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Study on Flow Characteristics in Right-Angled Pipe Fittings*

(2nd Report, On Case of Slurries in Hold up Flow)

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The authors made flow experiments on the three kinds of right-angled pipe fittings which had correctly geometrical shapes and were set in the horizontal plane. Their diameter ratios were 53.2/53.2, 53.2/42.1 and 53.2 mm/28.0 mm. Flyash and sand slurries were run through them in the "hold up flow" and in the various upstream velocities of the main flow and the flow conditions. Then the authors obtained the following conclusions:

(1) The densities of slurries in the downstream of the main flow and the branch flow are equivalent. At the same concentration in weight the coefficients of the pressure loss in the main flow and the branch flow are equivalent in the case of flyash and sand slurries respectively. Both coefficients of the pressure loss in the main flow and the branch flow increase with the increasing concentration in weight, but the incremental rate of the former is less than that of the latter.

(2) The pressure loss in the main flow is equivalent to that of a sudden enlargement and the pressure loss in the branch flow is the sum of a sudden contraction and a sudden enlargement as in the case of water flow. As the pressure loss in the branch flow, especially in slurry, is greatly influenced by the contraction, roundness at the branch corner may be very useful in order to reduce its pressure loss.

(3) The experimental results were expressed as the empirical formulae including also the previous experimental results with the water flow.

1. Introduction

The study on the pressure loss in hydraulic conveyor has been made on a straight pipe and a bend in detail, but almost no study has yet been made on a pipe fitting.

It is natural that a pipe fitting plays an important part in a pipe line and we must know its flow characteristic when a pipe line is designed. In this report are shown the experimental results of the flyash and sand slurry flowing through three kinds of right-angled pipe fittings which have correctly geometrical shapes, different area-ratios, are installed in the horizontal plane and are run under various flow conditions. The experimental results of the pressure losses in the right-angled pipe fittings are expressed by the empirical formulae. According to the streamline experiments, reported in the 1st

report⁽¹⁾, the mechanisms of the pressure losses in the dividing flow are described in this report.

[Nomenclature]

Subscripts 1, 2, 3: names of pipe lines (cf. Fig. 1)

13, 12: values between the upstream of the main flow and the downstream of the main flow and between the upstream of the main flow and the branch flow

m : value for slurry

s, w : value for particle and water

A_i : crosssectional area of a pipe

a : concentration in weight after discharging

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- d_i : inside diameter of a pipe
 $F_{r_{m1}}$: Froude Number $\{=V_{m1}^2/(gd_1)\}$
 g : acceleration of gravity
 l_i : distance from the dividing point N (an intersecting point of axis of the main pipe and the branch pipe), at which there is no disturbance for flow dividing
 m : area-ratio $(=A_1/A_2)$
 Δp_{Dm13} , Δp_{Dm12} : pressure drops between the upstream of the main flow and the downstream of the main flow and between the upstream of the main flow and the branch flow, expressed by Eqs. (1) and (3)
 Δp_{dm13} , Δp_{dm12} : total pressure drop due to flow dividing
 Δp_{ds13} , Δp_{ds12} : increment of pressure drop caused by mixed particles in water
 Δp_{dw13} , Δp_{dw12} : pressure drop caused by water flow
 Δp_{mi} : pressure drop for pipe friction, expressed by Eqs. (5), (6) and (7)
 Δp_{km13} , Δp_{km12} : differences of dynamic pressure between the upstream of the main flow and the downstream of the main flow and between the upstream of the main flow and the branch flow
 Q_i : flow discharge
 Re_{m1} : Reynolds Number $\{=(V_{m1}d_1)/\nu_w\}$
 V_{mi} : mean velocity of slurry calculated from the value after discharging
 W_s : weight of particles transported per unit time
 W_w : weight of water transported per unit time
 γ_{mi} : specific weight of slurry calculated from the value after discharging
 γ_w : specific weight of water
 ϵ_G : mixing-ratio in weight $(=W_s/W_w)$
 ζ_{m13} , ζ_{m12} : coefficients of the pressure loss of the main flow and the branch flow when slurry flows
 ζ_{s13} , ζ_{s12} : increments of coefficient of the pressure loss in the main flow and in the branch flow due to mixed particles
 ζ_{w13} , ζ_{w12} : coefficients of the pressure loss in the main flow and in the branch flow when water flows
 κ_i : correction factor for dynamic pressure (cf. appendix)
 λ_{mi} : coefficient of pipe friction when slurry flows
 λ_{si} : increment of coefficient of pipe friction due to mixed particles
 λ_{wi} : coefficient of pipe friction when water flows
 ϕ : velocity-ratio $(=v_s/v_w)$
 ν_w : kinematic viscosity of water

