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Notch Strengths of Rod and Plate Specimens*

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To investigate the relation between the notch strength of a rod and that of the corresponding plate mild steel specimens, fatigue and tensile experiments are carried out. The results are summarized as follows. (1) The notch strength of a rod is greater than that of a plate for the same profile. (2) An experimental relation between fatigue notch factors of a rod and those of a plate is given. (3) Static notch strength doesn't depend on the notch root radius ρ , and local yielding is induced at earlier stage before the apparent yielding. When the yielded zone connects the opposite notches, the stress is comparable with the lower yield point and doesn't depend on the notch shape. The notch factor of a rod estimated from that of a plate is greater than the measured one. (4) Relation between fatigue strength and static strength varies with ρ . (5) A notch constrains the deformation.

1. Introduction

The notch effect has been treated as a plane stress problem, in many cases, in the theoretical study. But a machine element has a solid geometry such as a rod. In such a practical case, the calculation of the stress concentration factor is troublesome generally. Then the relation between the stress concentration factors of a rod specimen with a notch and that of a plate specimen which had the same profile as the rod specimen was studied, and approximate relations were given for shallow⁽¹⁾ and deep⁽²⁾ notches respectively. However, actual notch strength does not always agree with the theoretical stress concentration factor due to the characteristic effect of material. Therefore, it may be necessary to investigate such a relation as described above for low carbon steel which is used as a structural material usually. On this occasion, if the fatigue and static experiments are carried out, some practical significance will be added to this work concerning easy test of fatigue strength by comparing these experimental results. It is also considered that the result of fatigue experiment together with the static result may be available to make clear the mechanism of strength⁽³⁾. These studies have apparently not been made to date. Therefore, tensile and fatigue experiments on a rod and the corresponding plate specimens made of same low carbon steel were carried out.

2. Preparation of specimens

The material used was commercial low carbon steel. The compositions are shown in Table 1. Heat treatments were carried out on this material and three series, A, B and C were prepared with different grain sizes respectively. The material that was prepared by heating at 1050°C for three hours and subsequent cooling to 600°C at the rate of 10°C per hour was named series A. The material that was prepared by heating at 950°C for two hours and cooling at the rate of 20°C per hour was named series B. The mother material was named series C. From these materials of which diameters were 19mm, fatigue and tensile specimens were made as shown in Fig. 1. For fatigue and tensile experiments, rod and plate specimens which contained notched and smooth specimens, respectively, were prepared. The geometrical shape of a plate specimen was the same as the profile of a rod specimen. The breadth or the diameter of a smooth specimen was made equal to the smallest one at the notch root of the corresponding notched specimen. For the machining of a notch, a cutting tool was used which was finished precisely with an oil stone. Notched specimens were inspected and their smallest cross sections at the notch roots were measured by using an enlarging projector of which the magnification factor was 20. The dimensions of notched specimens used in this study are listed in Table 2. The fatigue and tensile

Table 1 Chemical composition (%)

Carbon	Manganese	Sulfur	Silicon	Phosphorus
0.21	0.43	0.012	0.19	0.019

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specimens were heated at 910°C and 650°C, respectively, for one hour and cooled in the furnace to remove the initial stress that had been induced during the machining. Static tensile specimens were ground by hand with emery papers up to the degree 06 in order to make easy the observation of the plastic zone on the specimen. The microscopic investigations were carried out before the experiments, and the typical examples are shown in Fig. 2. The grain size numbers and the Vickers hardness values are listed in Table 3.

