The Vibration of a Cylinder Caused by Wake Force in a Flow*

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In this investigation three different experiments on the cylinder vibration caused by the wake force are undertaken. Namely, in the first place on the elastically supported cylinder in a wind tunnel and in the second place on the oscillated cylinder in a model basin experimental studies are undertaken. And in the third place on the oscillated cylinder in a wind tunnel an experimental research is made in which a fluctuating pressure on the cylinder surface is measured.

According to these experimental results it is concluded that the exciting mechanism is a self-induced vibration. In other words the exciting force is caused by the cylinder vibration itself under the special flow velocity, say, the nondimensional velocity of about 5.

In addition, the theoretical analysis under this consideration is shown to be able to explain these test results well.

1. Introduction

Tremendously many studies\(^{[1]}\)\(^{[2]}\) have been undertaken in reference to the vibration of the cylinder sustained elastically in a flow, but even now there are some unresolved problems in details. In almost all these studies the alternating lift force acting on a cylinder normal to the direction of a flow has been shown in the correlation to the Reynolds number. However, the ratio of the amplitude of the cylinder to its diameter seems to be a more important factor but studies from a point of view of such correlation are very few. Therefore even now the detail of the mechanism of excitation by a flow is not yet clear.

In this research the ultimate aim is to clarify the exciting mechanism and the correlation between the exciting force and the ratio of the vibration amplitude to the cylinder diameter. The study consists of the following three experimental works and the consideration based on these test results.

1) The wind tunnel test on an elastically supported cylinder 50 mm in diameter.
2) The two dimensional model basin test on a cylinder 25 mm in diameter oscillated with a crank mechanism.
3) The wind tunnel test on a cylinder 100 mm in diameter oscillated with a crank mechanism.

2. The wind tunnel test on an elastically supported cylinder

2-1 The purpose of the experiment

The condition of the occurrence of the vibration caused on a cylinder sustained in a flow has been studied by many investigators but problems in detail are still unsolved. Therefore in this experiment of which the apparatus consists of an elastically supported cylinder its aims are to outline the phenomena and to clarify the correlation between the exciting force which is calculated from test results and the ratio of the amplitude of the cylinder to its diameter.

2-2 The experimental apparatus

The wind tunnel is Eiffel type with a test section of 150 mm width and 720 mm height. In this test section a test cylinder with diameter of 50 mm and length of 150 mm is sustained by two cantilevers at its each end so as to be able to oscillate normal to the direction of a flow as shown in Fig. 1. The test cylinder is made of a brass sheet with thickness of 0.1 mm to lighten its weight for an experiment in the wide range of damping ratio as much as possible with varied damping force of the damper.

The spring constant is determined by the test weight sustained at midspan of the cylinder and its deflection. The correlation between the deflection of the cylinder and the output of strain gage set near the fixed point of the cantilever is calibrated previously, and in this way the amplitude of the cylinder during the experiment is measured with
the oscillograph.

Two kinds of cantilevers different in thickness are prepared and the following two sets of cylinder and spring system are used.

Spring constant in kg/cm 1.71 and 3.45
Natural frequency in c/sec 31.8 and 45.3
Equivalent weight of cylinder in kg 0.042 and 0.046

The damper is a flat plate type for the linearity of the damping force and it is filled with a mixed liquid of water and glycerin.

2-3 The experimental procedure

Filling the damper with a suitable damping liquid the logarithmic decrement is measured from the oscillogram of a free damping vibration of the cylinder caused by a shock. And then the wind tunnel is started and the range of the wind velocities causing a vibration of the cylinder is determined and varying in this range the velocity in fine steps the oscillograph recording is undertaken.

2-4 The experimental result

In Fig. 2 the experimental result for the natural frequency of the test system of 31.8 c/sec is shown. The horizontal scale is damping ratio $2m\delta/\rho d^2$ (where $m$; mass per unit length of the cylinder, $\delta$; logarithmic decrement, $\rho$; density of air) and the vertical scale on left side is the upper and lower limit of the nondimensional velocity of the vibration occurring range and the vertical scale on right side is the maximum amplitude of the cylinder in this range.

The vibration occurring range which is called the diagram of stability criterion is almost the same as in previous studies. However the variation of the maximum amplitude with the damping ratio considerably differs from the present results, namely at a damping ratio of about 15 the vibration amplitude of the cylinder suddenly decreases to nearly zero. Another side when the damping ratio decreases the amplitude inversely increases but at the damping ratio of zero the vibration amplitude seems not to be infinity and it is not recognized to be inversely proportional to the damping ratio. In other words it is considered that the vibration is not an ordinary forced vibration in which the amplitude is inversely proportional to the damping ratio under a constant exciting force.