2. Coefficients of pressure loss in the main flow and in the branch flow

Consider a pipe fitting which is installed in the

horizontal plane as shown in Fig. 1. The pressure drop between measuring positions I and III at which there is no turbulence due to the dividing flow and whose distances from the dividing point N are l_1 and l_3 respectively is expressed as follows;

$$\Delta p_{Dm13} = \Delta p_{m1} + \Delta p_{m3} + \Delta p_{km13} + \Delta p_{dm13} \quad \dots\dots\dots (1)$$

So, the total pressure drop between the upstream and the downstream of the main flow due to flow dividing becomes

$$\begin{aligned} \Delta p_{dm13} &= \Delta p_{Dm13} - (\Delta p_{m1} + \Delta p_{m3} + \Delta p_{km13}) \\ &= \Delta p_{ds13} + \Delta p_{dw13} \quad \dots\dots\dots (2) \end{aligned}$$

Similarly, the pressure drop between the measuring positions I and II at which there is no turbulence due to flow dividing and whose distances from the dividing point N are l_1 and l_2 is expressed as follows;

$$\Delta p_{Dm12} = \Delta p_{m1} + \Delta p_{m2} + \Delta p_{km12} + \Delta p_{dm12} \quad \dots\dots\dots (3)$$

So, the total pressure drop between the upstream of the main flow and the downstream of the branch flow due to flow dividing becomes

$$\begin{aligned} \Delta p_{dm12} &= \Delta p_{Dm12} - (\Delta p_{m1} + \Delta p_{m2} + \Delta p_{km12}) \\ &= \Delta p_{ds12} + \Delta p_{dw12} \quad \dots\dots\dots (4) \end{aligned}$$

The meanings of the symbols mentioned above are expressed as follows;

$$\begin{aligned} \Delta p_{m1} &= \lambda_{m1}(l_1/d_1)(\gamma_w/2g)v_{m1}^2 \\ &= (\lambda_{s1} + \lambda_{w1})(l_1/d_1)(\gamma_w/2g)v_{m1}^2 \quad \dots\dots\dots (5) \end{aligned}$$

$$\begin{aligned} \Delta p_{m2} &= \lambda_{m2}(l_2/d_2)(\gamma_w/2g)v_{m2}^2 \\ &= (\lambda_{s2} + \lambda_{w2})(l_2/d_2)(\gamma_w/2g)v_{m2}^2 \quad \dots\dots\dots (6) \end{aligned}$$

$$\begin{aligned} \Delta p_{m3} &= \lambda_{m3}(l_3/d_3)(\gamma_w/2g)v_{m3}^2 \\ &= (\lambda_{s3} + \lambda_{w3})(l_3/d_3)(\gamma_w/2g)v_{m3}^2 \quad \dots\dots\dots (7) \end{aligned}$$

$$\Delta p_{km13} = (1/2g)(\kappa_3\gamma_{m3}v_{m3}^2 - \kappa_1\gamma_{m1}v_{m1}^2) \quad \dots\dots\dots (8)$$

$$\Delta p_{km12} = (1/2g)(\kappa_2\gamma_{m2}v_{m2}^2 - \kappa_1\gamma_{m1}v_{m1}^2) \quad \dots\dots\dots (9)$$

$$\Delta p_{ds13} = \zeta_{s13}(\gamma_w/2g)v_{m1}^2 \quad \dots\dots\dots (10)$$

$$\Delta p_{ds12} = \zeta_{s12}(\gamma_w/2g)v_{m1}^2 \quad \dots\dots\dots (11)$$

$$\Delta p_{dw13} = \zeta_{w13}(\gamma_w/2g)v_{m1}^2 \quad \dots\dots\dots (12)$$

$$\Delta p_{dw12} = \zeta_{w12}(\gamma_w/2g)v_{m1}^2 \quad \dots\dots\dots (13)$$

After discharging v_w equals v_s , and in this case the dynamic pressure can be obtained from mean velocity. But when a slurry actually flows in a pipe, v_w is larger than v_s . So we must consider a correction factor κ , in order to obtain the true dynamic pressure, that is, the dynamic pressure when a slurry flows in a pipe, from the dynamic pressure based upon the mean velocity. (cf. appendix) When $\phi \rightarrow 1$, $\kappa \rightarrow 1$.

