:

\[
U = U_f \left( 1 + (a - 3) \left( \frac{z}{m_n} \right)^2 + (2 - a) \left( \frac{z}{m_n} \right) \right)
\]

Consequently, the momentum for the flow at section \( x = 0 \) is given by:

\[
M = \pi (d_f + c_v) \int_{m_n}^{m_f} \left( \frac{u}{m_n} \right) dB_{m_n} \left( d_f + c_v \right) \]

where \( B \) is a function of the rheological constant \( n \) only and

\[
B = \frac{4}{105} \left( s \right)^{2} \left( \frac{12}{6 + a} \right)^{2}
\]

Differentiating Eq. (44) with respect to \( z \), we have:

\[
\frac{dM}{dx} = -\pi B \left( \frac{c_v}{c_v} \right) \left( d_f + c_v \right) U_m \left( d_f + c_v \right)
\]

Then, the momentum equation becomes:

\[
2 \pi (d_f + c_v) \tau + \pi (d_f + c_v) c_v \frac{dp}{dx}
\]

\[
= -\pi B \left( \frac{c_v}{c_v} \right) \left( d_f + c_v \right) U_m \left( d_f + c_v \right)
\]

The shearing stress \( \tau \) at the surface is obtained from Eqs. (16) and (39) as:

\[
\tau = \frac{12}{6 + a} \frac{U_m}{m_n} \left( d_f + c_v \right) \left( d_f + c_v \right)
\]

Combining Eqs. (47) and (48), we get:

\[
\frac{dM}{dx} = -\frac{1}{2} \left( \frac{24a}{6 + a} \right) \left( \frac{c_v}{c_v} \right) \left( d_f + c_v \right) \frac{dp}{dx}
\]

Integrating Eq. (49) between \( x = x_i \) and \( x = l \), then, the pressure difference \( (p_i - p_f) \) from the merging point of the boundary layers to the rear of the float is given by
\[ p_l - p_w = \frac{\partial \psi}{\partial \theta} \left[ \left( \frac{12a}{6+a} \right)^n \eta \left[ 2 - (2n+1)\gamma (1+\psi) + \frac{2}{3} (2n+1)(n+1)\gamma^2 (1+\psi + \psi^2) \right] - B_\gamma \left[ 2 - 3\gamma (1+\psi) + 4\gamma^2 (1+\psi + \psi^2) - 5\gamma^3 (1+\psi + \psi^2 + \psi^3) \right] \right] \]  

(50)

On the other hand, the shearing stress acting on the float for the distance from \( x_1 \) to \( l \) is from Eq. (48),

\[ \pi d f \int_{x_1}^{l} \tau dx = \left( \frac{12a}{6+a} \right)^n \eta (1-n) \beta \left[ 1-n\gamma (1+\psi) + \frac{1}{3} n (2n+1) \gamma^2 (1+\psi + \psi^2) \right] - \frac{1}{6} (2n+1)(n+1)\gamma^2 (1+\psi + \psi^2 + \psi^3) \]  

(51)

It is necessary that the right hand side of Eq. (49) be negative for \( dp/dx < 0 \) as assumed above, so the following condition is introduced

\[ k R_w^{n^*} < \left( \frac{12a}{6+a} \right)^n \left( \frac{c_0}{c_r} \right)^{3n^*} = D \]  

(52)

The value of \( D \) in Eq. (52) becomes 5 when the working fluid is a Newtonian fluid, i.e., \( n = 1 \), and this value of \( D \) agrees with the results reported before\(^{40} \). The value of \( D \) is larger than unity when \( n \) is between 0.5 and 2 and that of \( k R_w^{n^*} \) is usually smaller than 0.1 for non-Newtonian working fluids, therefore it is considered that the velocity distribution can be approximated by Eq. (43) in the case of boundary layers merging into each other.

Therefore, the pressure difference and the frictional force acting on the float for the region between \( x = x_1 \) and \( x = l \) can be calculated from Eqs. (50) and (51), respectively.

2.2 Drag coefficient \( c_0 \) of the float and characteristic parameter \( \eta \)

When the float is at rest in the fluid flowing through the tapered tube, a force due to the pressure difference over the float and the shearing force acting on the float are balanced by the weight of the float \( W \) minus the buoyant force \( F \). Thus, putting \( W - F = W' \), we obtain the following equation

\[ W' = W - F = \left( p_w - p_l \right) \pi d f \left( \frac{12a}{6+a} \right)^n \eta (1-n) \beta \left[ 1-n\gamma (1+\psi) + \frac{1}{3} n (2n+1) \gamma^2 (1+\psi + \psi^2) \right] - \frac{1}{6} (2n+1)(n+1)\gamma^2 (1+\psi + \psi^2 + \psi^3) \]  

