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Steady and Dynamic Performances of a Variable Geometry Gas Turbine*

By Teruo SAWADA** and Akira NISHI***

A gas turbine which has a variable geometry free turbine can be expected to be improved not only in part load performance and acceleration capability but also in engine braking. Therefore it is suitable as the vehicular power plant from the above three view points. The control unit of such a gas turbine, however, becomes more complex than that of a conventional one.

In the present paper, the steady and dynamic performances of such a gas turbine are investigated by the nomogram method and analog simulation respectively.

The relations between part load performance, acceleration capability and nozzle angles are obtained, as well as the transient performance for a wide range of nozzle angles from the design value to engine braking. The results can be used as the basic data for the selection of turbine cycle, estimation of required component characteristics and design of controller at design phase.

1. Introduction

The gas turbine with a variable geometry free turbine can be expected to be improved not only in part load performance and acceleration capability but also in engine braking. Therefore it is suitable as the vehicular power plant from the above three view points. The control unit of such a gas turbine, however, becomes more complex than that of a conventional one, as the variable power turbine nozzle adds a new independent variable which must be controlled as well as fuel flow.

On the other hand, it is necessary to know the performances of the gas turbine for the control of both fuel flow and power turbine nozzle angle in order to use the gas turbine as the vehicular power plant.

Investigations of steady and dynamic performances of the gas turbine have been carried out by many designers of control unit or engine itself^{(1)~(3)}. The study of gas turbine, however, covering a wide operating range by changing the power turbine nozzle angle has not been made yet.

In the present paper the steady and dynamic performances of the gas turbine with a variable geometry radial power turbine are investigated by the nomogram method and analog simulation respectively.

As the result, the part load performance, acceleration capability and the transient performance by changing the nozzle angle from design to braking operation are made clear, and the combined characteristics of controls of both fuel flow and nozzle angle are presented clearly.

These methods and results are applicable as the basic data for the selection of turbine cycle, estimation of required component characteristics and design of controller during the design phase.

Symbols

A_n	: nozzle area	m^2
c_p	: specific heat at constant pressure	kcal/kg deg
E	: transfer function	
F	: function	
G	: mass flow of air and gas	kg/sec
G_f	: fuel flow	kg/sec
I	: moment of inertia	kg m sec ²
J	: equivalent work of heat	kg m/kcal
L	: output or absorbing power	PS
N	: revolutional speed of gas generator	rpm
P	: pressure	kg/m ²
Q	: torque	kg m
s	: Laplace operator	
T	: temperature	°K
t	: time	sec
α	: power turbine nozzle angle, measured from tangent	deg
η	: efficiency	
θ	: time interval from the time of fuel flow change to that of nozzle angle variation	sec
τ	: time constant	sec
ω	: angular velocity	1/sec
Suffixes		
c	: compressor	
ct	: compressor turbine	
d	: design value	
r	: regenerator	
1, 2, 3	: position (see Fig. 1)	

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** Professor, Faculty of Engineering, University of Osaka Prefecture, Sakai.

*** Lecturer, Faculty of Engineering, University of Osaka Prefecture.

2. Component characteristics

The gas turbine, as shown in Fig. 1, which has the regenerative cycle and a variable geometry radial power turbine is considered in the paper. The component characteristics at the design point of the gas turbine are presented in Table 1. In spite of the simplicity of its configuration in the regenerative gas turbine cycles, good performances over a wide operating range can be expected by using a variable geometry power turbine nozzle. So this gas turbine is promising as the vehicular power plant from the above view point.

Off design characteristics of the gas turbine components are assumed as follows:

(a) A single stage centrifugal compressor is employed and its characteristics are given in reference (4) as the typical centrifugal compressor characteristics.

(b) Pressure loss and combustion efficiency of the combustor are assumed as constant over a wide range of operation.

