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An Energetic Consideration of Fatigue Mechanism of Metal under Mean Stress

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It was observed that the propagation rate of fatigue crack is faster under the presence of tensile mean stress. The purpose of the present study is to give a theoretical basis of the observed characteristics of fatigue crack.

At first, the energies of a crack are evaluated, the elastic energy, the surface energy, the interaction energy with external stress field and the accumulated strain energy with stress cycles. Then, the energetic conditions of crack growth are discussed. It is suggested that the process of crack propagation is divided into nearly three stages, according to the size of crack, that is, the initiation, the growth of dislocation-type crack and the propagation of semi-macroscopic crack. It follows that, the mean stress have influence on every stages, which is approximately expressed in the form pV , where p and V are the hydrostatic component of stress and the volume of crack, respectively. Various other characteristics of fatigue crack are also discussed based on the energetic consideration.

1. Introduction

In the previous papers^{(1)~(3)} the microscopic aspects of the fatigue of metals under mean stress were studied adopting X-rays and microscope. The following two effects of mean stress were observed:

- (1) The microscopic deformation in fatigue process proceeds faster under tensile mean stress,
- (2) the propagation rate of fatigue crack is greater under tensile mean stress (Fig. 1).

It was reported^{(3)~(4)} that the factor (1) possibly results from the interaction between the excess vacancies produced in fatigue process and the hydrostatic component of applied stress. In the present paper, discussions are made on the factor (2), that is, the effect of mean stress on the propagation of fatigue crack. It is known from microscopic observation that the greater part of fatigue life is devoted to the propagation of crack, so it is important to reveal the nature of fatigue crack in discussing fatigue phenomena⁽¹⁾.

Several measurements^{(5)~(8)} have been made on the growth rate of fatigue crack under mean stress. It was found that the mean stress affects the growth rate, though the degree of the effect is different in different metals. In general, the growth

rate is given by

$$\frac{dl}{dN} = f(\sigma_a, \sigma_m, l) \dots \dots \dots (1)$$

where l and N are respectively the length of crack and the cycle number, and σ_a and σ_m are the stress amplitude and the mean stress, respectively. Few studies, however, have been made on the microscopic aspects of the role of mean stress, that is, the relation between the mean stress and the fatigue mechanism or the growth of fatigue crack.

The previous investigations^{(9)~(18)} on the nucleation and the growth of fatigue crack may be divided into the following two types. One is the

Fig. 1 Effect of mean stress on the growth rate of fatigue crack (measured with 0.25% carbon steel bending fatigue specimen which has a center hole of stress concentration factor $\alpha=2.1^{(11)}$)

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analysis of stress distribution around the fatigue crack based on the continuum mechanics^{(9)~(12)}, where the physical aspects of the crack growth is not taken into account. The other is the microscopic mechanism of the crack growth^{(13)~(18)} based on the dislocation theory or the observation with electron microscope, where usually little attention is paid to the stress distribution. It is difficult at present to make perfect theory of fatigue crack, especially to discuss the influence of mean stress, partly because of the wide character of fatigue. It seems important, however, to give some perspective of the problem. In this connection, the energetic consideration is applied, in the present study, to the growing process of fatigue crack, which is considered to combine the above two points of view, that is, the stress distribution and the physical process of the crack. Although the energetic treatment is somewhat phenomenological, it may be expected that the effect of mean stress and the correlation between the characteristic features of fatigue are clarified.

In the present paper, energetic consideration is made on the mechanism of fatigue fracture with relation to the effect of mean stress. At first, the energies concerning the fatigue crack are listed and then discussions are made on the growing process. [The word "crack" is used in the present paper in a wide sense including intrusion or pore⁽¹⁵⁾.]

2. Energies of crack

It is known that, in the energetic discussion on the fracture of metals, the interaction energy between the crack and the stress field is negligible when the growth rate of crack is fast and the fracture occurs under the atmospheric pressure. It is considered, however, that the interaction energy has to be taken into account in the discussion on the fatigue crack having a relatively low growth rate or the brittle fracture under high hydrostatic pressure, for instance. R.J. Dower⁽¹⁹⁾ made energetic discussion on the brittle fracture of metals under high hydrostatic pressure. It seems necessary that similar energetic consideration is applied to the growing process of fatigue crack.

