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Service Load and Fatigue Strength*

(Report 14, Crack Propagation under Repeated Varying Load)

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Using V-notched specimens of medium carbon steel, three kinds of fatigue tests under sinusoidally varying load are performed and following results are obtained.

The rates of crack propagations are accelerated and the superposition law of crack propagation is not applicable to these test results. The reduced lives of the final fracture are about 30 or 40% of the expected ones which are estimated using the superposition law of crack propagation.

Then, simple calculation method, namely multiplication law, is proposed to estimate the lives of final fracture under these varying load fatigue tests, whose method is based on the calculation using the fatigue lives under stepwise varying load and simple sinusoidally varying load fatigue tests.

From these test results, authors reaffirm that some kinds of loading pattern perhaps make the fatigue lives of specimens shorter to the same degree as those of practically fatigued members, such as car axles, driving shafts and so on.

1. Introduction

Authors have been investigating the crack propagation behaviours under continuously varying load conditions for the clarification of the fatigue behaviours under service load conditions^{(1)~(3)}. It has been clarified that the superposition law of crack propagation is not applicable to the crack propagation under continuously varying load conditions in which the crack propagates faster than expected.

Therefore, in this report the detailed observations are performed at first, namely the effects of maximum stress amplitude and the difference of maximum and minimum stress amplitudes are examined.

Next, in what kind of varying stress amplitude patterns the fatigue lives become shortest is investigated changing the maximum and the minimum stress amplitudes once or more in the process of fatigue progress. Namely, many alternating varying load step fatigue tests are performed. Thereafter, a simple calculation method is proposed for the

estimation of the fatigue lives under complex varying load conditions from those under the simple varying load conditions.

For the clarification of the fatigue behaviours under service load conditions, it is important to investigate the effect of varying load patterns on the crack propagation concerning the effects of the other types of stressing such as mini-sinusoidal stress⁽⁴⁾ or impulsive stress^{(5) (6)} which are superposed on the constant stress amplitude, because these kinds of stressing patterns which are investigated in this paper are similar ones, which are frequently observed in the stress history of car axles as mentioned in the previous paper⁽⁷⁾.

2. Experimental procedure

For this experiment V-notched shape 0.36% carbon annealed (900°C 5 hrs) steel specimens are used. The mechanical properties and the chemical compositions are indicated in Tables 1 and 2 and the dimensions of the test specimen are shown in Fig. 1.

Testing apparatus is an Ono type rotating

Fig. 1 Dimensions of test specimen (mm)

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bending fatigue machine with the varying load imposing equipment composed of oil pressure and electric controlled systems^{(8) (1)}.

The experiments are made in three kinds of varying stress amplitude patterns as shown in Fig. 2. They are explained as follows;

(1) As shown in Fig. 2(a), the stress amplitudes vary sinusoidally and its maximum, σ_{max} , and minimum, σ_{min} , stress amplitudes are constant in the fatigue process. This is the same as in the previous experiment⁽¹⁾. Authors name the stress wave pattern which is obtained by the fatigue testing machine the primary wave and the envelope of stress amplitudes the secondary one as indicated in Fig. 2(a).

(2) As shown in Fig. 2(b), the standard varying stress amplitude pattern is the same as that mentioned in (1), but σ_{max} and σ_{min} are changed one time in the fatigue process in this case. Namely, the envelope of σ_{max} or σ_{min} stress amplitude changes one time in the midst of fatigue test. This stressing pattern is named two varying load step test. The maximum and the minimum stress amplitude in the primary varying load are named σ_{1max} and σ_{1min} respectively, while those in the secondary varying stress amplitude are named σ_{2max} and σ_{2min} respectively. The numbers of cycles for the primary wave in the primary and secondary varying stress amplitude are named n_{1v} and n_{2v} respectively. In this report, the experiment is made under the following condition;

$$(\sigma_{1max} - \sigma_{1min}) = (\sigma_{2max} - \sigma_{2min}), \sigma_{1max} > \sigma_{2max}$$

(2) As shown in Fig. 2(c), the standard varying stress amplitude is selected the same as that in Fig. 2(a), but the imposed σ_{max} and σ_{min} stress amplitudes are changed periodically many times. Namely, the envelope of the stress amplitude changes stepwise many times. Authors call this type of stressing the alternating two varying load step fatigue test.