(c) A single stage axial flow turbine is used

Table 1 Design performance

Turbine output	$L=400$ PS
Pressure ratio	$P_2/P_1=3.33$
Mass flow at the compressor inlet	$G_1=2.87$ kg/sec
Revolational speed of gas generator	$N=30\,000$ rpm
Moment of inertia of gas generator	$I=2.04 \times 10^{-2}$ kg m sec ²
Polytropic efficiency of compressor turbine	$\eta_{ct}=0.85$
Combustion efficiency	$\eta_B=0.98$
Pressure loss of burner	$\Delta P/P_3=0.02$
Thermal efficiency of regenerator	$\eta_R=0.85$
Pressure loss of regenerator (air side)	$\Delta P/P_2=0.02$
Leak flow of regenerator (air side)	$\Delta G/G_2=0.04$
Time constant	$\tau=3$ sec

C: compressor
B: burner
CT: compressor turbine
PT: power turbine
HE: regenerator
G: gear box
L: load

Fig. 1 Gas turbine cycle

as the compressor turbine, and the relation between corrected mass flow (cmf) and pressure ratio is given in reference (5) as a typical characteristic,

$$\frac{G_3 \sqrt{T_3}}{P_3} = k \sqrt{1 - \left(\frac{P_3}{P_4}\right)^{-3.71}} \dots \dots \dots (1)$$

where $k=\text{constant}$, and its polytropic efficiency is given by

$$\eta_{ct}=0.85 \quad (\text{constant})$$

Under the assumption of the turbine characteristic as a single curve, some errors occur in the range of small revolutionary speeds. However, as the idling revolutionary speed of gas generator is about one half its design speed, the deviation of actual operating point from that given by Eq. (1) is not so large.

(d) A variable geometry radial inflow turbine is used as the power turbine, and its characteristics for the wide range of nozzle angles, presented in Fig. 2, are estimated by the experimental results⁽⁶⁾. Figure 2 (a) shows the relation between cmf and pressure ratio, the mass flow increases with increase of nozzle angle, but at the braking nozzle angle ($\alpha > 110^\circ$), mass flow decreases with decrease of nozzle throat area. In Fig. 2 (b) the relation between cmf and work ratio $[(T_4 - T_5)/T_4]$ is presented. As the energy is added to the turbine in the braking operation, $T_4 - T_5$ becomes negative. Both in Fig. 2 (a) and (b) the characteristics are given by the single curves for individual nozzle angle, and this means the output or absorbing power is expressed as gas horsepower, and more practical calculations will be done by using the actual relations between torque and revolutionary speed of the load.

(e) Efficiency and pressure loss of the regenerator are assumed as constant over a wide range of operation. And time lag phenomenon of the regenerator is very important, whenever the dynamic performance of the gas turbine is considered, and the transfer function of regenerator is given by the following equation as the first order approximation,

$$E=1/(1+s\tau) \dots \dots \dots (2)$$

3. Steady state performance

3.1 A method of calculation of steady state performance

Usually the revolutionary speed of gas generator (N), inlet mass flow (G_1) and maximum temperature (T_3) have been adopted as the variables for calculating the part load performance of a gas turbine, and the overall performance has been predicted for an arbitrary fuel control and for the given relations between the revolutionary speed of power turbine and the required torque of the load.

The gas turbine, considered here, has a variable geometry power turbine, and its variable nozzle

adds a new independent variable. Therefore such a gas turbine has two degrees of freedom for controlling it, and all independent variables are N , G_1 , T_3 , and α .

To simplify the calculation the nomogram method is applied as shown in Fig. 3. Firstly the gas turbine performance for $N=\text{constant}$ is considered.

I-plane in Fig. 3 shows the relation between work ratio $[(T_2-T_1)/T_1]$ and cmf ($G_1\sqrt{T_1}/P_1$) of the compressor, and if the inlet conditions of the compressor P_1 , T_1 are given and G_1 is assumed, the corrected revolutionary speed (crs) and cmf are

Fig. 2 Power turbine characteristics

obtained, so that points 1 and 2 are determined. II-plane presents the work balance of compressor and compressor turbine,

$$\frac{T_3-T_4}{T_3} = \frac{T_1}{T_3} \frac{T_2-T_1}{T_1} \frac{c_{pc}}{c_{pt}} \frac{1}{\eta_m} \frac{1}{1-\frac{\Delta G}{G_2}} \quad \dots (3)$$

By assuming the maximum temperature (T_3), points 4 and 5 are determined. III and IV-plane show the characteristics of compressor turbine, and cmf at the compressor turbine inlet ($G_3\sqrt{T_3}/P_3$) is determined by point 7.