In the following, a crack composed of dislocations with opposite signs shown in Fig. 2 is adopted as a microscopic model of fatigue crack (cf. Section 5.1). This type of crack was first proposed by E. Fujita⁽¹⁴⁾ for the nucleation of fatigue crack, and then also applied to the growing process of fatigue crack^{(15)~(18)}. The model of a semi-macroscopic crack shown in Fig. 3 is also adopted corresponding to the advanced stage of the crack propagation. (This type of crack is widely used in the analysis based on the continuum mechanics.)

2.1 Energy of crack and interaction energy between crack and stress field

It is known⁽¹⁹⁾ that the energy E_c of a crack is composed of the following four terms; the elastic strain energy E_1 of the crack, the surface energy E_2 , the interaction energy E_3 with the outer stress field, and the energy E_4 of the volume increase of the crack.

$$E_c = E_1 + E_2 + E_3 + E_4 \dots\dots\dots (2)$$

(Although the two-dimensional crack is supposed in the following discussion, similar analysis is applicable to a three dimensional penny-shaped crack by changing the coefficient of each energy slightly⁽¹⁹⁾.)

2.1.1 Elastic energy of stress field around crack: E_1

As the stress distribution around the crack shown in Fig. 2 has not been obtained, it is approximately represented in the present analysis with the dipole of edge dislocations. The energy of the dipole of edge dislocations per two dislocation lines is given⁽²⁰⁾ by

$$E_d = \frac{Gb^2}{2\pi(1-\nu)} \left\{ \ln \frac{\nu d}{b_0} - \frac{1}{2} \cos 2\theta \right\} \dots\dots\dots (3)$$

where G is the shear modulus, ν is the Poisson's ratio, b is the Burgers' vector and b_0 is the radius of dislocation core whose value is in the order of 10^{-8} cm; r_d is the distance between the two opposite dislocations and θ is the angle between the Burgers' vector and the direction connecting the two dislocation cores.

Let us consider the arrangement of dislocations just before the crack nucleation, as shown in Fig. 4.

Fig. 2 Microscopic fatigue crack composed of edge dislocations with opposite signs

Fig. 3 Semi-macroscopic fatigue crack

Fig. 4 Supposed arrangement of dislocations just before the crack formation

When the number of edge dislocations in each side of the crack is $n/2$, substituting $r_d=h$, $\theta=\pi/2$ into Eq. (3),

$$E_1^{(1)} \cong \frac{Gn^2b^2}{8\pi(1-\nu)} \left(\ln \frac{h}{b_0} + \frac{1}{2} \right) \quad (4)$$

If the crack shown in Fig. 2 is produced in the arrangement of dislocations (Fig. 4), the energy E_1 is given by exchanging the minimum radius b_0 of the elastic field in Eq. (4) with $nb/2$.

$$E_1^{(2)} \cong \frac{Gn^2b^2}{8\pi(1-\nu)} \left(\ln \frac{2h}{nb} + \frac{1}{2} \right) \quad (5)$$

which corresponds to the case of $nb/2 \leq h$. On the other hand, when the crack grows and becomes $nb/2 > h$, $nb/2$ and h in Eq. (5) have to be exchanged.

$$E_1^{(3)} \cong \frac{Gh^2}{2\pi(1-\nu)} \left(\ln \frac{nb}{2h} + \frac{1}{2} \right) \quad (6)$$

(Generally, the shape of crack is complex, so the height h of the crack is considered to express the mean value).

2.1.2 Surface energy of crack: E_2

The surface energy E_2 of crack is given by

$$E_2 = S\gamma' \quad (7)$$

where S and γ' are the surface area of the crack and the effective surface energy per unit area, respectively. Usually plastic deformation occurs ahead of the fatigue crack, so the effective surface energy γ' is given by the sum of the plastic work w and the intrinsic surface energy $\gamma^{(19)}$, that is, $\gamma' = \gamma + w$. The surface area S is given by $S \cong 2h + nb$ for the cracks shown in Fig. 2 and Fig. 5, while $S' \cong 2(l+h)$ for that shown in Fig. 3. The value of γ is dependent both on the crystal plane on which the crack grows and the atmosphere surrounding the crack. As is well known by experiments, the fatigue life is fairly affected by the atmosphere.