3. Experimental results and discussions

3.1 Fatigue experiment

Fatigue experiments were carried out using a Schenk type fatigue testing machine with a couple of devices for bending of rod or plate specimen. Alternating bending for a plate specimen was carried

out in the plane in which the surface of a specimen was included. Alternating speed was 3 000 c/min. The experimental results are shown in Figs. 3 and 4. From these results, fatigue strengths at 10^7 cycles were plotted against the notch root radius ρ as shown in Fig. 5. In this figure, the fatigue notch factors are also shown. According to this figure, the fatigue strength is greater for rod than for plate, and as the difference between them does not depend on the notch root radius the experimental results are represented by two parallel straight lines. Although the magnitude of the fatigue notch factor is only a little greater for rod than for plate, their variation with the variation of the notch root radius seems to be represented by the same straight line for rod and plate specimens. Thus, the fatigue strength is greater for rod specimen than for plate specimen but the fatigue notch factor [is more sensitive in the former than in the latter to the notch effect. The relation between the fatigue notch factors

Table 2 Dimension of notch (mm)

Material	Specimen		Notch root radius ρ	Notch depth d	Thickness t
Series C	Fatigue specimen	Plate specimen	0.2, 0.5, 1.0, 2.0	1.0	3
		Rod specimen	0.2, 1.0, 2.0	1.0	—
	Tensile specimen	Plate specimen	0.5	2.6	2
			0.2, 0.5, 1.0	1.3, 2.6	3
			0.5	2.6	4
		Rod specimen	0.2, 0.5, 1.0	1.3, 2.6	—
Series A, B	Tensile specimen	Plate specimen	0.5	1.3, 2.6	3

Table 3 Grain size number and hardness

	Series A	Series B	Series B
ASTM grain size number	6.4	7.1	8.2
Vickers hardness value	73	77	120

Fig. 2 Microscopic structure ($\times 100 \times 2/3$)

of rod and plate specimens is discussed a little more in detail as follows.

In this study, the notch depth d was taken to be $d/b=0.2$, where b is the radius of a rod or half breadth of a plate specimen. This is based on the following reasons. One of these reasons is that, according to the previous photoelastic study⁽⁴⁾, the notch depth in this study corresponds to a deep notch and the effect of ρ on the stress concentration is considerable for $d/b=0.2$. The other reason is that the variation of stress concentration with the notch depth is very considerable for a shallow notch but the depth in this study corresponds to the region just after the maximum of the variation curve and then the scattering of the strength due to the difference of the notch depth in machining can be reduced for this depth.

For the machining of the notch, the notch root radius ρ can be cut precisely using a cutting tool which has been finished under the inspection using a photo-enlarging projector. But the notch depth is apt to be irregular for all specimens. In this study, the error in notch depth was about ± 1 percent.

Thus the notch in this study is considered a deep notch, and it is studied whether the relation⁽²⁾ between the stress concentration factors of a rod and that of the corresponding plate for a deep notch,

$$\alpha_3 = 0.75\alpha_2 + 0.25 \quad \dots\dots\dots(1)$$

holds good in the case of fatigue notch factor, β . Equation (1) is represented as

$$\beta_3' = 0.75\beta_2 + 0.25 \quad \dots\dots\dots(2)$$

Fig. 3 S-N diagram for rod specimens

Fig. 4 S-N diagram for plate specimens

taking the fatigue notch factors β_3' and β_2 for a rod and the corresponding plate, respectively, instead of stress concentration factors. Comparing this β_3' with the experimental result for a rod specimen β_3 , it can be seen that $\beta_3 > \beta_3'$. The difference between the magnitudes of β_3 and β_3' is about 0.1. Although the difference is small, it increases a little with a decreasing magnitude of ρ as shown in Fig. 6. Then we must be careful when we want to evaluate the fatigue notch factor of a rod specimen using Eq. (2) for small magnitude of ρ . The straight line connecting experimental points for β_3 shown in Fig. 6 was given by experimental equation (3).

$$\beta_3 = 0.857\beta_2 + 0.226 \quad \dots\dots\dots(3)$$

It can be seen that the experimental results can be represented properly by this relation and then Eq. (1) or Eq. (2) must be modified to Eq. (3) in the case of fatigue notch factor.

3.2 Tensile experiments

3.2.1 Plate specimen

Tensile experiments were carried out using an oil pressure type testing machine, and the tensile speed was about 20/100 mm per minute in elongation.