On the other hand, V-plane shows the relation between pressure ratio and cmf of the compressor, and points 1' and 2' are determined by $N/\sqrt{T_1}$ and $G_1\sqrt{T_1}/P_1$ assumed above, and pressure ratio of the compressor is obtained from point 3'. Pressure losses in the combustion chamber and the regenerator are considered in VI-plane and the pressure ratio P_1/P_3 is determined by point 5'. By giving the maximum temperature as well as II-plane, (P_1/P_3) $\times \sqrt{T_3/T_1}$ is determined at point 7' in and VII-plane, and $G_3\sqrt{T_3}/P_3$ is obtained by

$$\frac{G_3\sqrt{T_3}}{P_3} = \frac{G_1\sqrt{T_1}}{P_1} \frac{P_1}{P_3} \sqrt{\frac{T_3}{T_1}} \quad \dots (4)$$

where it is assumed that the fuel added is equal to cooling air bled from the compressor. G_1 assumed initially is adjusted iteratively until the value which is given by Eq. (4) coincides with the value of point

Fig. 3 Nomogram

7 obtained by the procedure given by Fig. 3 (T_3 is not varied).

By the above calculations, an equilibrium operating point of the gas generator is determined.

P_3/P_4 is obtained from point 9, and P_4 is determined by point 4', as P_3 is known. P_4/P_5 is obtained by giving P_5 which is given by assuming the pressure losses of regenerator and exhaust pipe. Moreover, $G_4\sqrt{T_4}/P_4$ can be calculated, as T_4 is known from point 5.

Nozzle angle (α) can be obtained from point C in Fig. 2 (a) by giving P_4/P_5 , $G_4\sqrt{T_4}/P_4$, and it involves α' and α'' which correspond to the normal (operation of producing the output) and braking operations respectively. And two work ratios of power turbine $[(T_4 - T_5)/T_4]$ can be determined by giving $G_4\sqrt{T_4}/P_4$ and α , in Fig. 2 (b), and output (or absorbing power) can be calculated by

$$L = \frac{J}{75} G_4 c_{p4-5} (T_4 - T_5) \\ = \frac{J}{75} \frac{G_4 \sqrt{T_4}}{P_4} \frac{(T_4 - T_5)}{T_4} c_{p4-5} P_4 \sqrt{T_4} \text{ (PS)} \dots (5)$$

An equilibrium operating point for the combination of T_1 , G_1 and α is obtained by the above procedures, and the equilibrium operating points on the line of $N/\sqrt{T_1} = \text{const.}$ are also obtained by the change of T_3 .

Secondly, by repeating the above procedures for

Fig. 4 Steady state performance

the variation of $N/\sqrt{T_1}$, all of the steady state performances for an arbitrary fuel flow are determined for the combinations of N , G_1 , T_3 and α .

3.2 Steady state performance

The output or torque obtained by the above method is shown in Fig. 4 against crs of the gas generator. Torque in the figure is calculated under the assumption that the revolutionary speed of power turbine is constant. The solid and dotted curves in this figure indicate the performances for constant nozzle angle and for constant fuel flow respectively.

Larger power is produced in the region of smaller nozzle angle than $\alpha = \alpha_d = 15^\circ$ for the constant fuel flow, and this means that the part load performance can be improved by setting the nozzle angle smaller than the design value. The surging limits of the compressor are also shown in the same figure, and the smaller nozzle angle in part load operation brings the more dangerous of surging in spite of the higher efficiency (explained later).

