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Viscous Frictional Moment between Eccentric Rotating Cylinders when Outer Cylinder Rotates*

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Although many studies on journal bearings have been reported as regards eccentric rotating cylinders, there is no frictional moment (torque) measurement which has been done for various eccentricity ratios and radius ratios in condition that there is sufficient fluid to fill the clearance in journal bearing. Recently, Oguro, Coles and Van Atta reported that under certain conditions a pattern of alternating helical stripes of laminar and turbulent flow could be observed in the gap between concentric rotating cylinders, with the outer cylinder rotating. However the critical Reynolds number has not been investigated in detail.

In this paper the torque acting on the inner cylinder in eccentric rotating cylinders is measured over the range from the laminar flow to the fully developed turbulent flow, with the outer cylinder rotating. Torque measurements are done using water, glycerine-water solution and spindle oil, and increasing the fluid pressure in order to prevent the occurrence of cavitation. And we studied experimentally the relation between torque coefficient and a few parameters, such as rotating Reynolds number R_{ω} , eccentricity ratio etc., and obtained $R_{\omega K}$ at which transition to turbulent flow occurs.

1. Introduction

It is well known that the stability of a viscous flow in the small annular gap between concentric rotating cylinders, with the inner one rotating, was first considered experimentally and theoretically by Taylor⁽¹⁾. Recently many theoretical studies have been reported on Taylor instability in the case of a large annular gap without axial flow or a small annular gap with axial flow.

Concerning the case, in which the outer cylinder rotates and the inner one is stationary, Taylor⁽²⁾ and others^{(3) (4)} reported that transition from the laminar flow to the turbulent one could be observed in their measurements of viscous frictional torque, and that the torque in a turbulent flow was greater than its theoretical value in a laminar flow.

Subsequently Schultz-Grunow⁽⁵⁾ showed in his theoretical analysis and experimental observation that the flow between two concentric cylinders, of which the outer one rotated and the inner was at

rest, was completely stable. Then Schultz-Grunow stated that transition from the laminar region to the turbulent one should have been caused due to eccentric arrangement in the experiments by Taylor⁽²⁾ and by others^{(3) (4)}.

Recently, however, Coles⁽⁶⁾ and Van Atta⁽⁷⁾ reported that a pattern of alternating helical stripes of laminar and turbulent flow, i.e. spiral turbulence, could be observed in the gap between two concentric cylinders, of which the outer one rotated and the inner was stationary, or which counter-rotated, and that transition to the turbulent flow occurred when the number of revolutions increased.

Although many practical studies have been reported on journal bearings, it seems that no torque measurements were carried out for various eccentricity ratios and radius ratios in the condition that there was sufficient fluid to fill the clearance in journal bearing or between the eccentric rotating cylinders.

In this paper the frictional torque acting on the inner cylinder in the eccentric rotating cylinders was measured over the range from the laminar flow to the fully developed turbulent one, with the outer cylinder rotating. Torque measurements were done by the use of water, glycerine-water solutions and spindle oil, with an increased fluid pressure in order

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to prevent the occurrence of cavitation. And we studied experimentally the relation between torque coefficient and a few parameters, such as rotating Reynolds number R_{ω} , eccentricity ratio etc., and obtained $R_{\omega K}$ at which transition to the turbulent flow occurred.

Nomenclatures

r_o : inside radius of outer cylinder
 r_i : outside radius of inner cylinder
 $\delta = r_o - r_i$: mean clearance
 $r_m = (r_o + r_i)/2$: mean radius
 ω : angular velocity of outer cylinder
 e : eccentricity
 $m = e/\delta$: eccentricity ratio
 $R_{\omega} = \omega(r_o^2 - r_i^2)/(2\nu)$: rotating Reynolds number
 ν : kinematic viscosity
 $C_M : M/(2\pi r_m^4 \omega^2)$: coefficient of frictional moment
 M : frictional torque acting on the unit length of inner cylinder
 $R_{\omega K}$: critical Reynolds number
 Suffix c indicates the case of concentric rotating cylinders.