2-5 The consideration of the experimental result

As previously stated the mechanism of the excitation leaves some unknown problems but if it is an ordinary forced vibration the alternating force is calculated from the decrement and the cylinder amplitude as follows. The equation of motion of the system is:

$$M\ddot{y} + C\dot{y} + Ky = P \sin at$$  \hspace{1cm} (1)

where $M$: equivalent mass of the cylinder
$C$: damping coefficient
$P$: amplitude of equivalent exciting force
then in $a$ is the cylinder amplitude at resonance, $P$ is given by:

$$P \equiv \frac{1}{\pi} aK\delta$$  \hspace{1cm} (2)

The alternating lift coefficient is usually stated as a ratio of the exciting force to the product $F$, $\frac{F}{\rho d^2}$.

![Fig. 2 Experimental result of elastically supported cylinder](image)

![Fig. 3 Equivalent lift coefficient $C_L$](image)
of the velocity head and the sectional area of the cylinder normal to the flow. If \( l \) is the length of the cylinder, \( F' \) is defined as:

\[
F' = \frac{1}{2} \rho V^2 dl \tag{3}
\]

then the lift coefficient \( C_L \) is

\[
C_L = \frac{P}{F'} = \frac{2}{\pi} \frac{K_a \delta}{\rho V^2 dl} \tag{4}
\]

If the representative velocity of the flow is the value at nondimensional velocity of 5.0, from numerical values of the experimental apparatus \( C_L \) is

\[
C_L = 18.1a \delta \tag{5}
\]

The lift coefficient calculated by the Eq. (5) with experimental values of \( a \) and \( \delta \) is shown in Fig. 3. As seen in Fig. 3 the lift coefficient is not a constant value but variable with the cylinder amplitude. Namely when the cylinder amplitude is small the lift coefficient is small also and when the amplitude becomes too large the lift coefficient decreases agains; with still larger amplitude it is supposed to act as damping force.

From these considerations and the fact that the frequency of the cylinder vibration is constant during the test, as previously pointed out in some literatures it is presumed that such vibration is a self-induced type.

3. The two-dimensional model basin test

3-1 The purpose of the study

As stated in previous paragraph if it is considered that when the velocity of the flow becomes a suitable value, an exciting force is induced by the cylinder vibration itself, it is presumed that under such condition the flow around the cylinder i.e. an appearance of the vortex in the early wake behind the cylinder and the separation point of the boundary layer fluctuates in large scale.

In this experiment the purpose of the study is to offer a guide referring to the variation of the static pressure on the cylinder surface and the detail of the exciting mechanism.

3-2 The experimental apparatus

The experimental apparatus is shown in Fig. 4. The model basin is 185 mm in width and the depth of water is 50 mm. In the upstream side there are a large tank and a settling screen.

The diameter of the model cylinder is 25 mm and a circular plate of the diameter of 40 mm is fitted at the lower end of the cylinder. The cylinder is oscillated normal to the direction of the flow by a crank mechanism which is driven by an electric motor with the amplitude of 4.5 mm.

The crank shaft has a cam which is set at the optional phase angle divided in eight equal parts.
of one revolution and the cam drives the micro switch that is connected to the magnetic shutter of a camera.

3-3 The experimental procedure

The velocity of the water flow in model basin is adjusted to 50 mm/sec by water supply to the tank. In the next place the crank is driven at nondimensional frequency of 0.1, 0.2 or 0.3 and the stream line around the cylinder is photographed. Stream lines are recorded as a dispersion of alminium powder on the water surface and with a considerably long time exposure of 1/15 second in photography.

3-4 The experimental result

The appearance of the vortex behind the cylinder and the fluctuation of the separation point of the boundary layer on the cylinder surface are examined on enlargement of these photographs. Examples of photographs taken at nondimensional frequency of 0.2 are shown in Fig. 5 (a) to (d). Figure 5 (a) is the photograph recorded when the cylinder passes the top dead centre and (b) the forward mid-stroke, (c) the bottom dead centre and (d) the backward mid-stroke.

According to these photographs separation points on both sides of the cylinder are just lateral points when the cylinder passes both dead centres, however when the cylinder is passing the forward mid-stroke as shown in Fig. 5 (b) the separation point at upper side of the cylinder deviates to the down stream, while the lower side separation point deviates to the upstream.