Therefore, Eqs. (2) and (4) become

$$\begin{aligned} \Delta p_{dm13} &= \Delta p_{Dm13} - \frac{\gamma_w}{2g} \left\{ \lambda_{m1} \frac{l_1}{d_1} v_{m1}^2 + \lambda_{m3} \frac{l_3}{d_3} v_{m3}^2 \right. \\ &\quad \left. + \left(\kappa_3 \frac{\gamma_{m3}}{\gamma_w} v_{m3}^2 - \kappa_1 \frac{\gamma_{m1}}{\gamma_w} v_{m1}^2 \right) \right\} \\ &= \zeta_{m13}(\gamma_w/2g)v_{m1}^2 \\ &= (\zeta_{s13} + \zeta_{w13})(\gamma_w/2g)v_{m1}^2 \quad \dots\dots\dots (14) \end{aligned}$$

$$\begin{aligned} \Delta p_{dm12} &= \Delta p_{Dm12} - \frac{\gamma_w}{2g} \left\{ \lambda_{m1} \frac{l_1}{d_1} v_{m1}^2 + \lambda_{m2} \frac{l_2}{d_2} v_{m2}^2 \right. \\ &\quad \left. + \left(\kappa_2 \frac{\gamma_{m2}}{\gamma_w} v_{m2}^2 - \kappa_1 \frac{\gamma_{m1}}{\gamma_w} v_{m1}^2 \right) \right\} \\ &= \xi_{m12} (\gamma_w / 2g) v_{m1}^2 \\ &= (\xi_{s12} + \xi_{w12}) (\gamma_w / 2g) v_{m1}^2 \dots\dots\dots (15) \end{aligned}$$

3. Experimental apparatus and method

3.1 Experimental apparatus

The pipe fittings which were used for experiments have circular crosssection and their deflecting angles are 90° . They were installed in the horizontal plane as shown in Fig. 2.

Seamless drawn steel pipes (JIS G 3452) were used for an experimental pipe line, the nominal diameter of the main pipe being 2 B and that of the branch pipe 2 B, $1\frac{1}{2}$ B and 1 B. Their mean diameters are 53.2, 42.1 and 28.0 mm respectively. The true form of the right-angled pipe fitting is shown in Fig. 3. The cubic bodies of rolled steel, whose dimensions of side were 155 mm, were bored to their inside diameters in order to preserve their correctly geometrical shapes and then the insides of them were plated with hard chromium ($t=40\mu$) in order to protect from the wear of slurry flow. The joining surfaces of the main and the branch pipe make very sharp edges.

The diameters of pressure taps are 2 mm, lest they should be blocked by particles. Their positions are left-side of the main and branch pipe as viewed from upstream toward downstream. The distances of the first taps from the dividing point are 50 mm, and after of the taps are 100 mm, and fur-pitches are 300 mm when they are dividing point, but there is a little dif-
line 1. The static pressure at every induced by a vinyl tube into a bottle particles mixed in water are settled red by U-manometer.

When the inside diameter of the branch pipe is 42.1 and 28.0 mm, the pipe fitting with a branch pipe shown by dotted line in Fig. 2 is exchanged and joined to the downstream pipe by a diffuser at position E after the diameter of the branch pipe is enlarged from 42.1 or 28.0 mm to 53.2 mm. 3 B gas pipe (JIS 3432) is used for a suction pipe of a pump and 2 B gas pipe is used for a pipe line except for an experimental pipe line. When flyash-slurry is run through a pipe line, an agitating pump is used in the slurry pit lest the particles should settle. But when sand-slurry is run through a pipe line, a slurry

B_2, B_3 : Suspended balance
 D_2, D_3 : Device which divides flow
 T_2, T_3 : Measuring tank with meter for water level
 V_1, V_2, V_3 : Flow rate regulating valve
 Fig. 2 Arrangement of experimental apparatus

Fig. 3 Right-angled pipe fitting used for experiment

pit is built of board so that it may have the shape of the letter "V" and in this case an agitating pump is not used.