2. Experimental apparatus and method

The general arrangement of the apparatus is shown in Fig. 1. The inner cylinder, whose outside diameter is 63.49 mm, is divisible into three parts ①, ② and ③, and each part is supported by ball bearings respectively; ① is a section, whose length is 180 mm, to measure the torque, ② is an up-stream section, whose length is 130 mm, to eliminate the end effect in the case with no axial flow or the effect of inlet length in the case with axial flow, and ③ is a down-stream section, whose length is 51.5 mm, to eliminate the effect of the end and outlet. The part ① can rotate very easily and the clearance between ① and ② or ③ is very small

Fig. 1 Experimental apparatus

Table 1 Dimensions of rotating cylinders

Sign of sleeve	Inside radius of outer cylinder r_o mm	Mean clearance δ mm	Radius ratio r_o/r_i	Clearance ratio δ/r_o
A	32.165	0.420	1.013	0.013 06
B	32.569	0.825	1.026	0.025 33
C	33.214	1.470	1.046	0.044 26
D	35.047	3.303	1.104	0.094 24

(about 0.02 mm) in order to reduce the leakage which passes through the annular clearance.

The inside radius of outer cylinder, which is shown in Table 1, is changed by inserting a brass pipe into the outer cylinder ④.

The test fluids used in the experiments are water, 41.35 per cent and 53.58 per cent glycerine-water solutions and spindle oil.

In the experiments, the inner cylinder is kept stationary while the outer one rotates. Torque, which is transmitted from the outer cylinder to the inner one, is measured by means of strain-gage type torque meter. The temperature of test fluid is measured by thermometers ⑥ or a thermocouple which is installed on the wall of inner cylinder. After a short time of test runs, test fluid is allowed to flow axially in order to make no temperature difference in the place between the thermometer and the section ①, because the temperature of test fluid increases locally owing to heat generation due to metal-to-metal contact of mechanical seal.

During the test runs, fluid pressure in the gap between the inner and the outer cylinder is increased in order to prevent the occurrence of cavitation. Both ends of the cylinders are closed by the closing plate ⑦ in order to prevent the axial flow, which will occur due to the pressure distribution around the inner cylinder, if the ends of the cylinders are not closed by plates, in the experiments of nonconcentric rotating cylinders. For comparison, torque measurements in the case, where the ends of the cylinders are not closed, are done for the largest gap ($r_o/r_i = 1.104$).

The means for making the inner and outer cylinders concentric or eccentric are shown in Fig. 2, where ⑧ is a dialgauge, ⑩ is a bar made of steel which is inserted into a hole ⑨, ⑪ is an anchor bolt, ⑫ is a screw, by which the outer cylinder can

Fig. 2 Schematic view of the method of adjustment for eccentricity

be moved right and left, and ⑬ is a screw, by which the outer cylinder can be moved upward. Whereas the inner cylinder can be settled, the outer cylinder can be moved and positioned at any eccentricity ratio from $m=0$ to $m=1$ by the above-mentioned method (see Fig. 2). It is estimated that by means of this arrangement, the eccentricity ratio m can be set to within an accuracy of ± 0.001 .

3. Viscous frictional moment in the flow between concentric rotating cylinders

In the laminar flow the theoretical frictional moment M_c between concentric rotating cylinders can be given by $M_c = 4\pi\mu\omega r_o^2 r_i^2 / (r_o^2 - r_i^2)$, while using the coefficient of viscous frictional moment C_M defined by $C_M = M / (2\pi\rho r_m^4 \omega^2)$, C_{MC} is given approximately in the laminar flow by

$$C_{MC} = \frac{\left(1 - \frac{\delta}{r_o}\right)^2}{R_o \left(1 - \frac{1}{2} \frac{\delta}{r_o}\right)^4} = \frac{\left(1 - \frac{\delta}{r_o}\right)^2}{R_o \left\{1 - \frac{\delta}{r_o} + \left(\frac{\delta}{2r_o}\right)^2\right\}^2} \approx 1/R_o \quad (1)$$

where the terms $(\delta/r_o)^2$ can be neglected. It is estimated from Table 1 that Eq. (1) is within 1 percent accuracy even in the case of maximum radius ratio, so that Eq. (1) is correct in the case of the other radius ratio used in these experiments (see Table 1).