The maximum and the minimum stress amplitudes and the number of cycles under the higher varying load condition are σ_{1max} , σ_{1min} and Δn_{1v} respectively. In the same manner, σ_{2max} , σ_{2min} and

Δn_{2v} are taken as signs of lower varying load condition. In this experiment, the difference between σ_{1max} and σ_{1min} is chosen equal to that between σ_{2max} and σ_{2min} and then Δn_{1v} and Δn_{2v} are held constant throughout one fatigue test.

The effect of the frequency ω_2 does not appear on the same material as shown in the previous the paper⁽¹⁾, then only one case of ω_2 ($\omega_2 = 0.05$ c/sec) is selected in this report.

The crack depth is measured by the same method as mentioned in the previous paper^{(1) (6)}.

3. Test results and investigation

The arranging method and the investigation of results are based on the superposition law of crack propagation⁽⁹⁾. Then, the crack propagation curves under constant stress amplitude are shown in Fig. 3. A detailed investigation of this diagram was made in the previous paper already⁽¹⁾.

Fig. 2 Amplitude patterns of stressing

Table 1 Mechanical properties of testing material

Upper yield point	Lower yield point	Tensile strength	Actual breaking point	Elongation	Contraction
σ_{s0}	σ_{su}	σ_B	σ_T	$\psi\%$	$\varphi\%$
kg/mm ²	kg/mm ²	kg/mm ²	kg/mm ²		
28.0	27.3	53.2	83.7	23.3	42.1

Table 2 Chemical compositions

C	Si	Mn	P	S
0.36	0.27	0.53	0.011	0.014

3.1 Fatigue test under constant $\sigma_{\max}$ and $\sigma_{\min}$ conditions

As mentioned in the previous paper⁽¹⁾, the experiment under sinusoidally varying load conditions was performed at the stress level $\sigma_{\max}=35 \text{ kg/mm}^2$ and $\sigma_{\min}=15 \text{ kg/mm}^2$ for the same material, then more detailed results are obtained now with diagrams shown in Figs. 4 and 5. The ordinate is crack depth and abscissa is the number of cycles in primary wave. The lines in the diagrams are the expected crack growth curves which are calculated from the superposition law of crack propagation and the calculation method was described fully in the

Fig. 3 Crack propagation diagram under constant stress amplitude

Fig. 4 Crack propagation diagrams under repeated varying load conditions

Fig. 5 Crack propagation diagrams under repeated varying load conditions

previous paper⁽¹⁾. From these diagrams, it is clear that the crack propagations are accelerated and the theoretical and experimental crack propagations do not coincide with each other.

In Fig. 6, results about the final fracture are shown the abscissa being the difference $(\sigma_{\max}-\sigma_{\min})$ and the ordinate the ratio of the theoretical to the experimental final fracture life. Authors call this ratio the ratio of reduced fatigue life. As shown in this diagram, the experimentally obtained results are situated in the range of fifty to seventy percent of the expected ones, which means that the superposition law of crack propagation is not applicable to the whole range of crack propagation. The effect of the difference $(\sigma_{\max}-\sigma_{\min})$ is remarkable in the stress range of zero to 10 kg/mm^2 and the larger the difference the greater the reduction of fatigue life, while the effect of the difference of stress amplitude does not appear in the stress range of over 10 kg/mm^2 . The effect is more remarkable with the increase of $\sigma_{\max}$, then the maximum stress amplitude is considered to be the main effective factor for the fatigue in varying stress amplitude conditions⁽¹⁰⁾⁻⁽¹²⁾. From these results, it seems that the fatigue life does not decrease below 50% of the

expected one even if the difference of stress amplitudes amounts to 20 kg/mm^2 .

On the other hand, in the service load conditions car axles are often cracked and fractured very earlier than the expected life and the cumulative cyclic ratios, $\sum(n/N)$, often decrease nearly to 0.005. Structures and machine elements which are used in the service load conditions are fatigued earlier than experimentally predicted under the effect of environment, machining and so on, as well as the effect of the variations of stress amplitude. It is important, however, to investigate how much the fatigue life becomes shorter by the selection

of the application method of stressing because such a method is considered to be one of the main investigation methods to elucidate the fatigue behaviours under service load conditions. Then the following experiments are performed.

