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A Theoretical Approach to Two-Phase Critical Flow*

(5th Report, Several Problems on Discharging of Saturated Water through Orifices)

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Saturated water discharging through orifices is theoretically and experimentally studied and the following results are obtained.

(1) Saturated critical flow through frictionless channel can be easily treated by combining critical condition with such upstream parameters as static pressure and steam quality, and this method can be applied to predicting of discharging rate of saturated water through orifice except the cases where thermodynamical metastability is established.

(2) Ratio between back pressure and upstream pressure is not invariable, but this seems to form one of flow conditions.

(3) The magnitudes of orifice diameter and upstream pressure which can be treated by traditional orifice constant of around 0.6 are in a considerably wide range.

(4) In the case when orifice flow rate is calculated as homogeneous flow, orifice constant seems to converge around 0.4 as for large orifice size and high upstream pressure.

(5) The flow rate through orifice varies with a decreasing back pressure until the critical condition is reached.

1. Introduction

It is thought that a critical flow takes place when a liquid having large specific enthalpy is discharged into the atmosphere of low pressure and steam bubbles appear as a result of flashing under depressurization. In a long channel it is known that we can treat it as saturated flow^{(1)~(3)}. However, in a short channel like orifices especially in case of small diameter, critical phenomena are not observed because of the thermodynamical metastability. But, as reported⁽³⁾ before, steam bubbles appear due to flashing under depressurization in a reservoir tank when the saturated water is discharged from a reservoir of constant volume, and they are absorbed into the flow channel. In this case, thermodynamical metastability cannot be established as observed in the channel of small L/D ratio (where D is comparatively large). Therefore, it may not be useless to examine saturated water discharging through orifice. Especially, in the case of loss of coolant accident of Boiling Water Nuclear Reactor the above

mentioned phenomena may take place, and it is by all means necessary to predict the actual condition of discharging phenomena in order to presume the scale of the accidental effects. Here the trouble is that the relation between the critical condition seen at the critical point, and that of upstream has not been discussed adequately. The only example of analysis of flow rate from channel diameter and flow conditions at upstream and downstream is Moody's method⁽⁴⁾, which is based on the assumption that is not always precise, that is $\partial G/\partial s=0$ and $\partial G/\partial p=0$. Experimentally, examples are few that reported the state of discharge from reservoir through orifices under high temperature and high pressure conditions.

In this paper⁽²⁾ an analysis is to be made to combine the theory on two-phase critical flow (developed in the previous paper) with upstream flow condition in case of such a frictionless channel as orifice, developing as little contradictory fluid mechanics as possible in this field.

The experiments are conducted about saturated water discharging through orifices of up to 70 mm ϕ under the reservoir pressure up to 70 ata, and several problems encountered in such orifice flow are

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studied to make clear the predicting accuracy of the above theoretical model.

2. Nomenclatures

- A_p : sectional area of reservoir tank m^2
 A_0 : flow area of orifice m^2
 c : orifice constant defined as Eq. (22)
 C_i : specific heat of water $\text{kcal/kg}^\circ\text{C}$
 D, d : diameter of flow channel m
 F : a force acting on the inner wall of channel kg/m^3
 G : weight velocity $\text{kg/m}^2\text{sec}$
 G_T : Total weight velocity ($\equiv G$) $\text{kg/m}^2\text{sec}$
 G_f : flashing rate from saturated water kg/sec
 g : acceleration of gravity m/sec^2
 H : water head in reservoir tank m
 h : ($\equiv H - 0.34$) in Fig. 10, water head from orifice center m
 Δi : $i_0 - i_i$ kcal/kg
 i : specific enthalpy kcal/kg
 L : channel length m
 p : static pressure ata. or atg.
 Q : heat transferred from reservoir wall kcal/sec
 R : energy dissipation rate $\text{kcal/m}^3\text{sec}$
 s : slip ratio ($\equiv V_g/V_l$)
 t : time from start of blowing down sec
 T : temperature $^\circ\text{C}$
 u : specific internal energy kcal/kg
 V : flow velocity m/sec
 V_p : volume capacity of reservoir tank m^3
 v : specific volume of fluid m^3
 w : weight of fluid kg
 x : steam quality
 x_B : steam quality flowing out of reservoir, Eq. (18)
 z : channel axial position m
 γ : specific weight kg/m^3
 $\bar{\gamma}$: mean specific weight of Eq. (23) kg/m^3
- subscripts
 0, 1: at flow inlet of channel or in reservoir tank
 2~4: pointed position in Fig. 7
 c: critical point
 g: steam
 l: water
 p: reservoir tank