3.2 Experimental method

Before the experiment began, many hydraulic grade lines were found at various flow rate ratios and mean velocities of the upper stream of the main flow for the three kinds of right-angled pipe fittings

when water flowed in them. Then positions of pressure taps were decided where there was no disturbance due to the dividing flow. They are indicated by symbols 1-1, 1-2, ... as shown in Fig. 2 and it was also ascertained that there was no disturbance there due to the dividing flow when slurry flowed in right-angled pipe fittings. The differences of static pressure among positions 1-1, 1-2 and so on were measured by U-manometers and the gauge pressure was measured at position 1-1. Discharged weights were measured by suspended balances B_2 and B_3 and their quantities were measured by measuring tanks T_2 and T_3 using the dividing devices D_2 and D_3 . The temperature of slurry was measured in the slurry pit.

3.3 Slurries used for experiment

The particles were flyash and sand and they were mixed in water. Their mean diameters were about 15μ and 300μ and their specific weights were 2122 and 2670 kg/m^3 respectively. The sedimentation velocities of the flyash and sand in the water at 20°C were 0.0137×10^{-2} and $5.04 \times 10^{-2}\text{ m/sec}$

Fig. 4 Examples of experimental results

Fig. 5 Examples of experimental results

respectively. The mean diameter of the particles corresponded to the mode of distribution curve of particle diameters which was measured by a sedimentograph. The shape of flyash was spherical and that of sand was like a diamond which had sharp edges.

4. Experimental results and considerations

4.1 Experimental results

Some part of experimental results in the water have been described in the 1st report. The experimental results of slurries were shown in Figs. 4~11. The range of concentration in weight after discharging was 10 to 50 % for flyash slurry and 10 to 20 % for sand slurry. The experiments were made in the "hold up flow" and results of both slurries are the same at the same concentration in weight. The experimental range of sand slurry was limited because the particles of sand settled

Fig. 6 Examples of experimental results (Many curves express the mean value when flyash-slurry flows)

Fig. 7 Example of experimental results

Fig. 8 Examples of experimental results

more easily than those of flyash. When the sand-slurry flowed through the pipe line, experiments were made on the right-angled pipe fittings 53.2/53.2 and

Fig. 9 Examples of experimental results (Curve for $\alpha=20\%$ expresses the mean value when flyash-slurry flows)

Fig. 10 Examples of experimental results

53.2/42.1 in the concentration in weight 30 and 40%. But the static pressure fluctuated widely and irregularly under those conditions. So the experimental results of sand-slurry served as references. The experimental results were, however, considered to be the same as with flyash-slurry. The concentrations were scattered due to sealing water to the ground packing of slurry pump. And the data are scattered and not readily correlated. When the experimental results are calculated, the mean value of the scattered concentration is used and ϵ_G is taken for its corresponding mean concentration in weight.

In this experiment, the range of the upstream

Fig. 11 Examples of experimental results

velocity of the main flow was from 1 to 6 m/sec. In this experiment, the value of concentration in weight in the main and branch flow is nearly equal to that in volume. It is no more than 3 % when the concentration in weight in the main flow is 10 ~ 20 % and it is no more than 4 % when the concentration in weight in the main flow is 30 ~ 40 %. But in the pneumatic conveyor the flow rate-ratio in weight does not increase even if the flow rate-ratio in volume is larger than a certain value⁽²⁾. When the experimental results were calculated, κ was equal to 1, for φ was considered to be nearly equal to 1 in this experiment.

4.2 Consideration

(1) In the experimental range it is not observed that the coefficients of the pressure loss in the main flow and in the branch flow are not influenced by the upstream velocity of the main flow. (2) Although both coefficients of the pressure loss in the main flow and in the branch flow increase with the increasing concentration in weight, the incremental rate of the former is less than that of the latter. But both qualitative inclinations are equal to the water flow. (3) The coefficient of the pressure loss in the branch flow is greatly influenced by the increasing crosssectional area-ratio m and when the concentration in weight and v_{m2}/v_{m1} are equal, the greater m becomes, the greater its value becomes. But the coefficient of the pressure loss in the main flow is not influenced by m and concentration in weight fairly. (5) The incremental value of coefficient of the pressure loss in the branch flow for mixed particles increases with increasing velocity-ratio at the same concentration in weight.