(53)

where \( p_w \) denotes the pressure at the front of the float as shown in Fig. 1. \( \left( p_w - p_l \right) \) can be rewritten as

\[ \left( p_w - p_l \right) = \left( p_w - p_0 \right) + \left( p_0 - p_l \right) \]  

(54)

where \( \left( p_w - p_0 \right) \) is the pressure difference between the front of the float and the minimum clearance and is derived from Bernoulli's law as

\[ p_w - p_0 = \left( \frac{\rho}{2} \right) \left( U_{wo}^2 - U_r^2 \right) = \left( \frac{\rho}{2} \right) \left( 1 - \xi_0 \right) U_{wo}^2 \]  

(55)

where

\[ \xi = \left( 1 + 2\beta \right) \gamma \left( 8\beta \right) \gamma \left( 1 + 4\beta \right) \]  

(56)

We introduce, as usual, a dimensionless drag coefficient of the float defined by

\[ c_0 = \frac{8W'}{\partial \psi \partial \theta \pi d f \left( \frac{12a}{6+a} \right)^n \left( \frac{c_0}{c_r} \right)^{3n^*} } \]  

Substituting Eqs. (33), (40), (50), (51), (53), (54), (55) and (56) into Eq. (57), we then find

\[ c_0 = E \gamma + G \left( \beta \gamma \right) + F_1 \left( \beta \gamma \right)^2 + F_2 \left( \beta \gamma \right)^3 + \ldots \]  

(58)

where \( E \), \( F_1 \), \( F_2 \) and \( F_3 \) are

\[ E = \left( \frac{12a}{6+a} \right)^n \left[ \left( 2+8\beta \right) - (2n+1)\gamma + (2n+1)(3n+2) \right] \gamma^2 \frac{2}{3} - \frac{2n(2n+1)}{3} \left( \frac{c_0}{c_r} \right)^{3n^*} \]  

(59)

\[ G = \left( 1 - \xi_0 \right) + \frac{f_1}{2} + \frac{f_1}{2} (f_1 A + 0.5 + A + 1) \beta \gamma - 2 \left( \frac{12a}{6+a} \right)^n \eta_1 (1 + \beta) - B \gamma (2 - 3\gamma + 4\gamma^2) \]  

(60)

\[ F_1 = 8n \left( \frac{12a}{6+a} \right)^n \frac{\beta \gamma_1}{(2n+1)} + \frac{12a}{6+a} \frac{\gamma_1}{2} + B \gamma - \frac{12a}{6+a} \]  

(61)

\[ F_2 = 3 \left( \frac{12a}{6+a} \right)^n \frac{\beta \gamma_1}{(2n+1)} + \frac{12a}{6+a} \frac{\gamma_1}{3} - \frac{8n(2n+1)}{3} \beta + \frac{2(n+1)(2n+1)}{3} \frac{c_0}{c_r} \]  

(62)

\[ F_3 = \frac{B}{4} \left( \frac{12a}{6+a} \right)^n \frac{\beta \gamma_1}{(2n+1)(n+1)(2n+3)} \]  

(63)
We will now examine the range of the values of α, β and γ defined by Eqs. (34), (35) and (36) and those of \( F_1, F_2 \), and \( F_3 \) given by Eqs. (61), (62) and (63) for various combinations of tapered tubes and floats which are usually adopted in float-area-type flow meters. Since \( \alpha \ll 1, \beta < 0.1 \) and \( \gamma < 0.2, F_1, F_2 \), and \( F_3 \) are smaller than 10 when \( \gamma \) is between 0.5 and 2; and, since \( \gamma < 0.2 \) and \( \psi < 1, F_1(\psi \gamma), F_2(\psi \gamma)^2 \) and \( F_3(\psi \gamma)^3 \) may be neglected in the comparison with the terms \( E\gamma \) and \( G \), Eq. (58) reduces to
\[
\varepsilon_D = E\gamma + G
\]
(64)

On the other hand, from Eqs. (25), (37) and (57), \( \varepsilon_D \) can be expressed as a function of \( \gamma \) as
\[
\varepsilon_D = M\gamma^{(a-n)}
\]
(65)
where
\[
M = \left( \frac{8\pi}{\rho d^2} \right) \left( \frac{m_0 n+1\rho}{\mu^n} \right)^{1/(a-n)}
\]
(66)

Consequently, from Eqs. (64) and (65), we have
\[
M\gamma^{(a-n)} = E\gamma + G
\]
(67)

Since \( M, E \) and \( G \) in Eq. (67) are obtained when the geometrical dimensions of the tapered tube and the float (\( k, l, d \), and \( D \)), the weight of the float (\( W \)), the position of the float which is at rest in the tapered tube (\( m_0 \)) and the properties of the fluid employed (\( \rho, \mu \) and \( n \)) are given, the characteristic parameter \( \gamma \) of a float-area-type flow meter can be calculated from Eq. (67), graphically or numerically. Using this \( \gamma \), the drag coefficient \( c_D \) of the float is calculated from Eq. (64) or (65).

