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Shape of the Cyclic Stress-Strain Loop in the Low-Cycle Fatigue Tests*

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The stress-strain relations for nickel-chrome steel, aluminum, and low-carbon steel were investigated under various cyclic loading conditions, such as constant strain cycling, constant load cycling with and without mean stress. The following approximate formula to represent the shape of cyclic stress-strain loop was obtained for the whole range of life excluding the initial and final periods,

$$\frac{\varepsilon}{(\sigma_{\max})^p} = K \left(\frac{\sigma}{\sigma_{\max}} \right)^m$$

where σ and ε denote the changes of stress and strain in each half cycle measured from the tip of each loop, $\sigma_{\max}$ is the maximum tensile stress in each half cycle and K , m , and p are material constants.

Cyclic stress-strain curve proposed by Morrow can be deduced from the above formula in a special case.

1. Introduction

It is very important to represent quantitatively the behaviors of cyclic deformation and the shape of cyclic stress-strain loop for discussing synthetically the low-cycle fatigue strength of metals with the test results obtained under various loading conditions, as well as for designing the machine-elements which are operated under the cyclic loading conditions from the standpoint of low-cycle fatigue.

The results of study on the cyclic deformation behaviors and the fatigue strength in the constant load cycling test with and without mean stress have already been reported in some papers⁽¹⁾⁻⁽⁴⁾ by the authors, but those on the shape of cyclic stress-strain loop have just briefly been described in the authors' papers^{(1) (5)}. Hence, the shape of cyclic stress-strain loops in the whole range of life excluding the initial and final periods is discussed fully in this paper with use of the low-cycle fatigue test results on aluminum, low-carbon steel, and nickel-chrome steel. It is found that the cyclic stress-strain curve in each cycle can be represented quantitatively by the approximate formula proposed in this paper regardless of the stress ratio.

2. Specimens and test procedure

Aluminum (A1B 3), low-carbon steel (S 10 C), and nickel-chrome steel (SNC 2) were used as the testing materials. The shape of the specimens and the mechanical properties of these materials are shown in Fig. 1 and Table 1, respectively. Low-cycle

Table 1 Mechanical properties

Materials	Ultimate strength kg/mm ²	Elongation %	Reduction of area %
Aluminum (A1B 3)	11.5	—	73.9
Low-carbon steel (S 10 C)	35.9	36.8	68.2
Nickel-chrome steel (SNC 2)	91.0	38.9	62.4

Materials	d	R
A1B 3	6	17
S 10 C	4.5	16
SNC 2	5	24

Fig. 1 Fatigue test specimens

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fatigue tests were conducted at 10 c/min with a hydraulic push-pull fatigue test machine under the various cyclic loading conditions as shown in Table 2.

Cyclic stress-strain curve was obtained from measurement of the relation between the applied load and the change in the diameter of the specimen by an X-Y recorder.

Here, the longitudinal strain ε was calculated with use of the logarithmic radial strain ε_d from the following equation: $\varepsilon = -2\varepsilon_d + \{(m-2)/m\}(\sigma/E)$, where m is Poisson's number, E is Young's modulus, and σ is the true stress. This equation can be derived from simple calculations with the following assumptions: Total strains in longitudinal direction and radial direction are the sum of the plastic and elastic strains in each direction. The relation between the longitudinal elastic strain and the radial elastic strain follows the Hooke's law, and the relation between the longitudinal and radial plastic strains satisfies the constant volume condition.

3. Test results and discussions

As described previously, the

Table 2 Cyclic loading conditions

Materials	Stress ratio (Nominal)		
Aluminum (A1B3)	-0.9,	-0.75,	-0.5
Low-carbon steel (S 10 C)	-1.0,	-0.5,	
Nickel-chrome steel (SNC 2)	-1.0,	-0.5,	Constant strain cycling test

Fig. 2 Coordinate systems for representing cyclic stress-strain loop

cyclic stress-strain curves in the initial and final periods of load cycling are not discussed in this paper. Generally, there are two coordinate systems for representing the stress-strain relation in each half cycle, as shown in Fig. 2. The $\sigma' \sim \varepsilon'$ coordinate system, with the origin O' , gives the stress and strain increments at any instant during this cycle, and the $\sigma \sim \varepsilon$ coordinate system, with the origin O , gives the stress and the strain measured from the stress-free state O .