2.1.3 Interaction energy between outer stress field and crack: E_3

The elastic stress in the material is partly released by the presence of crack, which corresponds to the interaction energy E_3 between the outer stress field and the crack⁽²¹⁾.

$$E_3 = -\frac{\pi\theta\sigma_c^2l^2}{8G} \quad (8)$$

where σ_c is the normal stress perpendicular to the crack and θ is a constant. Although the value of θ depends on the shape of crack, it is approximately

Fig. 5 Advanced stage of the dislocation crack

given by $\theta \cong \nu^{(21) \sim (23)}$. It follows from Eq. (8) that the energy E_3 is independent of the sign of the stress σ_c . The length l of the crack shown in Fig. 2 is given by $l \cong nb/2$.

2.1.4 Energy of volume increase: E_4

Let us denote the volume change of the crack by the strain $(\epsilon_1, \epsilon_2, \epsilon_3)$. Assuming that the principal axes of stress coincide with those of strain, the energy E_4 of the volume expansion due to crack growth is given by

$$E_4 = -(\sigma_1\epsilon_1 + \sigma_2\epsilon_2 + \sigma_3\epsilon_3)V_c \\ = -p\epsilon V_c - (\sigma_1'\epsilon_1' + \sigma_2'\epsilon_2' + \sigma_3'\epsilon_3')V_c \quad (9)$$

where V_c is the volume of the crack and

$$\left. \begin{aligned} \sigma_i' &= \sigma_i - p, \quad p = (\sigma_1 + \sigma_2 + \sigma_3)/3 \\ \epsilon_i' &= \epsilon_i - \epsilon/3, \quad \epsilon = \epsilon_1 + \epsilon_2 + \epsilon_3 \end{aligned} \right\} \quad (i=1, 2, 3) \quad (10)$$

It follows from Eq. (9) that, $E_4 = -\sigma V_c$ when the crack increases its volume in the direction of the stress axis (σ), while $E_4 = 0$ when the crack propagation is in the direction perpendicular to the stress axis. If the volume increases uniformly in every direction, $E_4 = -pV_c$. Generally, the growth of fatigue crack is considered to be nearly uniform, otherwise the crack would collapse. Then, it is expected that

$$E_4 \cong -pV_c \quad (11)$$

It is noteworthy that the outer stress field affects the energy E_4 in the form of the hydrostatic stress component p . The volume of crack is given by $V_c \cong nbh/2$ in Fig. 2, and by $V_c' \cong hl$ in Fig. 3.

2.2 Energies supplied from the surroundings to crack

In the discussion of the nucleation and the growth of the microscopic fatigue crack, the energy W_c supplied from the surroundings to the crack during its growth has to be taken into account. It is considered that W_c is composed of the following two terms, provided that the motion energy of the dislocations is neglected; the strain energy W_1 of the dislocations before composing the crack, and the work W_2 done by the applied stress during the motion of the dislocations to the crack.

$$W_c = W_1 + W_2 \quad (12)$$

2.2.1 Strain energy of dislocation: W_1

As is well known, the strain energy of edge dislocation per unit length is given by⁽²⁴⁾

$$E_e = \frac{Gb^2}{4\pi(1-\nu)} \ln \frac{r_1}{b_0} \quad (13)$$

where r_1 is the range (outer radius) of the elastic stress field having the order of 10^{-4} cm⁽²⁴⁾. The energy W_1 of the n dislocations which are present in the material before composing the crack is given by

$$W_1 = nE_e = \frac{nGb^2}{4\pi(1-\nu)} \ln \frac{r_1}{b_0} \quad (14)$$

In the case of the semi-macroscopic crack shown in Fig. 3, it may be necessary to consider the portion of the highest dislocation density ahead of the crack, such as the subgrain boundary. Let us denote the strain energy and the density (length) of dislocation by $W_1(x)$ and $\rho_c(x)$, respectively, which are released into the crack when the crack grows by x . Then, from Eq. (13)

$$W_1'(x) = \frac{\rho_c(x)Gb^2}{4\pi(1-\nu)} \ln \frac{r_1'}{b_0} \quad (15)$$

where r_1' ($r_1' \leq r_1$) is the radius of the elastic field of dislocation when the interaction occurs with the neighboring field because of the high dislocation density.