3. Model of critical flow for frictionless channel⁽²⁾

3.1 Critical condition

As the author had already reported⁽⁵⁾, flow equations of two-phase flow can be approximately expressed by (about axial flow where slip ratio is assumed without treating separate momentum conservation in each phase):

$$\left\{ G \frac{d}{dz} \left\{ x \left(\frac{s^2 V_l^2}{2g} + i_g \right) + (1-x) \left(\frac{V_l^2}{2g} + i_l \right) \right\} \right\} = -R \quad \dots\dots\dots (1)$$

$$\left\{ \frac{d}{dz} \left\{ G \frac{(1-x+sx)V_l}{g} + p \right\} \right\} = -F \quad \dots\dots\dots (2)$$

$$\frac{dG}{dz} = 0 \quad \dots\dots\dots (3)$$

where

$$G = s\gamma_g \gamma_l V_l / \{x\gamma_l + s(1-x)\gamma_g\} \quad \dots\dots\dots (4)$$

And it has been confirmed that critical condition can be expressed in terms of "Eigenvalue method" as follows (since the assumption of $ds=0$ holds almost good):

$$\begin{aligned} & (s-1)^2 \{1 - (s+1)x\} \xi(V_l^2/g)^2 \\ & + 2[\gamma_l \{1 + 2(s-1)x\} - s\gamma_g \{2 - s + 2(s-1)x\}] \eta \\ & + \{1 + (s-1)x\} \xi(i_g - i_l) \\ & - M \{[2 + 3(s^2-1)x]/(2s\gamma_g) \\ & - [3 - s^2 + 3(s^2-1)x]/(2\gamma_l)\} (V_l^2/g) \\ & - 2M^2(i_g - i_l)/(s\gamma_g \gamma_l) = 0 \quad \dots\dots\dots (5) \end{aligned}$$

where

$$M = x\gamma_l + s(1-x)\gamma_g, \quad s \equiv V_g/V_l$$

$$\xi = x(\gamma_l/\gamma_g)(d\gamma_g/dp) + s(1-x)(\gamma_g/\gamma_l)(d\gamma_l/dp)$$

$$\eta = x(di_g/dp) + (1-x)(di_l/dp)$$

Using the following equation which was proposed by Fauske⁽¹⁾ as slip ratio

$$s_c = \sqrt{\gamma_l/\gamma_g} \quad \dots\dots\dots (6)$$

the assumption of $ds=0$ holds good⁽²⁾, so let us use Eq. (6). The relationship at critical point (critical relationship) obtained by substituting V_l given by Eq. (5) and Eq. (6) in Eq. (4) coincides with the experimental value very well, as the author has already reported in the previous reports⁽⁵⁾. Now let us develop the analysis where the critical condition is to be applied to frictionless flow.

3.2 Model

We may neglect friction loss in the flow which is running, through an orifice on the wall of a reservoir filled with saturated water, into a region of low pressure. There, if we express fluid velocity at the orifice inlet ("inlet" means the point far enough from orifice edge) by V_{l1} , pressure by p_1 , velocity of critical point which is thought to come after steep transition by V_{lc} , and critical pressure by p_c , then the relation $V_{l1} \ll V_{c1}$ may be true (the model is schematically shown in Fig. 1). Therefore, let us assume as follows.

Fig. 1 Schematic figure of frictionless orifice flow

- Assumption (i) neglecting energy transfer across system boundary and friction loss.
 (ii) neglecting kinetic energy in comparison with enthalpy.
 (iii) entrance velocity V_{i1} can also be neglected, in comparison with V_{ic} .

But as for critical condition, we use the conclusion of Eq. (5), with assumption (ii) alone.