Figure 5 (e) is the photograph of the stationary cylinder. As seen in this photograph when the cylinder is stationary, separation points do not shift and vortex sheets on both side in the wake behind the cylinder seem to be nearly equal to each other.

The variation of the separation point is shown in Fig. 6; as seen in top of Fig. 6 the deviation in clockwise direction is positive and the movement of the cylinder on upper side is positive. In this figure point marks indicate the upper side separation point and cross marks indicate the lower side and (a), (b) and (c) are test results at non-dimensional frequency of 0.2, 0.1 and 0.3 respectively. In Fig. 6 (b) the oscillation of the cylinder is shown simultaneously.

In Fig. 6 (a) for nondimensional frequency of 0.2 the variation of separation points is considerably large and its phase angle leads about π/2 as compared with the cylinder movement in Fig. 6 (b).

Such variation of separation points signifies an existence of a circulation around the cylinder. In other words for example in the case shown in top of Fig. 6 it is presumed that the lift force in forward direction must be induced. Therefore in Fig. 6 (a) the lift is in phase with the velocity of the cylinder oscillation.

Farther in Figs. 6 (b) and (c) in which non-dimensional frequencies are different from 0.2 the variation of separation point is not so large as (a) and the phase angle is close to that of the displacement of the cylinder.

According to these experimental results as stated in previous paragraph, it is considered that the mechanism of the excitation is self-induced type in which the exciting force is induced by the cylinder vibration itself only under the special range of the flow velocity.

4. The wind tunnel test on a cylinder oscillated by a crank

4-1 The purpose of the study

In almost all former investigations in regard to the lift acting on a cylinder in a flow, a load cell was used at the position of the cylinder support for the lift measurement. But in this way an inertia force of the cylinder appears in the output signal of the load cell when the cylinder is oscillated of course or it is supported elastically and even nearly fixed.
In this farther experiment the purpose is to get a precise explanation by a different experimental method. Namely in this case the circular cylinder is oscillated by a crank normal to the direction of the air flow and the static pressure variation on the cylinder surface is measured.

4.2 The experimental apparatus

The experimental apparatus is shown in Fig. 7. The wind tunnel is the same one as used in the experiment stated in chapter 2. The test cylinder made of brass sheet with thickness of 1.0 mm is 104 mm in diameter and 300 mm long.

Two static pressure holes are drilled on the cylinder surface at mid span and the cylinder is fixed on its end plate at an arbitrary angle to the direction of flow at each 30 degrees.

The exciting device consists of an electric motor, a variable speed transmission and a crank mechanism. The crank radius is selected as 20, 10, 5 or 2.5 mm and the forcing frequency is variable continuously from 10 c/sec to 25 c/sec.

The static pressure pickup is the same type as a condenser microphone shown in Fig. 8 but for the experimental purpose its frequency response is made uniform in a low cycle range and it is able to indicate a stationary pressure also. The membrane of the pickup is made of a thin rubber sheet so as to be able to respond to a very low pressure variation and a thin alminium leaf is affixed on the membrane as an earth plate. In this way the noise caused by the fluctuating stray capacitance induced by the cylinder vibration must be avoided, therefore the noiseless operation is checked with closed static pressure holes.

4.3 The experimental procedure

After these pressure pickups are previously calibrated the wind tunnel is operated and the flow velocity is measured by the manometer reading. At the proper flow velocity the crank is driven and the oscillograph recording is performed at various frequencies. The mark of the bottom dead centre is recorded simultaneously on the oscillogram by an electro-magnetic pickup on the flywheel.

4.4 The experimental results

A typical example of the oscillogram is shown in Fig. 9 (a). As seen in it the alternating static pressure synchronizes with the vibration of the cylinder. However the phase difference between the oscillation of the cylinder and the pressure fluctuation shifts with the nondimensional velocity. Therefore the pressure should be distinguished between an
in-phase component for the oscillating velocity of the cylinder and an in-phase component for the oscillating deflection, namely as the exciting force component and the restoring or inertia force component. Consequently the harmonic analysis is undertaken on continuous five cycles of each recording for an average value.

Some examples of these analysed results are shown in Figs. 10 (a) and (b). In this case the velocity of the main flow is 10 m/sec and amplitudes of the cylinder are 20 and 2.5 mm respectively.