Figure 3 shows the experimental relations in the $\sigma \sim \varepsilon$ coordinate system between the stress and the strain on a large number of hysteresis loops obtained from fatigue tests under three loading conditions on nickel-chrome steel. As the hysteresis loops adopted here were chosen at random, the maximum tensile stress and number of cycles for these hysteresis loops differed greatly. In spite of this, the stress-strain relation for a certain cyclic loading

Fig. 3 Experimental stress-strain relations on the $\sigma \sim \varepsilon$ coordinate system for nickel-chrome steel

Fig. 4 Experimental stress-strain relations on the $\sigma' \sim \varepsilon'$ coordinate system for nickel-chrome steel

condition may be considered to be represented by an approximate curve with small deviations. However, the stress-strain data under three cyclic loading conditions do not lie on a single curve, as clearly known from the figure. Therefore, it may be inconvenient

to use the $\sigma \sim \varepsilon$ coordinate system for representing synthetically the cyclic stress-strain relations under various cyclic loading conditions. Such behaviors shown in Fig. 3 of nickel-chrome steel are observed with aluminum and low-carbon steel, too. The rela-

tions for nickel-chrome steel shown in Fig. 3 are transformed into those in the $\sigma' \sim \varepsilon'$ coordinate system as shown in Fig. 4. It is found from this figure that the stress-strain relations in the constant load cycling test with mean stress roughly overlap those in the completely reversed load cycling tests. As a matter of course, the $\sigma' \sim \varepsilon'$ system may be considered to be more convenient for measuring the total strain range and the hysteresis area from the shape of cyclic stress-strain loop than the $\sigma \sim \varepsilon$ coordinate system. Now, let us examine the experimental results in Fig. 4 more minutely. Then we can see a fairly wide scatter of the experimental data, which is dependent on the maximum tensile stress $\sigma_{\max}$ during a cycle, at comparatively high values of σ' .

In the previous papers^{(1) (5)}, the authors suggested that the shape of cyclic stress-strain loop was dependent on the maximum tensile stress in each cycle, and proposed the $\bar{\sigma} \sim \bar{\varepsilon}$ coordinate system for representing the cyclic stress-strain curve in each half cycle, where $\bar{\sigma} = \sigma' / \sigma_{\max}$ and $\bar{\varepsilon} = \varepsilon' / \sigma_{\max}$. The cyclic stress-strain relations represented by the $\bar{\sigma} \sim \bar{\varepsilon}$ coordinate system for aluminum, low-carbon steel, and nickel-chrome steel are shown in Fig. 5 (a), (b), and (c) on a log-log basis, respectively. The scatter of the stress-strain relations in Fig. 5 (c) for nickel-chrome steel is considerably small in comparison with that in Fig. 4 in spite of the wide variations in the maximum tensile stress $\sigma_{\max}$ from 48.8 kg/mm² to 91.0 kg/mm² in these experiments. A similar tendency is also observed with aluminum and low-carbon steel.

(a) Experimental stress-strain relations on the $\bar{\sigma} \sim \bar{\varepsilon}$ coordinate system for aluminum in the constant load cycling tests

(b) Experimental stress-strain relations on the $\bar{\sigma} \sim \bar{\varepsilon}$ coordinate system for low-carbon steel in the constant load cycling tests

(c) Experimental stress-strain relations on the $\bar{\sigma} \sim \bar{\varepsilon}$ coordinate system for nickel-chrome steel in the constant load cycling tests

Fig. 5

Figure 6 is a schematic representation of the relations in Fig. 5. As indicated in this figure, we can divide the stress-strain relation into three parts, such as, I, II, and III. Here, the points A and B can be determined in the following way.