In such a model, by integrating Eqs. (1) and (2) and using Eq. (4), the following relationships between the properties at inlet and at critical point can be given.

$$x_c = x_1(i_{g1} - i_{i1}) / (i_{gc} - i_{ic}) + (i_{i1} - i_{ic}) / (i_{gc} - i_{ic}) \quad \dots\dots\dots (7)$$

$$p_c = p_1 - s_c \gamma_{gc} \gamma_{ic} (1 - x_c + s_c x_c) (V_{ic}^2 / g) / M \quad \dots\dots\dots (8)$$

Though the right hand side of Eq. (8) can be calculated by (V_{ic}^2 / g) , x_c and p_c , the right hand side of the equation given by substituting Eqs. (5) and (6) in Eq. (8) is a function of only p_c because x_c and p_c are given by Eq. (7), s_c and p_c by Eq. (6), and interrelation of (V_{ic}^2 / g) and x_c , p_c , s_c by Eq. (5). So, if inlet conditions of p_1 , x_1 are given, p_c is obtained as the solution to Eq. (8).

3.3 Theoretical solutions

Solutions evaluated about water-steam mixtures are shown in Figs. 2~4. In Fig. 2 critical pressure ratio (ratio of critical pressure to upstream pressure) is taken as vertical coordinate and entrance steam quality is taken as horizontal coordinate with entrance pressure as parameter. The smaller its compressibility and entrance pressure are, the larger the critical pressure ratio becomes. In the region where x_1 is small, difference caused by pressure p_1 is observed. And it decreases in accordance with increase of inlet steam quality x_1 , and settles to nearly a constant value without regard to pressure in the range of $p_c/p_1 = 0.45 \sim 0.5$, $x_1 > 30\%$.

Fig. 2 Relation between upstream quality and critical pressure ratio

Since it is confirmed^{(2) (9)} that p_c/p_1 becomes smaller owing to thermodynamical metastability in the case of small entrance quality x_1 especially in small orifice diameter, it does not seem proper to use this model in such a case. In Fig. 3 is shown the relation between steam quality at critical position x_c and entrance steam quality x_1 in the same case, where the quantity of x_c near $x_1 \sim 0$ is larger under high pressure. When x_1 is large this relation to pressure is reversed.

In Fig. 4 is shown a theoretical value of critical flow rate. Comparing with Moody's theoretical solution⁽⁴⁾ (expressed by the dotted line), our value

Fig. 3 Relation between upstream quality and critical quality

Fig. 4 Relation between critical flow rate and upstream conditions

of G_c is apparently smaller by about 20 percent. This solution, as is mentioned above, coincides with the experimental value very well in regard to the relation at critical position. Then we can see clearly that, in the case that friction loss of channel can be neglected, critical flow is obtained, without employing Moody's model⁽⁴⁾ that makes both constant entropy equation and energy conservation compatible with each other.

4. Experiments on saturated water discharging through orifice⁽²⁾

4.1 Experimental apparatus and its processes

As we have already reported⁽³⁾ about the test apparatus used in the previous reports, here let us briefly explain about them. We used a 125 mm ϕ discharging duct which connects a reservoir of 600mm ϕ inner diameter and 2 900 mm (about 0.8m³) height with a receiver of 42 m³ capacity, putting orifices in the duct. The duct has a gate valve operated by pneumatic piston, and two orifices are provided both in front and back of this valve. Series of experiments are made while changing combination of the diameters of these two orifices and the back pressure of the upstream orifice.

The experiments were done by making the piston valves open after heating water in the pressure reservoir to the given saturated temperature.

4.2 Purposes of experiments

The purposes of these experiments are as follows:

(i) To see the applicable limit of the predicting method by use of the usual orifice constant under the variable upstream pressure and orifice diameter.

(ii) To see whether critical phenomenon can be observed.

(iii) To see whether regularity can be observed in orifice back pressure (Isbin and others⁽¹⁰⁾ reported the back pressure is measured as $0.3 p_1$)

In the propositions made up to now, it has been understood that orifice flow can be explained by orifice constant⁽⁷⁾ (for single phase incompressible fluid) because it does not become critical flow owing to thermodynamical metastability phenomenon even if saturated water is introduced; but it has not been made clear about the case of larger orifice diameter or of high upstream pressure. Especially in case of flow from a pressure reservoir of constant volume the circumstance is complicated because steam bubbles are generated in the reservoir. Therefore, we examined how far the predicting procedure with orifice constant is applicable.