2.2.2 Work done by applied stress to motion of dislocations: W_2

The force f acting on the dislocation is given by $f = \tau_s b$, where τ_s is the shear stress on the slip plane. Let us assume that, when the dislocation crack composed of n dislocations (Fig. 2) is produced, n' dislocations ($n' \geq n$) move by the mean distance s in the perpendicular direction to the dislocation line under the shear stress τ_s . If the averaged frictional force to the motion of dislocation is τ_0 , the stress $(\tau_s - \tau_0)$ is supplied additionally to the dislocations in the process of crack formation. Then, the work W_2 done by the applied stress to the motion of dislocation during the crack formation is given by

$$W_2 = n'(\tau_s - \tau_0)bs \quad (16)$$

(It is considered that the energy W_2 is important especially in the nucleation and in the early stage of the crack growth⁽¹⁴⁾.)

It is apparent that some of the energies of the crack expressed by Eqs. (4)~(11) and (14)~(16) are related to the boundary conditions of crack, while the others are to the nature of metals.

3. Growth of crack

The growing process of fatigue crack may be divided into the following three stages according to the relative importance of the energies mentioned above; the crack nucleation, the growth of dislocation crack and the growth of semi-macroscopic crack (for the sake of simplicity, the uniaxial stress state is assumed in the following discussion).

3.1 Crack nucleation

In order to make the crack shown in Fig. 2, it is necessary for the n dislocations having the energy W_c to take the position shown in Fig. 4. W_c is given from Eqs. (12), (14) and (16) by

$$W_c = \frac{nGb^2}{4\pi(1-\nu)} \ln \frac{r_1}{b_0} + n'(\tau_s - \tau_0)bs \quad (17)$$

The energy $E_c^{(1)}$ of the arranged dislocations is given from Eq. (4) by

$$E_c^{(1)} = E_1^{(1)} \approx \frac{Gn^2b^2}{8\pi(1-\nu)} \left(\ln \frac{h}{b_0} + \frac{1}{2} \right) \quad (18)$$

[In Eq. (18) E_c is taken equal to E_1 , for in this stage a crack has not been produced yet.] The energetic condition is given by

$$W_c - E_c^{(1)} = n'(\tau_s - \tau_0)bs + \frac{Gnb^2}{8\pi(1-\nu)} \left[\ln \left\{ \left(\frac{r_1}{b_0} \right)^2 / \left(\frac{h}{b_0} \right)^n \right\} - \frac{1}{2} \right] > 0 \quad (19)$$

In the next stage, the crack (pore) is produced at the center of the arranged dislocations. The plastic deformation ahead of the crack may be small in this stage, that is, $w \approx 0$. The energy $E_c^{(2)}$ is given from Eqs. (2), (5), (7), (8) and (11) by

$$E_c^{(2)} \approx \frac{Gn^2b^2}{8\pi(1-\nu)} \left(\ln \frac{2h}{nb} + \frac{1}{2} \right) + (2h + nb)\gamma - \frac{\pi\nu\sigma_c^2h^2}{8G} - p_+ \frac{nbh}{2} \quad (20)$$

The hydrostatic stress component p_+ at the tensile peak of the applied stress cycle is used in Eq. (20), for the crack is thought to be nucleated at the tensile peak of the stress cycle, as is energetically expected from Eq. (21). The secondary condition of the crack nucleation is given from Eqs. (18) and (20) by

$$E_c^{(1)} - E_c^{(2)} = \frac{Gn^2b^2}{8\pi(1-\nu)} \ln \frac{nb}{2b_0} + \frac{\pi\nu\sigma_c^2h^2}{8G} + p_+ \frac{nbh}{2} - (2h + nb)\gamma > 0 \quad (21)$$

Consequently, the crack is nucleated if the number n of dislocations is present which satisfies both Eqs. (19) and (21) at the same time. The situation is schematically shown in Fig. 6, where the possible number of dislocations for the crack nucleation is shown by the hatching. Once a crack is nucleated, the difference between $E_c^{(1)}$ and $E_c^{(2)}$ increases with an increase in the number n [Eq. (21)]. Therefore, the crack is stable.