4.2.1 Mechanism of the pressure loss in the main flow

As mentioned in the 1st report, the pressure loss in the main flow for the water is similar to a sudden enlargement. When the difference of density between the particles and the water is small and v_{m1}

is very large, the slurry is conveyed in the hold up flow and when its concentration is small, the flow patterns near the dividing point are to be considered the same as the water flow. So the pressure loss in the part ㉠ → ㉡, shown in Fig. 6 of the 1st report, is expressed as follows;

$$\Delta p_{dm13} = \zeta_{m13} (\gamma_w / 2g) v_{m1}^2$$

in which

$$\zeta_{m13} = \xi_{m13} (1 - Q_{m3}/Q_{m1})^2 = \xi_{m13} (Q_{m2}/Q_{m1})^2 \dots (16)$$

where ξ_{m13} is a correction factor and its form becomes from Eq. (14)

$$\xi_{m13} = \xi_{s13} + \xi_{w13} \dots (17)$$

ξ_{w13} is a correction factor for the water flow and from the 1st report we can put $\xi_{w13} = 0.4$ in the experimental range for the right-angled pipe fittings used in the experiment. ξ_{s13} is a correction factor for the particles mixed in the water flow and we can put it as $\xi_{s13} = 0.2$ from Figs. 4 ~ 10. Figure 12 is a result calculated using them. When Q_{m2}/Q_{m1} is 0, the pressure loss in the main flow is due to turbulence which is caused at the joining surface of the main and the branch pipe (Fig. 13). When a pipe fitting is installed in the horizontal plane, the joining surface of the lower bottom has a strong effect on the pressure loss in the main flow, because the density of slurry becomes higher from the upper to the lower bottom.

The pressure loss in the main flow is not influenced more strongly than that in the branch flow at the same concentration in weight. The reason is explained as follows. In Fig. 6 of the 1st report, the particles which are conveyed to the dividing point in a velocity-ratio φ are held in check because the velocity of the water is decreased beyond the dividing point. So they give energy to the water. Then the particles which move with eddy at position F lose their energy because they are rubbed against the pipe wall and rubbed by or collided with themselves. So the incremental rate of the pressure loss of the main flow is less than that of the branch flow, for it is expressed as the difference

Fig. 12

Fig. 13 Result of flow pattern experiment⁽¹⁾ (Disturbance is observed at the bottom of the joining surface of main and branch pipe)

of them. From Figs. 4 ~ 11, the incremental value of the main flow is ignored rather than that of the branch flow when m is large and that inclination increases with increasing concentration in weight. But, from the measured results, it is considered that the incremental ratio of the pressure loss in the main flow becomes fairly large independently of m when the concentration in weight is large and Q_{m2}/Q_{m1} nears to 1.0.

4.2.2 Mechanism of the pressure loss in the branch flow

The coefficient of the pressure loss in this case is expressed by Eq. (15). But let us consider as follows in order to observe its mechanism of the pressure loss.

As mentioned in the 1st report, the coefficient of the pressure loss in the branch flow is considered to be the sum of a sudden contraction and a sudden enlargement; the former is expressed by $\textcircled{A} \rightarrow \textcircled{B}$ and the latter is expressed by $\textcircled{B} \rightarrow \textcircled{D}$. So we put

$$\Delta p_{dm12} = \zeta_{m12} (\gamma_w / 2g) v_{m1}^2$$

$$\zeta_{m12} = (\zeta_{vm12} + \zeta_{cm12}) (v_{m2} / v_{m1})^2 \dots \dots \dots (18)$$

in which

$$\zeta_{vm12} = \{(1/C_{vm})^2 - 1\} (1/C_{cm})^2 \dots \dots \dots (19)$$

$$\text{and } \zeta_{cm12} = \{(1/C_{cm}) - 1\}^2 \dots \dots \dots (20)$$

where C_{vm} is a coefficient of velocity and C_{cm} is a coefficient of contraction in the branch flow for slurry flow.