C_M versus R_o curves measured for $r_o/r_i = 1.013$, 1.026, 1.046 and 1.104 are shown in Fig. 3. Experimental results agree very well with Eq. (1) in the laminar flow region, as shown in Fig. 3. In the turbulent-flow region the experimental results of C_M decrease at the same value of R_o , as radius ratio increases. These experimental results agree with those obtained by Taylor⁽²⁾ and by others⁽³⁾⁽⁴⁾. Schultz-Grunow⁽⁵⁾ showed complete stability with perfect concentricity according to his observation, and considered that the transition to the turbulent flow, which was observed by Taylor⁽²⁾ and by

Fig. 3 Relation between C_M and R_o in the concentric

others⁽³⁾⁽⁴⁾, was caused by the imperfection in the geometrical shape, e.g. eccentric arrangement. However we also observed the transition to the turbulent flow even in our experiments, where the inner and outer cylinders were arranged with very precise concentricity.

4. Viscous frictional moment in the flow between eccentric rotating cylinders

4.1 Coefficient of viscous frictional moment in the laminar flow

When the outer cylinder rotates and the inner one is stationary, the frictional torque acting on the unit length of the inner cylinder is given in the case, where the effect of curvature can be neglected and Reynolds number is so small that the inertia effects can be neglected, as well as the case of the inner cylinder rotating and the outer cylinder being stationary⁽⁸⁾, by

$$M = 4\pi\mu\omega r_m^3 \frac{\sqrt{\delta^2 - e^2}}{2\delta^2 + e^2} \quad (2)$$

Expressing Eq. (2) by means of C_M gives

$$C_M = \frac{1}{R_o} \frac{\sqrt{1-m^2}}{1 + \frac{1}{2}m^2} \quad (3)$$

It is noticed from the above equation that C_M decreases linearly with an increase of R_o in the case where the inertia effects can be neglected, and that C_M decreases with an increase of m . The ratio of C_M in the case of eccentric rotating cylinders to C_{MC} in the case of concentric rotating cylinders is given, using Eqs. (1) and (3), by

$$\frac{C_M}{C_{MC}} = \frac{\sqrt{1-m^2}}{1 + \frac{1}{2}m^2} \quad (4)$$

Experimental results for $r_o/r_i = 1.013$, 1.026, 1.046 and 1.104, where axial flow which occurred due to the pressure distribution between the eccentric rotating cylinders was prevented by the closing plates which were furnished at the ends of the cylinders, are shown at various eccentricity ratios

Fig. 4 Relation between C_M and R_o in the eccentric rotating cylinders

in Figs. 4 through 7.

The experimental results obtained with a very close clearance ($\delta=0.420$ mm) are shown in Fig. 4, where the C_M - R_w curves which are perfectly linear are the curves of C_M given by Eq. (3). The experimental values of C_M for $m=0, 0.25$ and 0.5

Fig. 5 Relation between C_M and R_w in the eccentric rotating cylinders

Fig. 6 Relation between C_M and R_w in the eccentric rotating cylinders

Fig. 7 Relation between C_M and R_w in the eccentric rotating cylinders

agree very well with Eq. (3). On the other hand, the experimental values of C_M for $m=0.75$ are considerably larger than C_M given by Eq. (3).

The experimental results obtained with a clearance of $\delta=0.825$ which is about two times the clearance for Fig. 4 are shown in Fig. 5, where the values of C_M for $m=0, 0.25$ and 0.5 are in as close agreement with Eq. (3) as in Fig. 4. However, it is seen from more precise observation of C_M made for $m=0.25$ and 0.5 in Fig. 5 that the values of C_M agree very well with Eq. (3) in the range of small R_w , while C_M decreases gradually from the theoretical values given by Eq. (3) with an increase of R_w above $R_w=800$ for $m=0.25$ and above $R_w=600$ for $m=0.5$. And this tendency continues up to the transient regime. For $m=0.75$, on the other hand, C_M agrees well with Eq. (3) up to $R_w=240$, while in the range of R_w above 240 the values of C_M become greater than the values given by Eq. (3) with an increase of R_w .