In these figures the horizontal scale indicates the nondimensional velocity and the vertical scale indicates the amplitude of the static pressure. On upper side of each figure the cosine component of alternating pressure based on the bottom dead centre is shown and on lower side the sine component is shown.

From these experimental results the following points are concluded:

1) The sine component of the fluctuating static pressure becomes a considerably large value at the nondimensional velocity of about 6 to 7 and according to its phase angle the lift acting on the cylinder is in-phase to the velocity of the cylinder vibration, in other words the lift becomes an exciting force.

2) In addition the cosine component of the pressure affects the inertia force according to the phase angle consideration.

3) It is a very important fact that the amplitude of the pressure decreases with the diminishing amplitude of the cylinder as seen in Fig. 10 (a) and (b), and in addition when the cylinder is stationary in a flow, the fluctuation of the pressure can not occur as shown in Fig. 9 (b).

These experimental results prove that the mechanism of the excitation is a self-induced vibration.

5. The consideration of the mechanism of the excitation

5-1 Birkhoff's explanation on the Strouhal number

Birkhoff explained the Strouhal number considering the equilibrium between the hydraulic restoring force acting on the dead stream behind the cylinder and the inertia force of the dead stream. As this consideration is a starting point in this paragraph it may be better to describe the outline of it. Figure 11 shows a model of the flow around the cylinder. The angular oscillation of a dead stream with the centre of gravity at G about the axis of the cylinder is given by the following equation.

\[ I \frac{d^2\alpha}{dt^2} + k\alpha = 0 \] .................................(6)

where

\[ I : \text{the moment of inertia of the dead stream} \]

\[ = 2\rho h \left( \frac{d}{2} + l \right) \] .................................(7)

\[ k : \text{the coefficient of restoring moment acting on the} \]

Fig. 10 Amplitude of oscillating static pressure

Fig. 11 Birkhoff's model
dead stream = \frac{2\pi\rho V^3}{\frac{d}{2} + l} \cdot l \quad \cdots \cdots \cdot (8)

\alpha : \text{ the inclination of the dead stream}
2l : \text{ the length of the dead stream}
h : \text{ the width of the dead stream}

Then the natural frequency of the dead stream \( f_s \) is shown as follows

\[ f_s = \frac{\omega_0}{2\pi} = \frac{V}{\sqrt{4\pi h \left( \frac{d}{2} + l \right)}} \quad \cdots \cdots \cdot (9) \]

If it is assumed that \( h = 1.25d \) and \( l = 1.1d \) the Strohman number \( S_t \) is shown as follows

\[ S_t = \frac{f_s d}{V} = 0.2 \quad \cdots \cdots \cdot (10) \]

According to the photograph of the model basin test in chapter 3 these values of \( h \) and \( l \) may be proper, therefore Birkhoff's theory is natural for the explanation of the outline of this phenomenon.

5-2 The equation of the self-induced vibration

Adding to Eq. (6) the damping force \( c \frac{d\alpha}{dt} \) as shown in Fig. 12 which is a model of the elastically supported cylinder in a flow and the alternating force caused by the cylinder vibration \( y = a \sin \omega_t t \). Eq. (6) is described as follows.

\[ l \frac{d^2\alpha}{dt^2} + c \frac{d\alpha}{dt} + k(\alpha - \alpha_0) = \omega_t^2 I \alpha \sin \omega_t t \quad \cdots \cdots \cdot (11) \]

where

\[ \alpha_0 = \frac{\dot{y}}{V} = -\frac{\omega_0}{V} \sin \omega_0 t \quad \cdots \cdots \cdot (12) \]

and according to the proportion of the dead stream previously stated

\[ \dot{\alpha} = \frac{a}{d^2 + l} = 0.625 \frac{a}{d} \quad \cdots \cdots \cdot (13) \]

then with \( k/l = \omega_t^2 \)

\[ \frac{d^2\alpha}{dt^2} = \frac{c}{I} \frac{d\alpha}{dt} + \omega_t^2 \alpha \]

\[ = \sqrt{\left( 0.625 \frac{\omega_t^2}{V^2} \right)^2 + \left( \omega_t \frac{\omega_0}{V} \right)^2} \sin(\omega_0 t - \delta_t) \quad \cdots \cdots \cdot (14) \]

where

\[ \delta_t = \tan^{-1} \frac{\omega_t \frac{\omega_0}{V}}{0.625 \frac{a}{d}} \quad \cdots \cdots \cdot (15) \]