In order to obtain the slope at a certain point of the stress-strain relation in the $\bar{\sigma} \sim \bar{\epsilon}$ coordinate system on a log-log basis, we take two adjacent points on the stress-strain curve and denote these points as $(\bar{\epsilon}_1, \bar{\sigma}_1)$ and $(\bar{\epsilon}_2, \bar{\sigma}_2)$, respectively. Then the slope of the curve $\bar{n}$ at the point $(\bar{\epsilon}_1, \bar{\sigma}_1)$ may be approximately given by $\log(\bar{\sigma}_2/\bar{\sigma}_1)/\log(\bar{\epsilon}_2/\bar{\epsilon}_1)$. Figure 7 (a), (b) and (c) show the relations between $\bar{n}$ and $\bar{\epsilon}_1$ for aluminum, low-carbon steel, and nickel-chrome steel, respectively. The value of $\bar{n}$ is nearly constant for the value of $\bar{\epsilon}_1$ which is less than $\bar{\epsilon}_A$ in the figures. Beyond $\bar{\epsilon}_A$, $\bar{n}$ rapidly decreases at first, then gradually approaches a nearly constant value $\bar{n}_s$ for all values of $\bar{\epsilon}_1$ higher than $\bar{\epsilon}_B$.

Fig. 6 Schematic representation of stress-strain relation with the $\bar{\sigma} \sim \bar{\epsilon}$ coordinate system on a log-log basis

(a) Relation between $\bar{n}$ and $\bar{\epsilon}_1$ for aluminum

(b) Relation between $\bar{n}$ and $\bar{\epsilon}_1$ for low-carbon steel

(c) Relation between $\bar{n}$ and $\bar{\epsilon}_1$ for nickel-chrome steel

Fig. 7

Now let $\bar{\sigma}_A$ and $\bar{\sigma}_B$ be the values corresponding to $\bar{\epsilon}_A$ and $\bar{\epsilon}_B$ on the stress-strain curve represented in the $\bar{\sigma} \sim \bar{\epsilon}$ coordinate system, respectively. Then, $\bar{n}_s$, $\bar{\epsilon}_A$, and $\bar{\sigma}_A$ may be assumed to be the material constants, while the value of $\bar{\epsilon}_B$ is not defined so clearly. Table 3 shows the values of these material constants including the roughly estimated value of $\bar{\epsilon}_B$ from Fig. 7.

It may be considered that the straight part I, straight part III with slope $\bar{n}_s$, and the curved

section II in the cyclic stress-strain curve shown in Fig. 6 correspond to the elastic range, the plastic range, and the transition range from elastic to plastic, respectively. The straight part III with the constant slope $\bar{n}_s$ can be represented by the following form:

$$\bar{\sigma} = C(\bar{\epsilon})^{\bar{n}_s} \quad (1)$$

The value of C in this expression seems to be nearly constant for each material at first glance at Fig. 7 (a), (b), and (c), but checking again more precisely, you will see that C is dependent on the value of σ_{max} as described next. With use of the values of $\bar{n}_s$ in Table 3 and the values of $\bar{\sigma}$ and $\bar{\epsilon}$ on each cyclic stress-strain curve obtained experimentally, the value of C in the range III for each hysteresis loop can be calculated by Eq. (1). Figure 8 (a), (b), and (c) show the relation between the value of C thus calculated and σ_{max} for each material on a log-log basis. It is found

Table 3 Values of $\bar{n}_s$, $\bar{\epsilon}_A$, $\bar{\epsilon}_B$, and $\bar{\sigma}_A$

Materials	$\bar{n}_s$	$\bar{\epsilon}_A$	$\bar{\epsilon}_B$	$\bar{\sigma}_A$
AlB3	0.065	16×10^{-5}	40×10^{-5}	1.3
S10C	0.064	7.5×10^{-5}	30×10^{-5}	1.3
SNC 2	0.077	6.5×10^{-5}	30×10^{-5}	1.3

(a)

(b)

(c)