As mentioned previously a sinusoidally varying stress amplitude is often observed in the stress history of car axles; it is useful to make an experimental study of this phenomenon from the view point of practical use.

3.2 σ_{max} and σ_{min} change in the course of experiment

For the above reasons, two kinds of two step sinusoidally varying load are imposed on the specimens in the experiment of this section. They are

Fig. 6 Ratio of reduced fatigue life versus difference σ_{max} and σ_{min} stress amplitudes

Fig. 7 Relation between the crack depth λ_{1v} at the beginning of secondary varying load application and the remaining fatigue life n_{2v}

a two varying load step test and an alternating two varying load step test as shown in Fig. 2(b) and (c). The results are as follow:

3.2.1 Experiment under two varying load step fatigue test

For the experiment of this section, the sinusoidally varying load pattern is changed once in the course of fatigue process. The results are shown in Fig. 7, indicating on abscissa the number of cycles to final fracture under secondary varying stress amplitude, n_{2v} , on ordinate the crack depth at the starting point of secondary varying stress amplitude, λ_{1v} , namely the crack depth at point A in Fig. 2(b).

The solid lines in the diagrams represent the relation between the crack depth λ_{1v} , which is experimentally obtained, and the final fracture. Namely, these lines are obtained by supposing the crack propagation under secondary varying stress amplitude as following the crack propagation curves which are obtained by the application of sinusoidally varying load conditions, whose stress varying load conditions are the same as that of the secondary varying stress amplitude from the beginning of stressing. The existence of the effect of changing varying load is clear comparing the test results and the expected curves, namely it is considered that the effects of varying load do not appear when the experimental results coincide with the expected ones, and vice versa. Three kinds of stressing conditions are selected for this experiment as shown in Fig. 7(a), (b) and (c). The effect of changing load appears in Fig. 7(b) and (c) and the results are situated on the shorter side of the fatigue lives. From these results, it is clear that the crack propagation rates and the final fractures are controlled by the

choice of varying load conditions. It is considered that the effect of changing load does not appear on account of the relatively small difference between σ_{1max} and σ_{2max} in the case of Fig. 7(a).

Next, the crack propagation processes are investigated and the results are shown in Fig. 8. In this case, $\sigma_{1max} = 35 \text{ kg/mm}^2$, $\sigma_{1min} = 30 \text{ kg/mm}^2$, $\sigma_{2max} = 20 \text{ kg/mm}^2$ and $\sigma_{2min} = 15 \text{ kg/mm}^2$ and the crack depth λ_{1v} nearly equal to 0.2 mm. It is considered from this figure that the crack propagation under varying two step loading condition is faster than that under

constantly varying load conditions.

Then authors propose the following estimation method for the fatigue lives under varying two step load condition. This method is considered to be somewhat bold, but very simple. In Fig. 9(a), the method is explained schematically.

Authors consider that the varying load condition in Fig. 9(a-iii) is composed of the stressing conditions in Fig. 9(a-i) and (a-ii), namely stressing pattern in Fig. 9(a-i) means the envelope of the maximum stress amplitude in Fig. 9(a-iii) as shown with dotted line and that in Fig. 9(a-ii) means the sinusoidally varying load condition in Fig. 9(a-iii). Then, authors consider that the reduced fatigue life under the stressing pattern in Fig. 9(a-iii) will be able to be estimated from the reduced fatigue lives under the stressing patterns in Fig. 9(a-i) and (a-ii).

To put it more specifically as the superposition law of crack propagation is adopted in this report, the following method is used: First, two step

Fig. 8 Crack propagation diagram under two varying load step test

loading fatigue test is performed taking the primary and secondary stress amplitude equal to σ_{1max} and σ_{2max} , which are the maximum stress amplitudes in the two step varying load fatigue test as shown in Fig. 9(a-iii), then the reduced ratios of fatigue lives are obtained as R_{fs} . Namely, R_{fs} is the ratio between the experimentally obtained fatigue life and the expected one which is estimated from the superposition law of crack propagation.