2-3 Flow rate characteristics of the float-area-type flow meter

From the practical point of view, it is necessary to know the theoretical relation between the position of the float and the flow rate rather than \( c_D \).

By using Eq. (25), \( \gamma \) defined by Eq. (37) is written as
\[
\gamma = \frac{k}{2}\left( \frac{d}{d_0} \right)^{1-n} \left( \frac{U_{cd}}{U_{cd0}} \right)^{2-n}
\]
(68)
The value of \( \gamma \) is calculated from Eq. (67) in the same way as mentioned in the previous section when the position of the float is given, and the mean velocity \( U_{cd0} \) at the minimum clearance is obtained from this \( \gamma \) by using Eq. (68). Since \( m_0 \) is a function of the position \( X \) of the float in the tapered tube, the flow rate \( Q \) when the float is at rest in the tapered tube can therefore be calculated from
\[
Q = 2\pi m_0 (d - 2m_0 U_{cd0})
\]
(69)
and the flow rate characteristics, i.e. relations between \( X \) and \( Q \), are obtained.

It has been shown by the results of analyses made so far that the flow rate characteristics of this kind of flow meter can be theoretically obtained when the working fluid is a non-Newtonian fluid following the "power law", as in the case of a Newtonian fluid. Therefore, we will try to study next, the influence of the change of the rheological constant \( n \) on its flow rate characteristics.

Consider, as typical examples, non-Newtonian fluids with \( n=0.75 \) and 1.25 as the working fluid and the tapered tube and the float shown in Fig. 8 which are employed in the experiment described in section 3-2. We assume, additionally, that for the both working fluids the float is kept at the same position (\( X=5 \text{ cm} \)) in the tapered tube for a certain fixed flow rate (\( Q=0.15 \text{ g sec}^2/\text{cm}^4 \)), which is equivalent to specifying the pseudo viscosities of the fluids. The densities of the fluids, furthermore, are assumed to remain constant, i.e. \( \rho=1\times10^{-3} \text{ g sec}^2/\text{cm}^4 \).

The relations between the position of the float and the flow rate for these non-Newtonian fluids are theoretically calculated and are shown in Fig. 3. For comparison, the theoretical result for a Newtonian fluid fulfilling the same conditions as above is also included in Fig. 3.

It is seen from Fig. 3 that, for \( X>5 \), a pseudoplastic fluid (\( n<1 \)) requires a higher flow rate than a dilatant fluid (\( n>1 \)) in order to keep the float at the same position and that a Newtonian fluid (\( n=1 \)) requires a flow rate in between.

3. Experiment

In order to verify, experimentally, the performance of the float-area-type flow meter derived analytically in the previous chapter, when the working fluid is a non-Newtonian fluid following the "power law", experiments were carried out on its flow rate characteristics.

3-1 Working fluid and measurements of its physical properties

Although there are a number of non-Newtonian fluids following the "power law" expressed by Eq.
several-percent solutions of CMC (carboxymethylcellulose) were used in this experiment specifically because of ease in handling. As the rheological constant \( n \) of the solution of CMC is smaller than unity, they belong to the pseudoplastic fluids. Since a solution of CMC shows the so-called thixotropic phenomena, i.e. it gelatinizes with time and returns to its original behavior when stress is applied, the fluids used were thoroughly agitated prior to the experiments.

The pseudo-viscosity \( \mu \) and the rheological constant \( n \) of the working fluid were measured by means of a specially manufactured tube viscometer shown in Fig. 4. The distribution of the inner diameter \( D_i \) of the cylindrical tube used in the viscometer was measured by the method described below. Mercury was inserted in a thin horizontal glass tube about 5 mm in length and the length \( x_{th} \) of the mercury was measured by means of a Universal Measuring Machine with an accuracy of 1 micron, and the mean inner diameter over the length \( x_{th} \) of the thin glass tube was calculated by

\[
W_{th} = \frac{\pi}{4} D_i \left( x_{th} - \frac{1}{3} D_i \right)
\]

(70)

with the mercury moving in the tube gradually, the variation of \( x_{th} \) was measured and the distribution of \( D_i \) was calculated from \( x_{th} \) by means of Eq. (70). In Eq. (70), \( W_{th} \) denotes the weight of the inserted mercury and \( \gamma_{th} \) its specific weight. The geometrical dimensions of the glass tube used in this experiment were decided by taking account of the following points. In some cases, the value of \( n \) changes with the velocity gradient of the working fluid. Consequently, in order to obtain the value of \( n \) at velocity gradients of the same order of magnitude as in the clearance between the float and the tapered tube of the practical float-area-type flow meter, a glass tube of about 1.8 mm in mean inner diameter was employed. The depth of the reservoir was so decided that the head of the fluid could be varied between 20 cm and 70 cm. From the results of measurements, the mean inner diameter of the thin glass tube was found to be 1.868 mm with an accuracy of within 0.3%. Furthermore, in order for both the entrance loss and the kinetic energy of the emerging stream to be neglected, the length of the glass tube had to be more than 100 times its mean inner diameter, so the necessary length was 19.18 cm.