Experimental results for a larger clearance ($\delta=$

1.470 mm) are shown in Fig. 6. Although the tendency of C_M versus R_w in Fig. 6 is same as in Figs. 4 and 5, the discrepancy between the experimental results and the values given by Eq. (3) at large R_w is greater in this case than in the case of Figs. 4 and 5.

Figure 7 shows the experimental results for $m=0.375$ and 0.625 other than $m=0.25, 0.5$ and 0.75 , obtained with the largest clearance in our experiments. In the case of this clearance ratio, the values of C_M agree very well with C_M given by Eq. (3) for any eccentricity ratio in the range of smaller R_w . However the experimental C_M - R_w curves depart gradually downward from the C_M - R_w curves given by Eq. (3) with an increase of R_w in the range of larger R_w for $m \leq 0.625$, and depart gradually upward for $m=0.75$, as can be seen in Fig. 8, where C_M - R_w curves are shown for $m=0.375$ and 0.75 . It is estimated that this tendency becomes pronounced with an increase of clearance ratio, as can be seen in Figs. 4 through 7. On the

other hand, Fig. 9 shows the experimental results obtained with the same clearance as in Fig. 7 but with the ends of the cylinders not being closed.

Table 2 Comparison of C_M/C_{MC}

m	Theoretical value of C_M/C_{MC}	
	Neglecting the effect of curvature	Considering the effect of curvature
0.1	0.990 0	0.985 6
0.2	0.960 6	0.956 3
0.3	0.912 9	0.908 8
0.4	0.848 6	0.845 0
0.5	0.769 8	0.766 6
0.6	0.678 0	0.675 2
0.7	0.573 6	0.571 4

Fig. 8 Relation between C_M and R_w in the laminar-flow region Fig. 9 Relation between C_M and R_w in the eccentric rotating cylinders (in the case where both ends of eccentric rotating cylinders are not closed)

It is seen from comparison between Fig. 7 and Fig. 9 that only for $m=0.75$, C_M in the case without the closing plates is a little greater in the transient region than C_M in the case with the closing plates, but that C_M in both cases is in good agreement except in the range of R_w described above. Therefore it is estimated that the experimental values of C_M for the other radius ratios, at which the clearance is smaller than in Fig. 7 or 8, are not dependent on whether the ends of cylinders are closed or not.

When we take C_M/C_{MC} as the ordinate, m as the abscissa, we obtain Figs. 10 (a), (b) and (c) for $r_o/r_i=1.026$, 1.046 and 1.104. Experimental results agree with C_M/C_{MC} given by Eq. (4) for any radius ratio up to $R_w=200$, but do not agree for $R_w \geq 500$. And the discrepancy between experimental results and Eq. (4) increases with R_w at the same radius ratio and eccentricity ratio. With R_w increasing especially in the case of $r_o/r_i=1.104$, which is the largest clearance, experimental results of C_M/C_{MC} become much smaller than C_M/C_{MC} given by Eq. (4) for $m \leq 0.65$, while experimental results for $m=0.75$ become larger than Eq. (4). This tendency is common to all clearance ratios used in present experiments, and becomes more pronounced with an increase of radius ratio at the same R_w .

In Table 2, the theoretical values of C_M/C_{MC} in the case, where both the effects of inertia force and curvature are neglected, are compared with the theoretical values of C_M/C_{MC} in the case⁽¹⁰⁾, where the inertia effects are neglected but the effect of curvature is considered, for $r_o/r_i=1.104$. It is considered from Table 2 that the curvature influences C_M little over the range of radius ratio used in the present experiments, because the theoretical

values of C_M/C_{MC} in the former case are in close agreement with the theoretical values of C_M/C_{MC} in the latter case.