Second, two types of sinusoidally varying load fatigue test are performed, they have the same maximum and minimum stress amplitudes as those of the primary and secondary varying load conditions in Fig. 9(a-iii). Then, the reduced ratios of fatigue lives are named R_{fv} . R_{fv} means the ratio between the experimentally obtained fatigue life and the estimated one using the superposition law of crack propagation under sinusoidally varying load conditions. Therefore, the reduced ratio of the fatigue life, R_f , under two step varying load fatigue test is obtained as the multiplication of the ratios R_{fs} and R_{fv} , namely $R_f = R_{fs} R_{fv}$.

The value of R_{fv} , however, varies with the stressing conditions, namely with the difference of σ_{max} or σ_{min} , then R_{fv} is obtained conveniently as follows:

$$R_{fv} = \left\{ \frac{n_{1v}}{N_{1v}} R_{fv1} + \left(1 - \frac{n_{1v}}{N_{1v}} \right) R_{fv2} \right\} \dots \dots \dots (1)$$

The signs in this formula are adopted as follows;

N_{1v} : Number of cycles for the final fracture imposing primary varying load.

n_{1v} : Number of cycles under primary varying load condition

R_{fv1} : Reduced ratio of primary varying load

Fig. 9 Explanation diagrams for multiplication law of crack propagation

application

R_{fv2} : Reduced ratio of secondary varying load application

Then, in the case of m step varying load condition,

$$R_{fv} = \sum_{i=1}^{m-1} \frac{n_{iv}}{N_{iv}} R_{fvi} + \left(1 - \sum_{i=1}^{m-1} \frac{n_{iv}}{N_{iv}}\right) R_{fv_m} \dots\dots\dots (2)$$

N_{iv} , n_{iv} and R_{fvi} are the values for that in number i stressing step.

For example, taking $\sigma_{1\max} = 35 \text{ kg/mm}^2$, $\sigma_{1\min} = 30 \text{ kg/mm}^2$, $\lambda_{1v} = 0.2 \text{ mm}$, $\sigma_{2\max} = 20 \text{ kg/mm}^2$ and $\sigma_{2\min} = 15 \text{ kg/mm}^2$, R_f is calculated as follows. First, R_{fs} for the case of $\lambda = 0.2 \text{ mm}$ is obtained from Fig. 10 as 0.7. R_{fv} is calculated from the above equation, (1), substituting R_{fv1} and R_{fv2} which are obtained as 0.74 and 0.58 from Fig. 6 respectively and n_{1v}/N_{1v} as $(3.5 \times 10^3)/(1.1 \times 10^5) = 0.32$. Then, R_{fv} is equal to 0.63 and $R_f = R_{fs} R_{fv} = 0.44$. Performing similar calculation for the other values of λ_{1v} , the estimation curve is obtained as shown in Fig. 10 with dotted line. This estimation curve coincides with the test results which are shown in figure with sign $\odot$. From these results, it is possible in some extent to estimate the fatigue lives under complex stress conditions from those of simple stressing conditions using above calculation method—this is defined as multiplication law of crack propagation.

Moreover, the fatigue life in one case decreased to 40% of the estimated one in this two step varying load fatigue test and the lives were shorter

Fig. 10 Relation between the experimentally and theoretically obtained reduced final fracture (Two varying load step application)

than those of two step loading conditions which are shown in Fig. 10 with sign $\odot$. This seems to suggest the possibility of the reduction of fatigue life through selection of stressing conditions.

3.2.2 Experiment under alternating two varying load step fatigue test

In this experiment, the estimated fatigue lives are calculated by the same method as that of two step varying load fatigue test. Namely, the fatigue lives under stressing condition in Fig. 9(b-iii) are estimated from the multiplication of the reduced ratios of fatigue lives under the stress conditions in Fig. 9(b-i) and (b-ii).

The experiment in Fig. 9(b-i) is performed taking the maximum and the minimum stress amplitudes as $\sigma_{1\max}$ and $\sigma_{2\max}$ respectively, whose values are the same as the maximum values $\sigma_{1\max}$ and $\sigma_{2\max}$ in Fig. 9(b-iii) or Fig. 2(c). The number of cycles in one varying load cycle, Δn_1 or Δn_2 for higher or lower varying load condition is selected to satisfy the conditions of $\Delta n_1/N_1 = \Delta n_{1v}/N_{1v}$ and $\Delta n_2/N_2 = \Delta n_{2v}/N_{2v}$. N_1 and N_2 are the fatigue lives under high and low constant stress amplitude conditions $\sigma_{1\max}$ and $\sigma_{2\max}$ in Fig. 9(b-i) respectively. The signs N_{1v} , N_{2v} , Δn_{1v} and Δn_{2v} are defined in the 6th line from the bottom on the right column in this page and the right part of these formulas means the ratios of the number of cycles under one varying load step to that of the final fracture under high or low varying load condition.