Fig. 8 C values in Eq. (1) against σ_{max}

from these figures that every experimental plot lies on the straight line drawn in the figure. Consequently, this relation can be expressed by $C = \alpha(\sigma_{max})^\beta$. Then, Eq. (1) becomes the following form:

$$\frac{\sigma'}{(\sigma_{max})^{\eta_{III}}} = D_{III} \left(\frac{\epsilon'}{\sigma_{max}} \right)^{\bar{n}_s} \quad (2)$$

or

$$\frac{\epsilon'}{(\sigma_{max})^{P_{III}}} = K_{III} \left(\frac{\sigma'}{\sigma_{max}} \right)^{m_{III}} \quad (3)$$

where η_{III} , D_{III} , $\bar{n}_s$, P_{III} , K_{III} , and m_{III} are material constants. Table 4 shows these values for the materials tested. As known from Eq. (2) or Eq. (3), the stress-strain curve in the range of III can be represented with only three material constants. In the case of very large values of $\bar{\epsilon}$, the cyclic stress-strain relation in the whole range may be approximately represented by the above relations given by Eq. (2) or Eq. (3), just like the fully plastic static case where the stress-strain curve can be represented approximately by the n power hardening rule. But, in the range of a relatively small cyclic strain such as in the constant load cycling test with mean stress, the stress-strain curve corresponds to the range II in Fig. 6. Although there were various means to represent approximately the stress-strain curve in the range II, the straight line passing through the point A, which lay between the line $\overline{AB}$ and the tangential line of curve $\widehat{AB}$ at the point A, was used as a substitute for the real curve in the range II on account of the simplicity of the form. Then, the stress-strain curve in the range II may be given approximately by the following form:

$$\left(\frac{\sigma'}{\sigma_{max}} \right) = D_{II} \left(\frac{\epsilon'}{\sigma_{max}} \right)^{\bar{n}_{II}}$$

or

$$\left(\frac{\epsilon'}{\sigma_{max}} \right) = K_{II} \left(\frac{\sigma'}{\sigma_{max}} \right)^{m_{II}}$$

Table 4 Material constants

Part	Eq.		Aluminum (AlB3)	Low-carbon steel (S10C)	Nickel-chrome steel (SNC2)
III	(2)	η_{III}	0.86	0.856	0.707
		D_{III}	4.2	5.31	12.6
		$\bar{n}_s$	0.065	0.064	0.077
	(3)	P_{III}	3.15	3.25	4.81
		K_{III}	2.52×10^{-10}	4.8×10^{-12}	5.01×10^{-15}
		m_{III}	15.4	15.6	13.0
II	(4)	D_{II}	78.8	43.9	82.0
		$\bar{n}_{II}$	0.47	0.36	0.43
	(5)	K_{II}	9.7×10^{-5}	2.8×10^{-5}	3.5×10^{-5}
		m_{II}	2.12	2.78	2.33

where D_{II} , $\bar{n}_{II}$, K_{II} , and m_{II} are the material constants. Table 4 shows these values for the material tested. The above method of representing approximately the stress-strain curve in the range II may be considered to have some propriety based on the following observations: (1) The values of $\bar{\epsilon}_A$ and $\bar{\sigma}_A$ at point A in Fig. 6 were nearly constant for each material regardless of the wide variation in σ_{max} ; (2) the deviations of $\bar{\epsilon}$ in the range II at the same value of $\bar{\sigma}$ were comparatively small.

The "cyclic stress-strain curve" called by Morrow⁽⁶⁾, namely, the relation between the cyclic plastic strain and cyclic stress amplitudes in the constant strain cycling test may be deduced from Eq. (3) for a special case of $\sigma'/\sigma_{max} = 2$.

4. Conclusions

The stress-strain relations in nickel-chrome steel, aluminum, and low-carbon steel were investigated under various cyclic loading conditions, such as constant strain cycling, constant load cycling with and without mean stress. The approximate formula to represent the shape of cyclic stress-strain loop was obtained for the whole range of life excluding the

initial and final periods.

This approximate formula represented fairly well all of the stress-strain relations obtained for various cyclic loading conditions. Consequently, if the cyclic stress-strain relation is once determined by completely reversed load or strain tests, it may be applicable to obtaining the cyclic stress-strain curves under other loading conditions.

The "cyclic stress-strain curve" called by Morrow, namely, the relation between the cyclic plastic strain and cyclic stress amplitudes in the constant strain cycling test may be deduced from the above formula as a special case.

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