It is well known that the Lüders band appears suddenly across a specimen obliquely at the yield point in tensile test for mild steel. This phenomenon can be seen in the case of a smooth specimen but a very different process can occur in the case of a notched specimen and its typical example is shown

Fig. 5 The result of fatigue experiments

Fig. 6 The relation between β_2 and β_3

in Fig. 7. The extension of yielded zone on a notched plate specimen with an increase of load was observed as follows: (a) A very small plastic zone such as a whisker initiated at the notch root; (b) this yielded zone progressed gradually across the specimen; and (c) opposite notches were connected by this progressed zone. Up to this stage, no remarkable change could be seen in the movement of the pointer of the testing machine or on the recording paper. In this photograph, when a whisker is observed at the lower part than the line connecting the opposite notches, a corresponding whisker is observed at the upper part on the back surface of the specimen. At this stage, the load-elongation curve showed a linear increase. After the neighbourhood of the notches was hardened, (d) Lüders bands were produced suddenly from the vicinity of the notch roots to the opposite straight edge obliquely.

Fig. 7 Extension of plastic zone

Although the gradient of load-elongation curve was reduced slightly before or after this stage, it was still linear. To investigate these processes in detail, the displacement of the crosshead of the testing machine was measured using a dial-gauge. According to this result, the gradient described above was changed at the intermediate stage between (c) and (d). After this stage, (e) Lüders band appeared suddenly at the smooth part as can be seen in a smooth specimen. At this time, the pointer of the testing machine indicated a sudden drop in load and an obvious change could be seen on the recording paper. This corresponds to the yield point of a smooth specimen and to the subsequent, so called, "dance stage". Thereafter, Lüders bands extended all over the surface of the specimen, and were broken at last after the hardening.

These features did not depend on the notch shape and on the heat treatment. Therefore, nominal stresses were defined as σ_a , σ_c , σ_d and σ_e for the stages (a), (c), (d) and (e), respectively, and they are compared with each other among the tensile specimens and discussed below. In addition to this, the maximum tensile strength, σ_{max} , and the breaking stress calculated with the broken cross section, σ_f , are also considered. Figures 8 (a) and (b) show the variation of notch strength with the change of notch root radius, and (a) is a case of $d/b=0.2$ and (b) is one of $d/b=0.4$, respectively. Figure (c) shows the effect of thickness of a specimen in a case of $\rho=0.5$ for the deep notch, $d/b=0.4$.

Elastic stress concentration is considerably affected by notch root radius generally and decreases with an increasing notch depth for a deep notch. However the effect of ρ can not be seen distinctly in Figs. 8 (a) and (b) contrary to the elastic consideration. The magnitude of σ_f is affected by ρ , but it is considered that this is caused from the contraction of the cross section rather than the effect of stress concentration and this result is related to the deformation character that will be described later. On the other hand, in general, strength in Fig. 8 (b) is greater than that shown in Fig. 8(a) and the strength of a rod is greater than that of the corresponding plate, and these points agree with the elastic consideration. According to Fig. 8 (c), the variation of notch strength with the change of the thickness of a specimen is not so considerable but σ_{max} and upper yield point of the smooth specimen increase and σ_a of a notched specimen decreases with an increasing thickness.

The magnitudes of σ_c are comparable with each other in Figs. 8 (a) and (b), and the effects of ρ and d can not be seen. Nor the effect of t can be seen in Fig. 8 (c). Thus, it is considered that

the magnitude of σ_c does not depend on the shape of a specimen, and strength is comparable to the lower yield point of a smooth specimen. To make sure about this point, further experiments were carried out on specimens which had different grain sizes. The plastic zone of a heat-treated material looked like a rough skin, and the observation was slightly difficult. The appearance of an extending plastic zone on the smooth part of the specimen was different from that of the specimen for which the

material was not heat-treated. In this case, the plastic zone extended gradually due to the stress concentration at the front⁽⁵⁾ on the contrary to the non-treated material in which the plastic zone extended joining with the new Lüders bands appearing at the front of the plastic zone. In the case of a notched specimen, slip bands in crystal grains could be observed under a photomicroscope as shown in Fig. 9. These slip bands are oriented along the macroscopic direction of the progression of the plastic zone, and it is considered that 45°-shear type slip⁽⁶⁾ occurs in the specimen. The slips were many in the vicinity of the notch and diminished in the neighbourhood of the center line of the specimen, and this shows the effect of the notch. Multiple slips were observed in grains when the load was increased.