Now, if we give C_{cm} the values calculated from the analytical results based upon the theory of two-dimensional potential flow, reported in the 1st report, we can calculate the pressure loss in the part of the sudden enlargement. Because the pressure loss in the part of the sudden enlargement $\textcircled{B} \rightarrow \textcircled{C}$, as reported in Fig. 6 of the 1st report, is considered to be the same as the pressure loss in the main flow, the incremental rate of the pressure loss of a sudden enlargement is scarcely influenced by particles mixed in water independently of the concentration in weight and area-ratio m . The calculated results mentioned above are shown by dotted lines in Figs. 5 ~ 11. So the difference between the experimental result and the dotted line is the pressure loss for the sudden contraction. Then the increase in the pressure loss for mixed particles is shown as a decrease in the coefficient of velocity.

Then Fig. 14 shows the value of C_{vm} calculated as above mentioned for example. The features when particles are mixed in the water are shown as follows; (1) when particles are mixed in the water, C_{vm} decreases, (2) C_{vm} is influenced by m and Q_{m2}/Q_{m1} , (3) C_{vm} is strongly influenced by m and Q_{m2}/Q_{m1} , when Q_{m2}/Q_{m1} nears 0, (4) C_{vm} becomes a constant value for every m , when Q_{m2}/Q_{m1} is greater than 0.5 and (5) although C_{vm} decreases with

decreasing m and its inclination becomes remarkable with increasing concentration in weight, C_{vm} of $m = 1.6$ and 3.61 change their order, when Q_{m2}/Q_{m1} is greater than 0.5.

Here, let us consider the reason why C_{vm} shows the inclination mentioned above. As the flow between $\textcircled{A} \rightarrow \textcircled{B}$ is an accelerative flow independent of Q_{m2}/Q_{m1} , energy is required for acceleration of the particles and its value increases with increasing concentration in weight. From the calculated result based upon the theory of two-dimensional flow, the velocity of the flow along the wall DB shown in Fig. 6 of the 1st report must be a nearly equal value on the free streamline. For example, when $m = 1.0$ and $Q_2/Q_1 = 0.5$, at the place $1.6 h_1$ distant from the stagnation point D, the value of the velocity along the wall DB is greater than 99 % of that of the velocity on the free streamline. So the pressure loss along the wall DB is very large. Figures 15 and 16 show the calculated results of the free streamlines when $m = 1.0$ and 2.0. In them they approach linearly from position E to the wall DB and then turn abruptly to the downstream when Q_2/Q_1 is nearly equal to 0. As the particles cannot follow the water flow in this region, they rub the wall DB and are rubbed by themselves. So the pressure loss in the branch flow becomes great with increasing concentration in weight. From the observation for the forms of the free streamlines, their inclination is remarkable when m and Q_{m2}/Q_{m1} are small. When the slurry flows from the main pipe to the branch pipe the particles rub the upper and the lower bottom of both the pipes, and the pressure loss increases with increasing m . Finally, from the correlation of the forms of the free streamlines and the degree of the acceleration of particles, the order of $m = 1.6$ and 3.61 is considered to be reversed when Q_{m2}/Q_{m1} is greater than 0.5.

Fig. 14 Calculated results of coefficient of velocity C_{vm}

From the considerations mentioned above, it is a very useful method to give roundness at the branch corner in the same way as the water or air flow^{(4) (5)} in order to reduce the pressure loss in a pipe fitting.

5. Empirical formulae

When we express the experimental results, it is very convenient for practical usages that the coefficient of pressure loss in the main flow is the combination of linear equations and quadric equations and that in the branch flow it is quadric equations. The results of individual empirical formulae are expressed as follows:

(1) Total empirical formulae for slurry flow

$$\left. \begin{aligned} \zeta_{m13} &= (0.04 + 0.19\varepsilon_G) - 0.68(Q_{m2}/Q_{m1}) \\ &\quad + 1.4(Q_{m2}/Q_{m1})^2, \quad (0 \leq Q_{m2}/Q_{m1} \leq 0.5) \\ \zeta_{m13} &= (-0.31 + 0.19\varepsilon_G) + 0.72(Q_{m2}/Q_{m1}), \\ &\quad (0.5 \leq Q_{m2}/Q_{m1} \leq 1.0) \end{aligned} \right\} \dots (21)$$

$$\begin{aligned} \zeta_{m12} &= (1.0 + 0.5\varepsilon_G) \\ &\quad - m\{(0.34/m + 0.5/m^2) - (0.02 + 0.01m)\varepsilon_G\} \\ &\quad \times (Q_{m2}/Q_{m1}) + m^2\{(0.6 + 0.41/m^4) \\ &\quad + (0.16 + 0.05m)\varepsilon_G\}(Q_{m2}/Q_{m1})^2 \end{aligned}$$