When the critical flow is established the relations among critical parameters are also studied to

see the extent of applicability to actual orifice flow.

4.3 Measured properties and positions

Pressure and temperature were measured at the points shown ②, ③, ②' and ③' in Fig. 5. Since pressures of ② and ③ did not differ from those of ②' and ③' respectively we employed either of them.

Temperatures were recorded with mV recorder of large input impedance through chromel-alumel thermocouples as sensors, and pressures were measured with pressure transducers using coupling mechanisms of diaphragm and strain gauge.

When pressure of position ④ was as high as that of upstream we specified the friction of duct to be referred to afterward in interpreting the data.

4.4 Cases of experiments and their parameters

The following three kinds of experiments are conducted to answer the purpose.

(I) Measuring the change of condition in the pressure reservoir and steam quality flowing out of it.

(II) Experiments to see the influence of upstream pressure and the diameter of orifices on flowing down rate.

(III) Experiments to see the influence of back pressure of orifice ① on flow rate.

Experiment (I) was done in order to know the change of condition in the reservoir, or the relation between flashing rate and steam rate of flowing out which was necessary to know the quality of steam absorbed into orifice, and in order to see whether it could be reasonably regarded as saturated evaporation. And conditions of experiments of (II) and (III) were as listed in Table 1.

4.5 Results of experiments and discussions(I) :

Evaluation of change of condition in the reservoir and flowing out steam quality

In Fig. 6 is shown the recorded result of relation between temperature and pressure taken by an XY

Fig. 5 Shape of channel and measured positions

Table 1 Conditions of experiments

Case No.	Dia. of orifice ①	Dia. of orifice ②	Initial reservoir pressure
II	10, 17, 30, 50, 70 ϕ	125 ϕ (Without orifice)	10, 30, 50, 70 atg
III	10, 30, 50, 70 ϕ	10, 30, 50, 70 ϕ	10, 30, 50, 70 atg

recorder during the transient time of blowing down. The thick line in the figure represents the saturated relation recorded for several hours in the process of heating, while the thin lines were recorded for from several seconds to several minutes when blowing down experiment was conducted.

Figure 6 is the record of the run with orifice ① of 70 mm ϕ and orifice ② of 125 mm ϕ (removed) under the reservoir pressure of 70 atg, T_1 being output from thermocouple (always soaked in water) at the same level above the center of the orifices, and T_6 being the temperature at the point about 340 mm below the ceiling (it can be regarded as the same as that of steam). It can be affirmed by referring to steam table⁽⁹⁾ that the thick solid line represents saturating relation. By the thin solid lines we can see both T_1 and T_6 are kept invariable, while only the pressure swiftly goes down as soon as flow begins running, and then they change parallel to the saturation curve under a slightly superheated condition. Up to the point indicated by an arrow $\downarrow$, fluid is mainly composed of water, and after the point steam comes to go out. Only the change of condition which is observed while water is mainly flowing out (up to the arrow $\downarrow$), has to be estimated for the purpose of this experiment. And in this region, superheat of only 3~4°C, excepting the earliest stage where pressure alone goes down suddenly, is observed.

Moreover, since relation between pressure and temperature may be still reasonably regarded as saturated in treating with gradient of Fig. 6, the curve runs almost parallel to the saturation curve. Figure 6 shows the most swift transition in the whole runs, so the other cases can well be regarded as being closer to saturated condition than that.

The reason why the starting points of the two

Fig. 6 State variations in reservoir (Comparison between T_1 and T_6 with 70 mm ϕ , 70 atg.)

curves in Fig. 6 do not meet one another is because they are the results of three separate experiments.