\[ \alpha_0 = \frac{\omega_t}{V} \frac{\omega_0}{V} \sin \omega_0 t \quad \cdots \cdots \cdot (12) \]

then

\[ \alpha = \sqrt{\left( 0.625 \frac{\omega_t^2}{V^2} \right)^2 + \left( \omega_t \frac{\omega_0}{V} \right)^2} \]

\[ \times \sin(\omega_0 t - \delta_t) \quad \cdots \cdots \cdot (16) \]

where

\[ \delta_t = \tan^{-1} \frac{2\varepsilon \omega_0}{\omega_t^2 - \omega_0^2} \quad \cdots \cdots \cdot (17) \]

\[ 2\varepsilon = \frac{c}{I} \quad \cdots \cdots \cdot (18) \]

As the effective angular displacement which causes the lift by the Magnus effect is \( \alpha - \alpha_0 \), using Eq. (12)

\[ \alpha_{\text{eff}} = \sqrt{\left( 0.625 \frac{\omega_t^2}{V^2} \right)^2 + \left( \omega_t \frac{\omega_0}{V} \right)^2} \]

\[ \times \sin(\omega_0 t - (\delta_t + \delta_2)) + \frac{\omega_0}{V} \cos \omega_0 t \quad \cdots \cdots \cdot (19) \]

Dividing Eq. (19) in the term of \( \sin \omega_0 t \) and that of \( \cos \omega_0 t \)

\[ \alpha_{\text{eff}} = -A \sin(\delta_t + \delta_2) + \frac{\omega_0}{V} \sin \omega_0 t \quad \cdots \cdots \cdot (20) \]

where

\[ A = \sqrt{\left( 0.625 \frac{\omega_t^2}{V^2} \right)^2 + \left( \omega_t \frac{\omega_0}{V} \right)^2} \quad \cdots \cdots \cdot (21) \]

Then the lift \( F_\alpha \) is shown as follows.

\[ F_\alpha = f_m \frac{A}{\omega_0} \sin(\delta_t + \delta_2) - \frac{1}{V} \frac{dy}{dt} \quad \cdots \cdots \cdot (22) \]

\[ \equiv f \frac{dy}{dt} - F_\alpha \quad \cdots \cdots \cdot (23) \]

where \( f_m \) is the proportional constant between the lift \( F_\alpha \) and the effective angular displacement \( \alpha_{\text{eff}} \).

In the next place, as the direction of the drag \( F_d \) is identical to the inlet angle of the relative velocity \( \alpha_\nu \), the component of the drag \( F_\nu \) in direction of axis is shown as follows.

\[ F_\nu = -\frac{1}{2} p \left( V^2 + y^2 \right) c_d \frac{dy}{dt} \frac{dy}{dt} \quad \cdots \cdots \cdot (24) \]

As it is considered that when the cylinder amplitude is large \( y^3 \) is affected considerably as the damping force

\[ F_\nu = -\frac{1}{2} p \rho c_d V \frac{dy}{dt} \frac{1}{V} \frac{dy}{dt} \quad \cdots \cdots \cdot (25) \]

\[ \equiv -F_\nu \frac{dy}{dt} - F_\nu \frac{dy}{dt} \quad \cdots \cdots \cdot (26) \]

when these hydraulic forces (23), (26) affect the cylinder shown in Fig. 12 the equation of motion of the cylinder is expressed as follows.

\[ \frac{dy}{dt} = F_\nu - F_\nu \frac{dy}{dt} + F_\nu \frac{dy}{dt} + K + F \quad \cdots \cdots \cdot (27) \]
As seen in Eq. (27) when the coefficient of the term is positive the self-excited vibration must be induced and it is presumed that $F_1$ becomes a large value only at $\omega_c = 2\omega_b$.

As the space is not enough to show the result of the numerical calculation of Eq. (23) and Eq. (26), it is added only that the calculated result is able to explain these experimental phenomena substantially.

6. The conclusion and the acknowledgment

In this research three different experiments on the cylinder vibration caused by the wake force are undertaken. From these test results having a large end effect, it is concluded that the exciting mechanism is a self-induced vibration.

In other words, as there is a forced vibration on the dead stream behind the cylinder caused by the cylinder oscillation, when the forced vibration is in a resonance state the feedback loop which consists of the elastically supported cylinder and its wake becomes positive. In addition, the theoretical analysis under these considerations is shown to be able to explain these experimental results substantially.

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References