Fig. 15 Forms of free streamlines, $m=1.0$ (h_1 expresses breadth of main canal)

Fig. 16 Forms of free streamlines, $m=2.0$ (h_1 expresses breadth of main canal)

$$\begin{aligned} &= (1.0 + 0.5\varepsilon_G) \\ &\quad - \{(0.34/m + 0.5/m^2) - (0.02 + 0.01m)\varepsilon_G\} \\ &\quad \times (v_{m2}/v_{m1}) + \{(0.6 + 0.41/m^4) \\ &\quad + (0.16 + 0.05m)\varepsilon_G\}(v_{m2}/v_{m1})^2 \dots (22) \end{aligned}$$

(2) Total empirical formulae for water flow

$$\left. \begin{aligned} \zeta_{w13} &= 0.04 - 0.68(Q_{w2}/Q_{w1}) + 1.4(Q_{w2}/Q_{w1})^2, \\ &\quad (0 \leq Q_{w2}/Q_{w1} \leq 0.5) \\ \zeta_{w13} &= -0.31 + 0.72(Q_{w2}/Q_{w1}), \\ &\quad (0.5 \leq Q_{w2}/Q_{w1} \leq 1.0) \end{aligned} \right\} \dots (23)$$

$$\begin{aligned} \zeta_{w12} &= 1.0 - (0.34 + 0.5/m)(Q_{w2}/Q_{w1}) \\ &\quad + (0.6m^2 + 0.41/m^2)(Q_{w2}/Q_{w1})^2 \\ &= 1.0 - (0.34/m + 0.5/m^2)(v_{w2}/v_{w1}) \\ &\quad + (0.6 + 0.41/m^4)(v_{w2}/v_{w1})^2 \dots (24) \end{aligned}$$

Equations (23) and (24) can be obtained by putting $\varepsilon_G=0$ in Eqs. (21) and (22). We determined the constant value of Eq. (24), referring to Ito's empirical formula⁽⁶⁾. Figure 17 shows an example of the calculated results of Eqs. (21) and (22). Figure 18 shows the calculated results of Eq. (24).

6. Conclusions

We made flow experiments on the three kinds of right-angled pipe fittings which had correctly

Fig. 17. Calculated results of Eqs. (21) and (22) for $m=1.0$

geometrical shapes and where installed in the horizontal plane. Flyash and sand slurries were run through them in the "hold up flow" and at various upstream velocities of the main flow and the flow conditions. The following conclusions were obtained:

(1) The densities of slurries in the downstream of the main flow and the branch flow are equivalent. At the same concentration in weight the coefficients of the pressure loss in the main flow and in the branch flow are equivalent in the case of flyash and sand slurries respectively. Both coefficients of the pressure loss in the main flow and in the branch flow increase with increasing concentration in weight, but incremental rate of the former is less than that of the latter.

(2) The pressure loss in the main flow is equivalent to that of a sudden enlargement and the pressure loss in the branch flow is the sum of a sudden contraction and a sudden enlargement as in the case of water flow. As the pressure loss in the branch flow, especially in slurry, is greatly influenced by the contraction, roundness at the branch corner may be very useful in order to reduce its pressure loss.

(3) The experimental results were expressed as the empirical formulae including also the previous experimental results with the water flow.

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Fig. 18 Calculated results of Eq. (24)

Appendix: Correction factor κ for dynamic pressure

At the present time the value of dynamic pressure for slurry is calculated as the value after discharging. It has an error from the true value of dynamic pressure when slurry with velocity-ratio φ flows in a pipe. Let us consider a correction factor in order to find the true value of dynamic pressure from the value after discharging when slurry flows in a pipe. Then we name it the correction factor κ for dynamic pressure.