3.2 Growth of dislocation crack

When the length of crack is small, that is, $l = (10 \sim 10^4)b$, each dislocation coming into the crack may play a relatively important role in the crack growth. Let us consider the model of dislocation crack shown in Fig. 5 ($h \leq nb/2$). It is expected again in this stage that the plastic work ahead of the crack is small, that is, $w \approx 0$. The energy $E_c^{(3)}$

Fig. 6 Energetic condition of the crack nucleation (schematic)

of the dislocation crack is given from Eqs. (2), (6)~(8) and (11) by

$$E_c^{(3)} \cong \frac{Gh^2}{2\pi(1-\nu)} \left(\ln \frac{nb}{2h} + \frac{1}{2} \right) + (2h+nb)\gamma - \frac{\pi\nu\sigma_c^2 n^2 b^2}{32G} - p_m \frac{nbh}{2} \dots\dots\dots (22)$$

The averaged hydrostatic stress component p_m during one stress cycle is used in the term E_4 of Eq. (22), for the growth of the dislocation crack is considered to be continuous; p_m corresponds to the mean stress σ_m .

The energies supplied from the surroundings of the crack are given from Eqs. (12), (14) and (16). It is expected in the growing stage of crack that the frictional force to the motion of dislocations coming into the crack is small ($\tau_s \cong \tau_0$). Therefore, the number of piled-up dislocations is negligible ($n' \cong n$).

$$W_c = \frac{nGb^2}{4\pi(1-\nu)} \ln \frac{r_1}{b_0} + n(\tau_s - \tau_0)bs \dots\dots\dots (23)$$

Consequently, the condition of the crack growth ($dl/dN > 0$) is given from Eqs. (22) and (23) by

$$\frac{d}{dn}(W_c - E_c^{(3)}) = \frac{Gb^2}{4\pi(1-\nu)} \ln \frac{r_1}{b_0} + (\tau_s - \tau_0)bs + \frac{\pi\nu\sigma_c^2 nb^2}{16G} + p_m \frac{bh}{2} - \frac{Gh^2}{2\pi(1-\nu)n} - b\gamma > 0 \dots\dots (24)$$

3.3 Growth of semi-macroscopic crack

With the growth of the crack ($l \gg b$), the individual dislocation coming into the crack loses its contribution. The crack shows a semi-macroscopic elasto-plastic nature^{(22) (23)} (Fig. 3) and it becomes possible to observe the crack by microscope. In this stage the crack grows by itself, that is, accompanied by the plastic deformation ahead of the crack tip⁽¹⁰⁾. Among the terms of the energy E_c' of the crack [Eqs. (2)], E_3' is comparatively large, for it contains l^2 , while the first term E_1' is negligible. Then, from Eqs. (7), (8) and (11),

$$E_c' \cong E_2' + E_3' + E_4' \cong 2(l+h)(\gamma+w) - \frac{\pi\nu\sigma_c^2 l^2}{8G} - p_+ hl \dots\dots\dots (25)$$

(The symbols having prime-mark correspond to the energies of the semi-macroscopic crack. In the term E_4' of Eq. (25), p_+ is used, for the crack often propagates stepwise in this stage⁽²⁶⁾.)

The stress concentrates at the tip of the crack corresponding to the interaction energy E_3' . The plastic deformation ahead of the crack may accumulate with the stress cycles and contribute to the successive stage of the crack growth. The strain energy W_1' accumulated at the tip of the crack is a function of the crack length l and the cycle number N . (Some parts of the plastic work w done ahead of the growing crack may also contribute to the successive growth.) In this stage,

W_2' is considered to be small and negligible as compared with W_1' . Then, from Eq. (15),

$$W_c'(x, l, N) \cong \frac{\rho_c(x, l, N)Gb^2}{4\pi(1-\nu)} \ln \frac{r_1'}{b_0} \dots\dots\dots (26)$$

where ρ_c is the dislocation density ahead of the crack. In general, the value of ρ_c decreases with an increase in x (Fig. 7), as is expected from the stress distribution ahead of the crack.