As can be seen in Fig. 8, Eq. (3) can not be applied in the range of R_w over a certain value, because the experimental results go upward from the C_M - R_w curves given by Eq. (3) for $m \geq 0.65$ and downward for $m < 0.65$. Although it can be noted that one of the possible reasons for the aforementioned fact is the inertia effects of the fluid between eccentric rotating cylinders, the actual fact is not

clarified.*

4.2 Coefficient of viscous frictional moment in the turbulent flow.

As can be seen in Fig. 4, where experimental values of C_M for $r_0/r_i=1.013$, $m=0.25$ are shown, C_M increases sharply at the transient R_w , at which the transition from the laminar flow to the turbulent flow occurs. After C_M increases with R_w for a short range of R_w , C_M decreases gradually for $R_w \geq 3000$. This is the same tendency as for $m=0$. For $m=0.5$, the increase of C_M at the transient R_w is less sharp. For $m=0.75$, C_M decreases gradually with an increase of R_w for $R_w \leq 1200$, while C_M is constant (about 6.5×10^{-4}) for $R_w \geq 1200$. At the

same R_w for $R_w \geq 6000$, C_M decreases with an increase of eccentricity ratio. The aforementioned tendency is same as for $r_0/r_i=1.026$ and 1.046 . Further we should like to add that an indicator of strain meter vibrated violently in the transient region from the laminar to the turbulent flow for any radius ratio and eccentricity ratio.

For $r_0/r_i=1.104$, a phenomenon of great interest was observed. At the same R_w for $R_w \geq 30000$, C_M decreased with an increase of eccentricity ratio and decreased gradually with an increase of R_w just like in the case of other radius ratio. In the range of R_w for $3000 \leq R_w \leq 10000$, however, we could observe the under-mentioned phenomenon that could not be observed for other radius ratios except $r_0/r_i=1.104$ and varying eccentricity ratio. That is, as can be seen in Fig. 11, for $m=0.25$, C_M increases sharply at the transient R_w . In the range of R_w greater than the transient R_w , C_M decreases with an increase of R_w just like in the case for the other radius ratios. Namely C_M moves from region I into region III with an increase of R_w , as shown in Fig. 11. Region I is the laminar flow region, and the regions II and III are turbulent flow regions, as shown for $m=0.5$, 0.625 and 0.75 in Fig. 11. But for $m=0.5$, after C_M increases sharply with the occurrence of turbulence, C_M decreases up to $R_w \approx 3300$, and again begins to increase at $R_w = 6000$. Namely in this case, region II exists between region I and II; region II is taken over the range from R_w , at which a sharp increase of C_M can be seen first with the occurrence of turbulence, to R_w , at which a sharp increase of C_M can be seen secondly (at $R_w = 6000$ for $m=0.5$). For $m=0.625$, this tendency

Fig. 10 Relation between C_M/C_{MC} and m in the laminar flow

* We concluded in reference (9) that viscous frictional moment is not influenced by the inertia.

Fig. 11 Relation between C_M and R_w in the transient region

is more pronounced and region II becomes larger. For $m=0.75$, although R_w , at which C_M moves from region I into region II, can't be decided clearly ($R_w=800\sim 1800$), it is seen that region II becomes larger, and that the second sharp increase of C_M is very remarkable. It could not be decided only by these experimental results, whether the flow is perfectly turbulent or not in the region II. Considering that the indicator of strain meter vibrated violently at R_w between region I and region II, or in the region II, however, we should estimate that both the laminar and turbulent flows exist in the region II. In the case of eccentricity ratio, at which region II does not exist, such as $m=0.25$ and 0.375 , we observed a vibration of indicator of strain meter at R_w between region I and III. In the case with-

out the closing plates, C_M is a little different from C_M in the case with the closing plates in the region II, but is nearly the same as C_M in the latter case in the region III.

Relationship between C_M and m in the turbulent flow is shown in Figs. 12 (a)~(d). C_M decreases with an increase of eccentricity ratio at any radius ratio and R_w , but this tendency of C_M is dependent on R_w . C_M decreases with an increase of m at the same R_w in the region of very great R_w (e.g. $R_w \geq 20\,000$ for $r_o/r_i=1.046$ and $R_w \geq 40\,000$ for $r_o/r_i=1.104$), where C_M decreases in proportion to $R_w^{-0.35}$ for any eccentricity ratio, as can be seen in Figs. 6 through 8.