Various studies have been made about the influence of grain size on the strength of materials⁽⁷⁾⁻⁽¹¹⁾. In this study, it is the purpose to know whether the consideration about σ_c described above

Explanation of symbols in Fig. 8

Symbol	Rod specimen		Plate specimen	
	Smooth specimen	Notched specimen	Smooth specimen	Notched specimen
Tensile strength	●	●	■	■
Upper yield point	●		■	
Lower yield point	●	○	■	■
Breaking strength	○	○	■	■

Fig. 8 The results of tensile experiments

holds good when the grain size is different rather than to know the grain size dependence, and the strength is plotted against $D^{-1/2}$, where D is the mean ferrite grain size. The result is shown in Fig. 10 and (a), (b) and (c) are the cases of smooth specimen, notched specimen $d/b=0.2$ and $d/b=0.4$, respectively. In these figures, the scattering of breaking stress of smooth specimen is considerable, but the other strengths are almost linear. The gradients of the curves are almost the same. Strength in Fig. 10 (c) is greater than that of (b) except σ_c , and this agrees with the above consideration. The magnitude of σ_c does not depend on the notch depth for each grain size and is almost the same as the lower yield point of the smooth specimen. Thus it may be said that strength σ_c does not depend on the geometrical shape of the specimen though the grain size is different.

Now, as the initiation of plastic zone was observed by naked eyes for σ_a of notched specimen, it might depend on the precision of the observation; and what stage of slip corresponds to the upper yield point will remain as a question.

3.2.2 Rod specimen

In the case of a rod specimen, the smooth part was ground with emery paper but the observation of the initiation of the plastic zone at the notch root was difficult. For a rod specimen, the appearance of the Lüders band at the neighbourhood of the shoulder of the notch was observed at first, and the band looked like a parabola curve (strictly speaking, this is a part of an ellipse), and the photograph is shown in Fig. 7 (i). It is considered that the strength at this stage corresponds to σ_a of the notched plate specimen. The reason for this is that the extrapolation of this Lüders band reaches the vicinity of the notch root and the change of the gradient of the load elongation diagram can be seen at this stage and these correspond to the case of plate

Fig. 9 Slip bands at the vicinity of a notch root ($\times 400 \times 3/4$)

specimen. To make sure about this point, a tensile test was carried out using a specimen which was slashed through the center line of the rod specimen, as shown in Fig. 11, and it was observed that the initiation of above Lüders band corresponds to σ_d of a plate specimen. After this stage, slip band as can be seen in a smooth specimen appeared obliquely to the axis at the smooth part and a hesitation period in load elongation diagram could be seen with an increase of tensile load, and after the hardening the specimen was broken. As described above, the first observation was made for σ_d and σ_a could not be decided in the case of rod specimen, then the magnitude of σ_a was estimated as follows. If the ratio $\sigma_a/\sigma_d \doteq 0.656$ for $d/b=0.2$ in the case of plate specimen also holds good for the corresponding rod specimen, $\sigma_{a\text{rod}} \doteq 26.3 \text{ kg/mm}^2$ is obtained. The ratio σ_a/σ_d for a plate hardly depends on the notch depth, and by applying this relation to a rod specimen with a deep notch, $d/b=0.4$, $\sigma_{a\text{rod}} \doteq 36.2 \text{ kg/mm}^2$ is obtained, and this is comparable to the upper yield point, 36 kg/mm^2 , of the smooth rod specimen. However, the magnitude of σ_a is reduced with an increasing thickness of a specimen, where the condition approaches that of a rod, as shown in Fig. 8 (c), then the yielding may be induced at earlier stage than estimated above in the case of a notched rod specimen. Consequently, the initiation of yielding at the notch root of a rod specimen could not be decided in this study and further study must be continued about this point.