Measurements were taken under several different conditions with the head of the working fluid in the reservoir shown in Fig. 4 being varied between 30 cm and 70 cm. The value of the flow rate \( Q \) was determined by means of a chemical balance measuring the weight (of about 10 g) of the fluid flowing out from the thin glass tube to a vessel during a certain time interval, which, in turn, was measured with a stop watch. Let \( dp \) be the product of the head of the working fluid and its specific weight. The relation between \( dp \) and \( Q \) is then given by

\[
\log \left( \frac{32Q'}{\pi D_i^3} \right) = \frac{1}{n} \log \left( \frac{D_i dp}{4L} \right) + \log \left( \frac{4}{n+3} \mu \right)
\]

(71)

Consequently, by plotting the experimental results with the aid of the relation given by Eq. (71) as in Fig. 5, \( n \) was obtained from the slope of the solid line, and then \( \mu \) was calculated from Eq. (71).

The specific weight of the working fluid was measured by means of a Baume's hydrometer.

3-2 Experimental apparatus and procedure

Figure 6 shows schematically the experimental apparatus arrangement. The working fluid is drawn from the lower reservoir to the head tank by a gear pump, and the head of the working fluid is stabilized by over-flow. The working fluid is conducted to the tapered tube through a valve which is used to adjust the flow rate. The value of the flow rate was determined by measuring the weight (of about 600 g) which was passed through the
tapered tube during a certain time interval. The temperature of the working fluid which was passed through the tapered tube was measured by installing a mercury thermometer of 0.1°C accuracy at the exit of the tapered tube.

The tapered tube used for these experiments was 35 cm long, 1.343 cm in mean inner diameter and 4.167 × 10⁻³ in gradient, and was made of glass. The distribution of the inner diameter of the tapered tube was precisely measured by means of a specially designed air micrometer as shown in Fig. 7, since the gradient and the inner diameter of the tapered tube had great influence on the accuracy of this kind of flow meter, as described in the previous chapter.

The float, 11.269 g in weight and 1.450 cm³ in volume and having the shape and geometrical dimensions as shown in Fig. 8, was employed in this experiment; it was made of stainless steel.

3.3 Experimental results and discussions

Typical examples of the flow rate characteristics of the float-area-type flow meter are shown in Fig. 9. In Fig. 9, the open circles and squares denote respectively the experimental results for two different several-percent solutions of CMC with n=0.877 and μ=2.78 × 10⁻⁴ sec g⁻¹ cm⁻² and with n=0.790 and μ=6.59 × 10⁻⁴ sec g⁻¹ cm⁻². In Fig. 10, the open circles denote the experimental result for a denser solution of CMC, i.e. one with n=0.765 and μ=1.23 × 10⁻³ sec g⁻¹ cm⁻² at a smaller flow rate than in Fig. 9. The solid lines in these figures show the theoretical values calculated by the analysis described in section 2. In Figs. 9 and 10, satisfactory agreement between the theoretical curves and the experimental values is seen and the appropriateness of the analysis developed in section 2 is proved. Data on the flow rate characteristics for floats of different shapes are not cited because the flow rate depends on the geometrical dimensions of

---

**Fig. 6** Arrangement of the experimental apparatus

**Fig. 7** Apparatus for measuring the inner diameter of the tapered tube

**Fig. 8** Float

**Fig. 9** Flow rate characteristics

**Fig. 10** Flow rate characteristics
the float \((l \text{ and } d_f)\) and the weight of the float \((W)\).

4. Conclusion

The fluid-dynamic analysis in the annular clearance between the float and the tapered tube of the float-area-type flow meter was performed in the case when the fluid employed was a non-Newtonian fluid following the "power law" model.

It was shown, as a result, that the drag coefficient \(C_d\) of the float was analytically expressed by a function of the characteristic parameter \(\eta\), with the dimensionless groups which were obtained from the geometrical dimensions of the float and the tapered tube, the pseudo-viscosity \(\mu\) and the rheological constant \(n\) as parameters. Moreover, the procedure for obtaining the flow rate characteristics for this kind of flow meter was theoretically formulated.

The theoretical results were compared with those of experiments which were carried out with several-percent solutions of CMC and good agreement was found to exist between them.

The authors express their cordial thanks to Prof. Y. Tomita, Tokyo Institute of Technology, for his kind advice and valuable suggestions in performing the experiments.

References