In the case of orifice diameter 10 ϕ under upstream pressure 70 atg, the thin lines and the thick line overlapped each other, so we could not distinguish one from the other. We can see by this that, when flow goes out slowly as in case of orifice diameter being 10 ϕ it is regarded as almost complete saturated variation. In this experiment it should be noted that the thermometer follows a sudden temperature variation from 0 to 100°C by 80% response with 0.25 seconds time delay. Thus we can affirm that the state change in the reservoir can be approximated by saturated change. So let us develop the relation between state change in the reservoir, flow-out steam quality and flow rate in order to prepare for arranging purposes of empirical data.

As for the conservations in the reservoir, mass conservations can be written by:

$$dW_l/dt = -G_T(1-x_B) - G_f \text{ (liquid)} \dots\dots\dots (9)$$

$$dW_g/dt = -G_T x_B + G_f \text{ (gas)} \dots\dots\dots (10)$$

and energy conservations are

$$d(u_l W_l)/dt = -i_l G_T(1-x_B) - i_g G_f + Q_l \text{ (liquid)} \dots\dots\dots (11)$$

$$d(u_g W_g)/dt = -i_g G_T x_B + i_g G_f + Q_g \text{ (gas)} \dots\dots\dots (12)$$

And since the volume capacity is constant the following equation is satisfied,

$$v_g W_g + v_l W_l = V_p \dots\dots\dots (13)$$

where V_p is the capacity of the reservoir; Q_l and Q_g are calories supplied from a wall or something like that into liquid phase and gas phase respectively; x_B is flowing-out steam quality. Assuming the thermally insulated inner wall we shall neglect Q_l and Q_g .

Differentiating Eq. (13) by time and substituting it into Eq. (9) and (12), we can express flashing rate G_f kg/sec in terms of pressure change and x_B as follows.

$$G_f = v_l G_T / (v_g - v_l) + G_T x_B - W_l B (dp/dt) \dots\dots\dots (14)$$

where

$$B \equiv \{ (dv_l/dp)_{\text{sat}} + (W_g/W_l) (dv_g/dp)_{\text{sat}} \} / (v_g - v_l).$$

And by use of the full height of the pressure reservoir L and water head in it, $W_l = A_p H / v_l$, $W_g = A_p (L - H) / v_g$ are formulated and the relation $W_g/W_l = (L - H)v_l / (Hv_g)$ is deduced.

And, Eq. (11) and (12) can be rearranged into

$$c_l W_l (dT/dt) = -(i_g - i_l) G_f \text{ (where } i_l = u_l \text{ is assumed)} \dots\dots\dots (15)$$

$$W_g (du_g/dt) = pv_g (dW_g/dt) \dots\dots\dots (16)$$

Since heat capacity at the steam side of the reservoir is quite small comparing with water, and since Eq. (16) is difficult to treat we neglect Eq. (16) assuming saturation relation (Relation of Clapeyron and Clausius):

$$dT/dp = (T + 273.16) (v_g - v_l) / (i_g - i_l) \dots\dots\dots (17)$$

Thus flowing-out quality x_B is given as follows from Eq. (15) by use of Eq. (14)

$$x_B = \frac{(-dp/dt)}{(-dH/dt)} \left\{ \frac{c_i(T+273.16)(v_g-v_i)}{(i_g-i_i)^2} - B \right\} - \frac{v_i}{v_g-v_i} \quad \dots\dots\dots(18)$$

where total weight velocity G_T expressed by the following approximation is used

$$G_T \sim A_p(-dH/dt)/v_i \quad \dots\dots\dots(19)$$

And G_f can be also expressed as the following equation only in terms of pressure variation in the reservoir.

$$G_f = \frac{C_i A_p H (T+273.16)(v_g-v_i)}{v_i(i_g-i_i)^2} \left(-\frac{dp}{dt} \right) \quad \dots\dots(20)$$

In case of discharging rate being larger compared with capacity of the pressure reservoir, flashing rate is large and absorbed steam weight rate is also large. In such a case, as discharging time is short, Eq. (18) by neglecting transferred heat has high accuracy. In case of discharging time being long, heat transfer through wall comes into question, and presuming x_B by use of Eq. (18) seems not to be so reasonable. In such a case, it seems that rate of flashing is small and absorbed steam is also small. As steam is absorbed into orifice by acceleration of pressure difference, it is feared that fluid of small density is more selectively absorbed. Because of such complicated phenomena, more appropriate method of evaluating x_B than Eq. (18) cannot be found here. Therefore, the explanations are made by use of x_B calculated from Eq. (18) with empirical data of $-dH/dt$ and dp/dt . But we can not estimate, by this method, steam rate generating at the points which do not give influence upon state variation of reservoir.