3.3 Examples of engine braking and part load performance

Firstly, the engine braking operations are considered. After the fuel flow is decreased until near the idling point, the nozzle angle is changed to braking angle rapidly. For example, the fuel flow is decreased from the design value to 20 percent of it and the nozzle angle is changed from the design value to $\alpha = 165^\circ$ rapidly. Output power or torque of this example is presented in Fig. 4 by a curve of O-P-Q.

Secondly, the part load performances are considered. In Fig. 5 the curves of A and B which are the functions of revolutionary speed are assumed as two examples, the temperatures in various parts which are T_3 , T_4 and T_5 and the specific fuel consumptions (sfc) obtained by the above nomogram method are also given in this figure for such examples. The smaller nozzle angle in part load operation produces the higher turbine inlet temperature (T_3) and smaller sfc, but the equilibrium running line on the compressor characteristic presented in Fig. 6, has a smaller surge margin.

When the practical operations are considered, the operating line of acceleration on the compressor characteristic passes through the region of smaller surge margin than the equilibrium running line^{(7) (8)}, then the equilibrium running line can not be allowed to come near the surge line when the rapid acceleration is required. So the nozzle angle is limited by the surging in part load operation.

The output against the revolutionary speed of gas generator is also shown in Fig. 4 by the curves of A and B, and from this figure the limits of surging can also be predicted.

4. Dynamic performance

4.1 A method of estimating the dynamic or transient performance of the gas turbine (analog simulation)

A number of techniques have been proposed in the past for estimation of dynamic performance of gas turbine. Analog simulation technique presented

Fig. 5 Part load performance

Fig. 6 Simulation of compressor characteristic

here offers a very simple procedure and it gives many good operating conditions and dynamic performances for various types of gas turbines. And so, this technique may be used for selecting the gas turbine cycles, for predicting the necessary component characteristics and for designing the controllers during the design phase of gas turbine.

In this method the dynamic or transient performance is obtained using the steady state component characteristics. This means that the moment of inertia of air or gas passing through the gas turbine components is assumed as negligible as compared with that of rotating parts of gas turbine components, and this is ascertained by the experiments in reference (8).

4.1.1 Dynamic balance of gas generator

The following relation is introduced between the required torque of compressor and the supply torque of compressor turbine by neglecting the mechanical losses of bearings,

$$Q_{ct} - Q_c = Q \approx 0 \quad (6)$$

where $Q > 0$ represents the accelerating operation of the system consisting of compressor and compressor turbine, $Q = 0$ represents the steady state and $Q < 0$ represents the decelerating operation.

From the equation of motion

$$I\dot{\omega} = Q, \quad \dot{\omega} = d\omega/dt \quad (7)$$

Equation (6) can be expressed as follows;

$$\frac{Q}{IN} = \frac{Q_{ct}}{IN} - \frac{Q_c}{IN} = \frac{\pi}{30} \frac{\sqrt{T_1}}{N} \frac{d(N/\sqrt{T_1})}{dt} \quad (8)$$

Integrating by t ,

$$\int_0^t \frac{Q}{IN} dt = \frac{\pi}{30} \left[\ln \left(\frac{N}{\sqrt{T_1}} \right) \right]_{N/\sqrt{T_1}, t=0}^{N/\sqrt{T_1}, t=t} \quad (9)$$

where Q_c/IN and Q_{ct}/IN can be expressed as follows respectively,

$$\frac{Q_c}{IN} = \frac{G_1 \sqrt{T_1}}{P_1} \frac{T_2 - T_1}{T_1} \frac{T_1}{N^2} \frac{30 J c_{p1-2} P_1}{\pi I \sqrt{T_1}} \quad (10)$$

$$\frac{Q_{ct}}{IN} = \frac{G_3 \sqrt{T_3}}{P_3} \frac{T_3 - T_4}{T_3} \frac{P_3}{P_1} \sqrt{\frac{T_3}{T_1}} \frac{T_1}{N^2} \times \frac{30 J c_{p3-4} P_1}{\pi I \sqrt{T_1}} \quad (11)$$

4.1.2 Simulation of gas turbine components

A good many diode function generators are used to simulate all the component characteristics. The compressor characteristics are represented by a family of curves of two variables, therefore three diode function generators are used to simulate them. This method is presented in reference (8) in detail, and it can not give exact simulations over the complete characteristics, but with care the results can be obtained over a satisfactory range with good accuracy. As an example of the simulation, the relation between pressure ratio and cmf is shown in Fig. 6, and the circuits configuration of function generator is shown in Fig. 7.