5. Critical Reynolds number

The value of C_M increases sharply at some R_w , where the transition from the laminar flow to the turbulent one occurs, and at the same time the indicator of strain meter vibrates violently. We define R_w , at which C_M increases sharply, as the

Fig. 12 Relation between C_M and m in the turbulent flow

critical Reynolds number $R_{\omega K}$. Relationship between $R_{\omega K}$ and δ/r_0 is shown in Fig. 13. $R_{\omega K}$ can be decided in the case of $m=0$ and 0.25 for $r_0/r_i=1.026$ and $m=0, 0.25$ and 0.5 for $r_0/r_i=1.046$ as shown in Figs. 4 through 6. In the case where a sharp increase of C_M can not be observed clearly, for example $m=0.5$ for $r_0/r_i=1.013$, we took the upper and lower critical Reynolds numbers. In the case of $m=0.75$ for $r_0/r_i=1.046$, $R_{\omega}=2000$ was taken as $R_{\omega K}$, because the indicator of strain meter vibrated violently at $R_{\omega}=2000$. This $R_{\omega K}$ is independent of eccentricity ratio at the same radius ratio.

In the case of $r_0/r_i=1.104$, as can be seen in Figs. 7 and 8, there is one critical Reynolds number for $m \leq 0.375$, for which C_M moves from region I into region III. But, as region II exists between region I and region III for $m \geq 0.5$, we can take two critical Reynolds numbers in such a case. Therefore we define R_{ω} , at which C_M moves from region I into region II, as first critical Reynolds number $R_{\omega KI}$, and R_{ω} , at which C_M moves from region II into region III, as second critical Reynolds number $R_{\omega KII}$. Although both $R_{\omega KI}$ and $R_{\omega KII}$ are nearly independent of eccentricity ratio, $R_{\omega KII}$ is about two times larger than $R_{\omega KI}$. Experimental results by Taylor⁽²⁾ and by Couette⁽¹¹⁾ are compared with our experimental results in Fig. 13. Our experimental results are a little smaller than the results by Taylor⁽²⁾ and by Couette⁽¹¹⁾ for $\delta/r_0 \leq 0.04426$ ($r_0/r_i=1.046$). For $\delta/r_0=0.09424$ ($r_0/r_i=1.104$), both $R_{\omega KI}$ and $R_{\omega KII}$ exist. The former locates on the extension to a straight line which is drawn through points of the present experimental results for $\delta/r_0 \leq 0.04426$, while the latter is in close agreement with the results obtained by Taylor⁽²⁾ whose

results increase sharply for $\delta/r_0 \geq 0.06$.

6. Conclusions

Experimental results are summarized as follows.

(1) In the flow between concentric rotating cylinders, we also observed the transition from the laminar to the turbulent flow, which had been observed in the torque measurements by Taylor⁽²⁾ and by others^{(3) (4)}, in spite of arrangement with very precise concentricity. The tendency of C_M in the turbulent flow is in close agreement with the experimental results reported previously.

(2) In the eccentric rotating cylinders, the experimental results are summarized as follows. When the clearance between cylinders is very small in comparison with the radius of outer cylinder, or when R_{ω} is small in spite of larger clearance, C_M agrees very well with Eq. (3). With an increase of R_{ω} , C_M-R_{ω} curve departs downward from Eq. (3) for $m \leq 0.65$ and upwards for $m > 0.65$. This tendency becomes pronounced with an increase of radius ratio.

In the turbulent flow, particularly at very great R_{ω} , C_M decreases with an increase of eccentricity ratio at the same radius ratio and R_{ω} . Near the critical R_{ω} , relationship between C_M and R_{ω} is very great depending on radius ratio and eccentricity ratio. Particularly for $r_0/r_i=1.104$, region II exists, as can be seen in Fig. 11.

(3) Critical Reynolds number is independent of eccentricity ratio at the same radius ratio. For $r_0/r_i \leq 1.046$, the critical Reynolds number observed in the present experiments increases in proportion to δ/r_0 . For $r_0/r_i=1.104$, two critical Reynolds numbers ($R_{\omega KI}$, $R_{\omega KII}$) can be obtained. $R_{\omega KI}$ locates on a straight line which is drawn through the present data points for $r_0/r_i \leq 1.046$ on log-log chart. On the other hand, $R_{\omega KII}$ is about two times larger than $R_{\omega KI}$ and agrees with the critical Reynolds number obtained by Taylor⁽²⁾.

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Fig. 13 Relation between critical Reynolds number and clearance ratio