Equation (19) must be regarded as lowest limit value of x_B .

4.6 Results of experiments and discussions

(II): Influence of diameters of orifices and upstream pressure

From the experiments done with constant diameter of orifice ② (125 mm ϕ : orifice is removed actually) and selected orifice ① among 10, 17, 30, 50 and 70 mm ϕ under the upstream pressure of 10, 30, 50 and 70 atg. the following results are obtained as shown in Fig. 7 and so forth.

Weight velocity and orifice constant are given as follows respectively in these figures.

$$G = (A_p/A_0)\gamma_1(-dH/dt) \quad \dots\dots\dots(21)$$

$$c = G/\sqrt{2g\gamma_1(p_1-p_2)} \quad \dots\dots\dots(22)$$

Relation between weight velocity and pressure drop in orifice is shown in Fig. 7, where the dotted line is calculated by taking orifice constant as 0.61 on the assumption that only the same quantity of water flows across the orifice under acceleration of

the same pressure difference. If there are not any steam bubbles around the inlet of orifice, and if there is maintained thermodynamical metastability as far as a vena contracta, the empirical weight velocities will almost correspond to the dotted line. (In the higher pressure region with large orifice diameter, this is not yet established even by the past studies on single phase fluid flow, it seems). Empirical data of orifice diameter 10 mm ϕ (marked by 0) are plotted nearer the dotted line; and weight velocity decreases with increasing orifice diameter. The other lines in Fig. 7 are as follows; the thick dotted line shows the empirical data by H. Nariai⁽⁶⁾ with the channel diameter of 2 mm ϕ and 4 mm ϕ in the pressure range up to 7 atg, and thick solid line represents M. W. Benjamin's data⁽⁷⁾ with orifice of 6.3~22 mm ϕ up to 23 atg. Benjamin et al. had also pointed out the decreasing tendency of orifice constant with increasing pressure and orifice diameter, while this tendency is much more remarkable here because of the wider range of pressures and orifice diameters, and because of superposed

Fig. 7 Measured discharging rate of saturated water through orifice

Fig. 8 Arrangement by orifice constant procedure

effect of absorbed steam into orifice inlet.

Figure 8 shows the orifice constant by supposing a homogeneous state (where slip ratio is equal to unity) with flow-out steam quality x_B given by Eq. (18), that is the orifice constant calculated by substituting the following mean specific weight for γ_i in Eq. (22).

$$\bar{\gamma} = \gamma_g \gamma_i / \{x_B \gamma_i + (1-x_B) \gamma_g\} \dots \dots \dots (23)$$

From this figure it can be deduced that a considerably clear tendency can be observed because of successful compensation of the effect of existing steam by use of mean specific weight $\bar{\gamma}$, but that Eq. (23) may be applied with $x_B \sim 0$ to a discharging case through small orifice because there is still the above mentioned problem in evaluating x_B . There is also pointed out the decreasing tendency of orifice constant c with increasing pressure difference across the orifice. And orifice constant of large diameter under higher pressure may be expected to converge around $C \sim 0.4$, because there is not so wide discrepancy between the plotted cases of 50 mm ϕ and 70 mm ϕ in the higher pressure region.

The reason for orifice constant being affected by orifice diameter or pressure difference may be the steep depressurization around the center of flow axis because of existence of considerable radial distribution of static pressure in the sectional area of the orifice. This effect is superposed by existence of steam resulting from the increasing flow resistance in case of increasing orifice diameter and pressure difference.