3.2.3 The relation between the notch strength of rod and plate specimens in tensile experiment

Whether the relation (1) holds good for the result of tensile experiment or not is investigated as follows. The strength σ_a has close relation to α based on the elasticity, but the breaking strengths are compared with each other as in the case of fatigue experiment because σ_a could not be obtained for a rod specimen. At first, the case of $d/b=0.2$ is

Fig. 11

considered. From the experimental results, taking the breaking strength for plate and rod smooth specimens as $\sigma_{f,\text{smooth}}=110 \text{ kg/mm}^2$ and $\sigma_f=45 \text{ kg/mm}^2$ for the notched plate specimen, $\beta_2=\sigma_{f,\text{smooth}}/\sigma_f$ is calculated as $\beta_2 \doteq 2.4$. From this result, $\beta_3'=2.1$ is obtained using Eq. (2). However, according to the experimental result $\sigma_f=67 \text{ kg/mm}^2$, then $\beta_3=1.6$ is obtained, and $\beta_3 < \beta_3'$ can be seen. Thus, calculated result for a rod using Eq. (2) become an overestimation. The similar result can be seen in the case of $d/b=0.4$. Furthermore, the similar result is obtained even if the maximum tensile strength is considered instead of breaking strength.

3.3 The relation between the fatigue strength and the static strength

Although fatigue tests in this study were not alternating tension but the alternating bending, the relation between these results and tensile results is studied as follows. The ratio between fatigue and tensile strength for the same magnitudes of ρ/b , d/b and thickness is different for different ρ , because the fatigue strength varies with ρ linearly but static strengths are hardly affected by ρ . The fatigue strength is small for small ρ . Therefore we must be careful when the fatigue strength is estimated from the static strength. In this study in which $d/b=0.2$ and $t=3 \text{ mm}$ for a plate, the experimental equation to give the fatigue strength w is obtained as

$$w = \{0.173(\rho/b - 0.045) + K\} \times 29 \quad \dots\dots\dots (4)$$

where $K=0.345$ for a rod and $K=0.28$ for a plate respectively, and the lower yield point was taken as 29 kg/mm^2 .

3.4 Deformation character

The contraction percentage of area and the elongation percentage for the gauge length, 80 mm , are shown in Figs. 12 and 13, respectively. In Fig. 12, the elongation percentage is smaller for a notched specimen than for a smooth specimen and the deformation is constrained by the presence of a notch, and the effect of ρ on the elongation percentage is not so considerable. The reason for this may be that as the elongation percentage is calculated for total elongation of the gauge length it contains not only the local elongation but also the uniform elongation and the local effect of a notch is not so considerable. The magnitude of elongation is greater for a greater cross section of the specimen and it is greater for Fig. (a) than for Fig. (b). But the magnitude is greater for a plate than for a rod. The reason for this is considered to be that the constraint for the deformation due to a notch is caused by a couple of sides for a plate but it is caused from all around the surface in the case of rod. In Fig. (c), elongation percentage of a smooth specimen

increases with an increasing thickness and this is also understood as above, that is, it is due to the increase in the cross section. The effect of a notch on the deformation in the neighbourhood of a notch

is greater for the breadth than for the thickness, then the magnitude of elongation percentage is hardly changed by the thickness when the notch depth is kept constant, namely the distance between the opposite notches is constant.