Noting that $dl = dx/2$, the condition of the crack growth ($dl/dN > 0$) is given from Eqs. (25) and (26) by

$$\frac{d}{dl}(W_c' - E_c') = \frac{Gb^2}{2\pi(1-\nu)} \ln \frac{r_1'}{b_0} \frac{d\rho_c(x, l, N)}{dx} + \frac{\pi\nu\sigma_c^2 l}{4G} + p_+ h - 2(\gamma+w) > 0 \dots\dots\dots (27)$$

Let us consider the physical process of the growth of the semi-macroscopic crack. The plastic deformation occurs at the tip of the crack owing to the stress concentration, and the dislocation density increases. Although the averaged dislocation density saturates after a certain number of stress cycles, the dislocations rearrange with the following stress cycles, that is, a substructure develops (a subgrain boundary becomes clear)⁽²⁵⁾. In the latter process, the dislocation density of the subgrain boundary increases, which helps the crack growth by supplying the strain energy in the successive stress cycles.

Although the difference in the crack behavior due to its length l was considered in the above discussion, it is expected that the magnitude of the applied stress also affects the behavior of the crack, for the term E_3' in Eq. (25) contains the normal stress σ_c^2 besides the crack length l^2 .

3.4 Rate of crack growth

Let us consider the rate (velocity) of crack growth based on the above conditions of the crack growth [Eqs. (24) and (27)]. The growth rate of the dislocation crack shown in Fig. 2 is possibly determined with the number of dislocations coming into the crack, and is relatively independent of the crack length. Therefore, the number and the velocity of the dislocations produced in the surroundings of the crack are important (cf. Appendix). This idea is supported by the fact that many small cracks

Fig. 7 Schematic distribution of the strain energy (dislocation density) ahead of the crack tip

are observed on the surface of a smooth fatigue specimen⁽¹⁾.

On the other hand, the growth rate of the semi-macroscopic crack shown in Fig. 3 is considered to be dependent on the strain energy W_c' stored ahead of the crack. Let us consider the stepwise growth of the crack having length l and denote the strain energy released at the tip of the crack by W_c' ($x, l, N+\Delta N$), where ΔN is the incremental stress cycle measured from a certain cycle number N when the crack growth stops instantaneously. If,

$$\frac{d}{dl}\{W_c'(\Delta l/2, l, N+\Delta N)-E_c'\}=0 \quad \dots\dots\dots(28)$$

is satisfied at $x=\Delta l/2$, the rate of crack growth is given by

$$dl/dN=\Delta l/\Delta N \quad \dots\dots\dots(29)$$

The rate of crack growth is mainly determined by the functional form of $W_c'(x, l, N)$ or $\rho_c(x, l, N)$. The function, however, has to be determined based on a more definite mechanism of the growth of fatigue crack than the energetic one mentioned above^{(5) (9) (11)}.

4. Effect of mean stress

One of the purposes of the present study is to clarify the relation between the mean stress and the growth of fatigue crack. Among the four terms of the energy E_c of the crack [Eq. (2)], the terms E_3 (or E_3') and E_4 (or E_4') are affected by the mean stress. Only the latter term E_4 (or E_4'), however, is dependent on the sign of the mean stress. As mentioned previously (Fig. 1), the tensile mean stress accelerates the crack growth, while the compressive one delays it (Fig. 1), which is considered energetically to result from the term $p_m V_c$ in Eq. (22) or $p_+ V_c'$ in Eq. (25). It may be

concluded that the mean stress affects the growth rates of both the dislocation crack and the semi-macroscopic crack, although its contribution is larger in the latter stage.