As the compressor turbine is represented by the single-line characteristic assumed before, the single function generator is used to simulate it.

Fig. 7 Arrangement of function generators

Fig. 8 Simulation of power turbine

The power turbine characteristics shown in Fig. 2 (a) and Fig. 2 (b) are the bivariant functions, and the curves are obtained by sliding a "design" curve along abscissa. The curves of work ratio is simulated by using a multiplier and comparator as shown in Fig. 8. By this method, the curves in the braking range, especially large nozzle angle, become symmetrical with the corresponding curves in the normal range, therefore they involve some errors (explained later). And more accurate simulations may be accomplished by applying the method used in the simulation of compressor characteristic to that of power turbine.

The time lag of regenerator is given by a lagging circuit.

4.1.3 Flow and block diagrams

Flow and block diagrams are shown in Figs. 9 and 10 respectively. The thermodynamic

Fig. 9 Flow diagram

Fig. 10 Block diagram

relationship require a large number of multipliers, as indicated in Eqs. (10) and (11), so that all thermodynamic quantities are treated as logarithmic forms, to replace multiplications and divisions by additions and subtractions respectively. So, this requires a good many logarithmic and anti-logarithmic function generators, and all of them are constructed by diode function generators, and these are expressed by the following equation,

$$100X = 100 \frac{1}{100} \{50[\log X + 2]\} = 1.0471^{50[\log X + 2]} \dots (12)$$

As in the interest of accuracy, all variables are desirably to have voltages held within the range of ± 20 to ± 80 volts for our simulator^{(9) (10)}.

4.2 Dynamic performances and their discussions

4.2.1 Acceleration

In the "reset" mode of analog simulator, all voltages corresponding to all engine components are set at the design values. To obtain the accelerating performance, the voltage corresponding to the fuel flow is decreased until the idling condition only by turning the potentiometer dial on the simulator, and as the next step, this voltage is increased according to the acceleration schedule. Examples of simulated response of accelerating performance are presented in Fig. 11 (b) and Fig. 11 (c) for the schedule shown in Fig. 11 (a). Figure 11 (b) shows the differences of accelerating time of gas generator due to the change of nozzle angle or nozzle throat area, and Fig. 11 (c) represents the outputs for each nozzle angle corresponding to Fig. 11 (b). There are decreases in accelerating time of gas generator with increases in nozzle angle or nozzle throat area. The final value of output, however, decreases considerably.

4.2.2 Transient performance for the variation of nozzle angle

A few examples of transient performances for the variation of nozzle angle from design to braking operations are discussed. In this case the following steps are considered, firstly the fuel flow is decreased from the design value to $G_f/G_{fd}=0.2$, and

secondly the nozzle angle is varied from the design value to $\alpha=165^\circ$ rapidly. The responses of revolutionary speed of gas generator, mass flow, output and temperatures of various parts are shown in Fig. 12 with the parameters of time intervals (θ) from the moment of fuel flow change to that of nozzle angle variation. For small θ , as the nozzle angle is changed to braking range during large value of T_3 , caused by the time lag of regenerator, the revolutionary speed of gas generator is accelerated rapidly and exceeds the design value, because of the

Fig. 12 Transient performance

Fig. 11 Acceleration performance

pressure drop of power turbine being decreased during the change of nozzle angle. Therefore large braking power is produced by the increased revolutionary speed and higher turbine inlet temperature, and it is dangerous for the vehicle or engine itself, in spite of the short duration time. It is possible to prevent the danger, however, easily by using the braking mechanism which produces a comparatively large time interval (θ).