Figure 9 shows the relation between $\Delta p = p_1 - p_3$ and reservoir pressure p_1 , where the data of 70 mm ϕ have a different tendency from others because the

Fig. 9 Tendency of pressure drop across orifice

pressure drop in the down stream becomes comparable to the upstream pressure drop of orifice with increasing steam absorption. Arrow mark shows increasing time. Thus experimental variation of back pressure for orifice diameter 70 mm ϕ is different from others because of the effect of down stream resistance. Meanwhile from Fig. 10 the pressure ratio across the orifice is well illustrated to vary by the combination between orifice diameter and pressure difference. Though Isbin pointed out⁽¹⁰⁾ an almost invariable ratio of back pressure to upstream pressure for 0.5 in ϕ orifice diameter, there is given a result for back pressure of orifice to be arbitrarily varied by choosing the combination of flow rate and the capacity of down stream area.

Prediction of two phase orifice flow with an analogy to single phase flow (by use of orifice

Fig. 10 Tendency of measured pressure ratio across orifice

Fig. 11 Explanation by critical flow concept

constant method) is rather difficult as shown above because of absorbed inlet steam, so we are going to try to explain the effect of compressibility by taking the criticality into consideration. The results are illustrated in Fig. 11, where parameters of Fig. 4 are substituted by x_B given from Eq. (18), and G_{ex}/G_{ca1} (empirical and predicted values are written as G_{ex} and G_{ca1} respectively) is taken as vertical coordinate and horizontal one is reservoir pressure p_0 . The horizontal line running on unity of vertical coordinate means the relation $G_{ca1}=G_{ex}$. G_{ex} of orifice diameter 10 mm ϕ is twice as large as G_{ca1} (this case can be almost explained by same orifice constant around 0.6 as single phase flow), this discrepancy between theoretical prediction as critical flow and experimental flow rate decreases with increasing orifice diameter resulting in $G_{ex}/G_{ca1} \cong 0.7$ in case of 70 mm ϕ . Underestimation of the flow rate comes from an improper model of saturate phase change because of existence of thermodynamical metastability, while overestimation is thought to be caused by misvaluation of flow resistance by local steam quality which is supposed to exist in such a position that should not affect the state variation in pressure reservoir, the latter is supposed to come from radial pressure distribution around orifice inlet. The case of 10 mm ϕ is thought to belong to the former case where inlet steam quality can be reasonably neglected resulting in a thermodynamical metastable flow especially in the lower region of pressure, on the other hand there must exist a higher steam quality than x_B from Eq. (18) around orifice inlet of 70 mm ϕ because of credible radial pressure distribution, that is there seems not to

exist thermodynamical metastability.

As seen above, there are still such two difficult problems as thermodynamical metastability and unsuccessful estimation of x_B from radial distributions of parameters in explaining two phase orifice flow by critical phenomena.

4.7 Experimented results and discussions(III) :

Effects of orifice back pressure

Experiments discharging saturate water through one of orifices whose diameter are 10, 30, 50 and 70 mm ϕ at the position ② in combination with variable upstream orifice ① are conducted resulting in variable back pressure of orifice ①. The purposes of these experiments are to investigate the feasibility limit to use orifice constant procedure, to see the existence of criticality and to study the process to reach the critical flow.

The results are illustrated in Figs. 12~15, taking pressure ratio p_3/p_1 as vertical coordinate and diameter of orifice ① as parameter, under almost constant pressure in reservoir tank. Orifice constant procedure with around 0.6 can be applied to small orifice diameter in spite of variable pressure condition (though Fig. 12 shows that there is observed a tendency of flow rate slightly decreasing in the higher region of pressure even for 10 mm ϕ orifice), but there is seen a larger difference in the effect of back pressure in case of larger orifice diameter. And also there is clearly pointed out the existence of criticality with large orifice diameter and large pressure difference. Under such a critical condition, critical pressure ratio (which can be defined as the limit pressure ratio p_3/p_1 where the flow rate does not increase any more under decreasing pressure

Fig. 12 Relation between back pressure and flow rate ($p_1 = 60 \sim 67$ ata)

Fig. 13 Relation between back pressure and flow rate ($p_1 = 44 \sim 49$ ata)

ratio) can be understood to depend on orifice size and upstream pressure p_1 in such a way that critical pressure ratio increases with increasing orifice size, but the position of critical point seems to be difficult to determine from these figures. Even in single phase flow, the critical pressure ratio of compressible orifice flow cannot be determined so clearly⁽¹¹⁾, so there must be the same ambiguous tendency in two-phase orifice flow.