Figure 8 shows those energetic correlations for the semi-macroscopic crack schematically. The influences of mean stress σ_m on E_3' , E_4' and E_c' are shown in Figs. 8(a), (b) and (c), respectively. The rate dl/dN of the crack growth may have such tendency as shown in Fig. 8(d), which is expected from Eqs. (27)~(29)^{(5) (6)}. As a result, the mean stress affects the fatigue limit or the fatigue life as shown in Fig. 8(e), which is well known by the fatigue tests⁽²⁷⁾.

Let us consider the ratio of the energies E_4' to E_3' in Eq. (27). Putting $\pi\nu\cong 1$ and taking $p_+\cong\sigma_c/3$ for the sake of simplicity, we have

$$\frac{dE_4'/dl}{dE_3'/dl}\cong\frac{G}{\sigma_c}\frac{h}{l} \quad \dots\dots\dots(30)$$

As the sign of the mean stress affects only the term E_4' , it is expected from Eq. (30) that the effect is different according to the shape h/l of the crack, which is probably related to the fact that the effect of mean stress is different in each metal or alloy.

G. Sines⁽²⁷⁾ found that the fatigue lives under various combinations of stress amplitude and mean stress are well explained in terms of the sum (N_1+N_2) of the normal stresses acting on the two slip planes (the planes of the maximum shear stress), which is perpendicular to each other. The sum (N_1+N_2) is equivalent to the hydrostatic stress component p , for $N_3=0$ at the surface. Therefore, the above discussion is supported by the Sines' results. In the case of torsional fatigue, $p=0$, so $E_4\cong 0$ ($E_4'\cong 0$), and consequently little effect of mean stress is observed in the torsional fatigue life^{(2) (27)}.

5. Discussions

5.1 Model of fatigue crack

In the above discussion, the fatigue crack composed of dislocations with opposite signs shown in Fig. 2, is supposed. It is known, however, that the crack is also produced by a pile-up of dislocations of the same sign as shown in Fig. 9. Then, it is necessary to decide which is the more suitable model of the microscopic fatigue crack (the dislocation crack), again from the energetic view point.

The elastic energy E_1'' of

Fig. 8 Relation between the energies of crack, growth rate, fatigue life and mean stress (schematic)

the stress field of the crack shown in Fig. 9 is given by ⁽²⁸⁾,

$$E_1'' = \frac{Gn^2b^2}{4\pi(1-\nu)} \ln \frac{4r}{l'} \quad (31)$$

where r is the outer radius of the elastic stress field of the crack. Generally, r is in the order of 10^{-4} cm and is greater than the other terms, that is, $4r/l' \ll 2h/nb$ (for l' , $2h$ and nb are in the same order). Then, it follows from Eqs. (5) and (31), that $E_1^{(2)} \ll E_1''$. Namely, in the case of the fatigue fracture which occurs at relatively low stress level, the model shown in Fig. 2 is more suitable than that shown in Fig. 9. This suggestion supported by the fact that the fatigue crack is initially produced along the slip line ⁽²⁶⁾. Furthermore, a pile-up of dislocations is hardly expected under the alternating stress cycles.

5.2 Direction of crack growth

Several characteristic features were observed related to the direction of the growth of fatigue crack ^{(1) (2) (26) (29)}. In bending (or push-pull) fatigue, the crack is produced initially along the slip line and gradually propagates in the perpendicular direction to the stress axis. On the other hand, two types of crack are observed in torsional fatigue, that is, one inclined 45° to the specimen axis (the principal stress direction), the other in the axial or the tangential direction. These features may be explained from the energetic point of view in the following way.

Although the dislocation crack propagates initially along the slip line, the crack gradually shows the semi-macroscopic nature. Accordingly, as the term E_3' becomes much greater than the other terms with an increase of the crack length l , the crack propagates in the direction which makes E_3' maximum. It is known ⁽²⁹⁾ that the energy E_3' of the crack subjected to the shear stress τ_c (Fig. 10) is given by almost the same formula as that subjected

Fig. 9 Crack produced by pile-up dislocations

Fig. 10 Crack under shear stress

to the normal stress σ_c (Fig. 3).