The turbine inlet temperature (T_3) decreases slowly with decrease of fuel flow, and it decreases rapidly for the increase of nozzle throat area, as the mass flow increases largely. In braking nozzle angle, the energy added from the outside of the turbine (from the vehicle) is converted to heat in the power turbine, and it is recovered by the regenerator, then T_3 increases again. So that the final value of T_3 in braking range is higher than that of normal one for the same fuel flow.

The outlet temperature of power turbine (T_5) shows the same tendency as T_3 .

The relations between gas generator revolutionary speed and output for various time intervals are shown in Fig. 13.

The dangers of overspeed of gas generator can be prevented by using the regenerator bypass mechanism, as T_3 can be decreased faster than the

Fig. 13 Transient performance for the variation of nozzle angle

conventional turbine. And this operation can be simulated easily by cutting off the circuits of regenerator and its time lag; an example of this result is shown in Fig. 13 by a dotted line.

A transient operating line on the compressor characteristic is presented in Fig. 14. In the figure it is shown that the transient operating line passes through a more stable region than the equilibrium line for the decrease of fuel flow. For the change of nozzle angle, mass flow increases faster than pressure ratio, and by the decrease of nozzle throat area again, which corresponds to the braking nozzle angle, mass flow changes faster than pressure ratio, and then the operating curve forms a loop. As the mass flow of the equilibrium operating point at $\alpha=165^\circ$ is smaller than that of $\alpha_d=15^\circ$, the final point reaches into the region of smaller surge margin.

A few examples have been discussed under the assumption of engine braking; in practical operations, however, many other operations except the above examples can be encountered, and these operations do not always produce the same results as above, but some of them may be estimated from the above results and all of them may be obtained by the same method as mentioned above.

4.3 Accuracy of simulation

In order to obtain the steady state performance, the various input variables such as fuel flow, ambient temperature and pressure are set at any desired values by setting the potentiometer dials on the simulator. After the input variables have been set, a steady state operating condition can be obtained by changing the fuel flow very slowly, and voltages corresponding to the output variables

Fig. 14 Operating line on the compressor characteristic

are measured.

By comparison of the results obtained by the above procedure with ones obtained by the nomogram method explained before, the accuracy of simulation can be checked. The results obtained by both methods are presented in Fig. 14. They agree with each other fairly well, and this means that the circuits of the analog simulator have little errors in principle. As the errors due to analog computer components have been found less than 0.3% by experience, major parts of them may be introduced from the simulator of engine component.

The steady state performances for the change of nozzle angles are shown in Fig. 13. For $\alpha \leq 140^\circ$, good agreement is obtained between the results of analog simulation and those of nomogram method; at $\alpha > 140^\circ$, however, some errors are indicated. Most parts of these errors may be traced to the simulator of power turbine explained in 4.1.2. In this case the maximum error of output is about 12% for $\alpha = 165^\circ$.

The errors of gas turbine components involved in dynamic performances may be in the same order as in the steady state performances.

As all the circuits consist of the electronic components and the recorder of this simulator has little time lag, the errors of transient response time are regarded as very small.

5. Conclusions

The vehicular gas turbine with a variable geometry radial power turbine is considered. Firstly the steady state performances are obtained by the nomogram method over a wide range of nozzle angles with involving the braking operations.

Secondly the dynamic or transient performances are obtained by the analog simulation for the rapid change of fuel flow and nozzle angle. From the results of these calculations and simulations, the followings are analysed:

(a) Steady state and dynamic performances over a wide range of nozzle angles.

(b) The improvements of part load performances by setting the nozzle angles smaller than design value, and the relations between these nozzle angles and surge margin.

(c) The improvements of acceleration capabilities by the change of nozzle angle during the accelerating operations.

(d) The differences of transient performance by changing the time intervals from the time of fuel flow change to that of nozzle angle variation in engine braking operations.

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