Figure 16 and Fig. 17 show the relation between orifice constant from Eq. (22) and back pressure about the above same data. The orifice con-

stant procedure with $c \cong 0.6$ may still be feasible with lower pressure region even for such a large orifice diameter as 70 mm ϕ (Fig. 17), but orifice constant c decreases with increasing orifice size and reservoir pressure p_1 because of effect of absorbed steam bubbles. Figure 17 includes some special cases where the empirical region of pressure ratio p_3/p_1 is limited in such a way that the data of 50 mm ϕ are limited to $p_3/p_1 \sim 0.5$, and those of 70 mm ϕ are also limited to ~ 0.8 . So there is no feasibility to apply orifice constant $c \sim 0.6$ to the region of smaller pressure ratio than the above values in case of orifice diameter 50 mm ϕ and 70 mm ϕ .

From these discussions it can be confirmed that orifice constant around 0.6 can be practically applied to prediction of flow rate through orifice of less than 10 mm ϕ under reservoir pressure up to 70 ata, and through orifice of less than 70 mm ϕ under the reservoir pressure up to 10 ata, by the same procedure taken in treating single phase orifice flow. And it is also concluded that there is seen criticality in two-phase orifice flow, and that the ratio of back and upstream pressure of orifice is understood

Fig. 14 Relation between back pressure and flow rate ($p_1 = 28 \sim 30.5$ ata)

Fig. 15 Relation between back pressure and orifice ($p_1 = 10 \sim 11$ ata)

Fig. 16 Relation between back pressure and orifice constant (I)

Fig. 17 Relation between back pressure and orifice constant (II)

not to be necessarily determined from given fluid conditions but to be given as arbitrary parameter ruling orifice flow conditions. It must be generally considered that the back pressure of orifice does not coincide with the critical pressure, but we cannot help taking back pressure of orifice as correlating parameter because of considerable difficulties in measuring pressure at the critical point.

5. Conclusions

Saturated water discharging through orifices is theoretically and experimentally studied resulting in the following conclusions⁽²⁾.

(1) Saturated critical flow through frictionless channel can be easily treated by combining critical condition with such upstream parameters as static pressure and steam quality, and critical pressure, exit quality and flow rate can be solved by thermodynamical method without contradiction. The model developed in this paper uses assumed expression of interphasic slip ratio of $s_c = \sqrt{\gamma_1/\gamma_g}$ ⁽¹⁾. The calculated results are compared with empirical data resulting in successful coincidence in case of large orifice size, while the case of small quantity of steam absorbed into orifice inlet cannot be well explained by this model because of improper assumption of saturated condition. In case of existence of rather many steam bubbles around orifice inlet, however, a certain tendency can be observed, which encourages us.

(2) Back pressure of orifice is thought to be arbitrarily varied without regard to upstream pressure. For there is not given any measuring result to show the invariable ratio of back pressure to upstream pressure.

(3) The orifice diameter which can be treated by orifice constant around 0.6 ranges up to 10 mm ϕ under the upstream pressure of less than 70 ata, and up to 70 mm ϕ under the upstream pressure of less than 10 ata, that is, covers rather wide ranges of orifice size and upstream pressure.

(4) The flow rate through orifice varies with decreasing back pressure until the critical condition is reached. At the criticality the flow rate cannot be affected by decreasing back pressure any more, when critical pressure ratio seems to increase with increasing orifice size.

(5) In the case that orifice flow rate is to be

calculated with orifice constant procedures as homogeneous flow with such a mean specific weight as Eq. (23), orifice constant seems to converge around 0.4 as in the case of large orifice size under high upstream pressure.

The theory developed in chapter 3 can be successfully applied to evaluating orifice flow of saturated water in case of high steam quality around orifice inlet, but this theory cannot be successfully applied to the range of infinitesimally small quality because of either ambiguity of evaluating method of inlet steam quality or improper model of saturated flow. And the range of orifice sizes explained by traditional orifice constant procedure is also clarified for saturated orifice flow. The author considers there must be developed the theoretical or experimental investigations about these transition phenomena to criticality and about thermodynamical metastability by means of thermo-hydrodynamical procedures including state physics.

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