$$(E_3')_\sigma \cong R\sigma_c^2, \quad (E_3')_\tau \cong R\tau_c^2 \quad (32)$$

where R is a coefficient. Let us consider the case of reversed stress amplitude, for the sake of simplicity. When the tension-compression stress with the amplitude σ_a is applied, $E_3' \cong R\sigma_a^2/2$ for the crack propagating in the direction inclined 45° to the stress axis (the slip direction), while $E_3' \cong R\sigma_a^2$ for that propagating perpendicular to the stress axis. Therefore, the direction of the crack growth changes gradually from the 45° direction (slip direction) to the perpendicular direction to the stress axis. In the case of torsional fatigue, the energy is given by $E_3' \cong R\tau_a^2$, both for the crack propagating in the axial (or the tangential) direction, and for that inclined 45° to the specimen axis (the principal stress direction). These results are qualitatively consistent with the experimental results mentioned above.

6. Conclusions

Energetic considerations were made on the nucleation and the growth of fatigue crack and on the effect of mean stress. The characteristic features of the fatigue of metals were also discussed energetically. The results may be summarized as follows:

(1) The energies of fatigue crack are evaluated separately. They are the energy of the crack itself, the interaction energy between the crack and the stress field, and the energy supplied from the surroundings during the crack formation.

(2) The growing process of the fatigue crack is divided into the following three stages according to the relative importance of the respective terms of the energies: the crack nucleation, the growth of dislocation crack and the growth of semi-macroscopic crack.

(3) The dislocation crack grows along the slip line receiving the energy from the dislocations coming into the crack. On the other hand, the semi-macroscopic crack propagates mainly by itself, that is, the plastic deformation occurs ahead of the crack due to the stress concentration, which contributes to the crack growth in the successive stress cycles.

(4) The mean stress affects the energy of the volume increase during the crack growth, approximately in the form of the hydrostatic stress component p .

Appendix

As an example of the definite mechanism of the growth of fatigue crack, the S-N curve is deduced in the following way, assuming that the effect of the dislocation velocity is dominant in the growth

of the dislocation crack. Let us consider the two-dimensional crack of the unit height shown in Fig. 5. The number $\Delta n/2$ of dislocations coming into the crack in a half cycle is given by

$$\frac{\Delta n}{2} = c \int_0^{T/2} \rho_m^{1/2} v dt \quad (33)$$

where ρ_m is the density of mobile dislocations, v is the dislocation velocity, T is the cycle period, and C is a constant ($C \leq 1$) related to the probability of the dislocations coming into the crack, which is dependent on the energetic condition [Eq. (24)].

For the sake of simplicity, it is assumed that ρ_m is constant during fatigue process and independent of the magnitude of the applied stress. It is reported⁽³⁰⁾ that the dislocation velocity v in bcc metal is given by

$$v = (\tau/D)^m \quad (34)$$

where D and m are the material constants. When the stress changes in sine-wave, substituting Eq. (34) into Eq. (33), we have⁽³¹⁾

$$\frac{\Delta n}{2} = A \frac{1}{f} \tau_0^m \quad (35)$$

$$A = \frac{\Gamma\left(\frac{m+1}{2}\right)}{\Gamma\left(\frac{m+2}{2}\right)} \frac{c \rho_m^{1/2}}{2\sqrt{\pi} D^m}$$

where τ_0 is the shear stress amplitude on the slip plane and f is the frequency. The increase Δl of the crack length in a cycle is given by $\Delta l = \Delta n b/2$. Denoting the crack lengths at the cycle numbers N_0 and N_1 by L_0 and L_1 , respectively,

$$L_1 - L_0 = A b \frac{1}{f} \tau_0^m (N_1 - N_0) \quad (36)$$

Let us assume that the fracture occurs at the cycle N_c when the crack length reaches a certain value L_c , and that $N_c \gg N_0$. If the frequency f is constant, the following relation is obtained from Eq. (36).

$$\log \tau_0 = c - \frac{1}{m} \log N_c \quad (37)$$

where c is a constant. Equation (37) expresses the S-N curve having a slope of $1/m$, which is known

to be in good agreement with the fatigue life of low-carbon steel⁽³¹⁾.

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