The stressing condition in Fig. 9(b-ii) is similar type of stressing condition to that mentioned previously in the case of two step varying load fatigue test. Namely, from this test, the reduced ratios caused by the sinusoidally varying load fatigue tests are obtained.

The reduced ratios caused by the test conditions in Fig. 9(b-i) and Fig. 9(b-ii) are defined as R_{fs} and R_{fv} . Then, the reduced ratio, R_f , caused by the fatigue test in Fig. 9(b-iii) is obtained from the multiplication law of crack propagation as $R_f = R_{fs} R_{fv}$.

As R_{fv} is considered to vary with the stressing condition in general, this is calculated from the following equation;

$$R_{fv} = \frac{1}{\Delta n_{1v}/N_{1v} + \Delta n_{2v}/N_{2v}} \left\{ \frac{\Delta n_{1v}}{N_{1v}} R_{fv1} + \frac{\Delta n_{2v}}{N_{2v}} R_{fv2} \right\} \dots\dots\dots (3')$$

In this equation, Δn_{1v} is the number of cycles in one varying load cycle and N_{1v} is that for the final fracture and R_{fv1} is the reduced ratio of fatigue life in higher varying load application, while Δn_{2v} , N_{2v} and R_{fv2} are those under lower varying load condition. Extended equation for R_{fv} is mentioned as

follows ;

$$R_{fv} = \sum_{i=1}^m \frac{\Delta n_{iv}}{N_{iv}} R_{fvi} / \sum_{i=1}^m \frac{\Delta n_{iv}}{N_{iv}} \dots\dots\dots (4)$$

For example, taking $\sigma_{1\max} = 35 \text{ kg/mm}^2$, $\sigma_{1\min} = 20 \text{ kg/mm}^2$, $\sigma_{2\max} = 30 \text{ kg/mm}^2$ and $\sigma_{2\min} = 15 \text{ kg/mm}^2$, estimation is made as follows :

First, a fatigue test under two step alternating load condition as shown in Fig. 9(b-i) is performed selecting $\sigma_{1\max}$ and $\sigma_{2\max}$ as 35 kg/mm^2 and 30 kg/mm^2 respectively. Then R_{fs} is obtained in this case as 0.63 from the result in Fig. 11. R_{fs} is calculated from the above equation (3) using $R_{fv1} = 0.48$ and $R_{fv2} = 0.56$ which are obtained from Fig. 6 and $\Delta n_{1v}/N_{1v}$ and $\Delta n_{2v}/N_{2v}$ are selected as 0.1. Thus, $R_{fv} = 0.52$ and $R_f = 0.33$. The dotted line in Fig. 11 is obtained from the above calculation method and the signs, $\odot$, denote the test results. It seems that the multiplication law of crack propagation is applicable to these fatigue tests. As shown in Fig. 11, the fatigue life dropped to 30% of the expected one in one case. Then, it is considered that the fatigue life can be reduced to a considerably large extent by the selection of stressing conditions.

From these experiments, it is considered possible to calculate the fatigue life under complex stressing conditions using the results under simple varying load conditions. Hereafter, it is important to continue these types of fatigue test and clarify from

the view points both practical and basic and to investigate in what stressing conditions the fatigue lives become shorter and why such phenomena appear.

4. Conclusions

Using V-notched specimens of medium carbon steel, three kinds of fatigue test under sinusoidally varying load are performed and a multiplication law of crack propagation is proposed for these test results with the following results obtained.

(1) Superposition law of crack propagation is not applicable for the experiment of this report.

(2) The multiplication law of crack propagation is applicable to the crack propagation under two step sinusoidally varying load fatigue test.

(3) The fatigue lives under these stressing conditions are reduced to about 30 or 40% of the expected one, and it is considered that the fatigue lives can be reduced considerably by the selection of stressing.

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Fig. 11 Relation between the experimentally and expectedly obtained reduced final fracture (Alternating two varying load step application)