(a) Notched specimen ($d/b=0.2$)

(b) Notched specimen ($d/b=0.4$)

(c) $d/b=0.4$, $\rho=0.5$ mm

Fig. 12 Deformation characters

The contraction percentage of area is greater for a smooth specimen than for a notched specimen and it is also constrained by a notch. As the contraction depends on the local condition in the neighbourhood of the breaking part, the constraint is strong for small ρ , and the effect is more considerable for a rod than for a plate which is similar to the above elongation percentage. According to Fig. (c), the magnitude does not depend on the thickness, but the magnitude for a notched specimens reduces with an increasing thickness. The reason for this is considered as follows. The reductions in deformations for the breadth and for the thickness increase with an increasing thickness, and consequently the reduction in the cross section increases with an increasing cross section and then no change can be seen in the contraction percentage of the area in the former case. In the latter case, the reduction in the thickness increases a little with an increasing thickness but the reduction in the breadth is almost constant for every thickness because the depths d and ρ are kept constant, and then the change in the cross section is very small compared with the increase of the cross section. Thus, the magnitude is reduced with an increase of thickness.

In addition to the above consideration, the effects of grain size on the deformation characters are considered from Fig. 13. The experimental results are plotted against $D^{-1/2}$ in the same manner as the strength. According to Fig. 13 (a), which shows the results of a smooth specimen, the deformation characters are greater for the wide specimen, which corresponds to a notched specimen of which $d/b=0.2$, than for that of $d/b=0.4$. The change of elongation percentage due to the grain size is very small and the magnitude reduces a little with a decreasing grain size, but the contraction percentage of area increases with a decreasing grain size. The reason for this is considered as follows. The elongation percentage is great for the coarse grain in which the deformation is easy. For the contraction percentage of area, the fracture is caused mainly by the slip which is at 45 deg. to the direction along the thickness and this fracture is caused at an earlier stage in the case of coarse grain, therefore the contraction in the breadth is small and the contraction percentage is reduced.

As described above, it is interesting to note that the changes of elongation percentage and contraction

percentage of area, which are used to know the deformation character, do not always show a similar tendency. These deformation characters, and especially the contraction percentage of area are understood by considering both deformations in breadth and in the thickness.

4. Conclusions

The results of this study are summarized as follows.

Fig. 13 Deformation characters versus grain size (gauge length 80 mm)

(1) The fatigue and static strengths of a rod specimen are greater than those of the corresponding plate specimen for the same notched profile.

(2) The magnitude of the fatigue notch factors is almost the same for a rod specimen and for the corresponding plate specimen, and decreases linearly with the increase of the notch root radius. But we must be careful when the fatigue notch factor of a rod specimen is calculated from that of the corresponding plate specimen using the conventional approximate equation, because the calculated result is smaller than the experimental result. The experimental equation is given for the case of this study.

(3) The static tensile strength does not depend on the notch root radius. The strength in stress unit is greater for a deep notch than for a shallow notch. Although the effect of the thickness of the specimen is not considerable, the stress for the initiation of a plastic zone at a notch root decreases with an increasing thickness.

(4) The nominal stress, which corresponds to the stage where the plastic zone connects the opposite notches, does not depend on the geometrical shape of the specimen, and is comparable to the lower yield point of a smooth specimen. Similar results could be obtained for other grain sizes.

(5) The yielding initiates at a considerably earlier stage than the apparent yielding, that is observed by the appearance of the "dance stage" in the load elongation diagram, (as in the case of a smooth specimen) in the case of notched plate specimen. From this result, it is considered that the first yielding will be induced at the earlier stage before the apparent yielding in the case of a rod.

(6) For the static tensile strength, the magnitude of the notch factor for the breaking stress of a rod is overestimated when it is calculated from that of the corresponding plate specimen using the approximate equation for the stress concentration factor.

(7) The relation between the fatigue and the static strength varies with ρ , because the fatigue strength varies linearly with ρ but the static strength does not depend on ρ . The experimental equation is given for this study.

(8) A notch constrains the deformation. The deformation of a notched plate specimen is constrained by a couple of sides but a rod is constrained from all around the surface, and accordingly the latter deformation characters are smaller than that of a plate. The variation of elongation percentage does not always agree with that of the contraction percentage of area, and these deformation characters can be understood by the consideration of the

deformation in thickness and in breadth.

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