Let us consider a long straight pipe and ΔL which is a portion of it. The flow in it is steady and particles included in it are scattered in the water thoroughly. Then the specific weight of slurry γ_{mc} in conveying state becomes

$$\gamma_{mc} = \frac{(W_s/v_s)\Delta L + (W_w/v_w)\Delta L}{(W_s/\gamma_s v_s)\Delta L + (W_w/\gamma_w v_w)\Delta L} = \gamma_w \frac{1 + (\varepsilon_G/\varphi)}{1 + (\varepsilon_G \gamma_w)/(\varphi \gamma_s)} \quad (25)$$

and the mean velocity v_{mc} is

$$v_{mc} = \frac{(W_s/\gamma_s) + (W_w/\gamma_w)}{(W_s/\gamma_s v_s) + (W_w/\gamma_w v_w)} = v_w \frac{1 + (\varepsilon_G \gamma_w/\gamma_s)}{1 + (\varepsilon_G \gamma_w)/(\varphi \gamma_s)} \quad (26)$$

where v_s : mean velocity of particles, v_w : mean velocity of water and γ_s : specific weight of particles. After discharging, $\varphi \rightarrow 1$. So the dynamic pressure p_{km}' calculated as the value after discharging is

$$p_{km}' = \frac{1 + \varepsilon_G}{1 + (\varepsilon_G \gamma_w)/\gamma_s} \frac{\gamma_w}{2g} \{v_w(\varphi=1)\}^2 = \frac{\gamma_m}{2g} v_m^2 \quad (27)$$

in which

$$\gamma_m = \gamma_w (1 + \varepsilon_G) / \{1 + (\varepsilon_G \gamma_w)/\gamma_s\} \quad (28)$$

As the dynamic pressure is "the kinetic energy per

Fig. 19

unit discharge," if we ignore the rotating energy of particles, the true dynamic pressure when slurry flows in a pipe is expressed by

$$\begin{aligned} p_{km} &= E_{km}/Q_m = (E_{ks} + E_{kw})/(Q_s + Q_w) \\ &= \frac{(1/2g)(W_s v_s^2 + W_w v_w^2)}{(W_s/\gamma_s) + (W_w/\gamma_w)} \\ &= \frac{1 + \varepsilon_G \varphi^2}{1 + (\varepsilon_G \gamma_w)/\gamma_s} \frac{\gamma_w}{2g} v_w^2 \dots\dots\dots (29) \end{aligned}$$

where E_{km} : kinetic energy of slurry, E_{ks} : kinetic energy of particles and E_{kw} : kinetic energy of water through an arbitrary section in ΔL respectively; Q_m : discharge of slurry, Q_s : discharge of particles and Q_w : discharge of water through an arbitrary section of ΔL respectively. So κ is

$$\frac{p_{km}}{p_{km'}} = \frac{1 + \varepsilon_G \varphi^2}{1 + \varepsilon_G} \left(\frac{v_w}{v_m} \right)^2 = \kappa \dots\dots\dots (30)$$

Then, as we can put $\varphi = 1 - \sqrt{\xi_s} (v_t/v_w)^{(3)}$, $(v_w/v_m)^2 = \xi_s (v_t/v_m)^2 \{1/(1-\varphi)^2\}$, where ξ_s : friction factor

between particles and pipe wall and v_t : sedimentation velocity of particles. When the slurry is conveyed in the "hold up flow," we can assume $v_w/v_m \approx 1$. If we take $\xi_s = 0.5$ for flyash and sand, the calculated results for κ are shown in Fig. 19.

References

- (1) S. Iwanami et al.: This Bulletin, p. 1041.
- (2) Y. Niitsu et al.: *Trans. Japan Soc. Mech. Engrs.*, Vol. 25, No. 153 (1959), p. 426.
- (3) S. Iwanami et al.: *Bulletin of JSME*, Vol. 7, No. 27 (1964), p. 552.
- (4) G. Vogel et al.: *Mitt. Hydr. Inst. T. H. München*, Ht. 1 (1926), S. 75; Ht. 2 (1928), S. 61; Ht. 3 (1929), S. 98; Ht. 4 (1931), S. 70.
- (5) Y. Sato: *Trans. Japan Soc. Mech. Engrs.*, Vol. 28, No. 192 (1962), p. 881.
- (6) H. Ito: *Interim Report of Research Committee for the static flow characteristics in a divided and confluent flow*, (1966), Japan Soc. Mech. Engrs.
- (7) I. Oki and T. Kawaguchi: *Trans. Japan Soc. Mech. Engrs.*, Vol. 17, No. 60 (1951), p. 146.