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On the Pulsation Phenomena in Gas-Liquid Two-Phase Flow*

(Relationship between Pulsating Pressure and Flow
Pattern in Upward Two-Phase Flow)

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An analysis on the statistical characteristics of static pressure pulsation in upward air-water flow through a smooth tube of 26.0 mm I.D. and 5.2 m long is made to have a correct understanding of the essentials of two-phase gas-liquid flow. The pulsating pressure is detected at five different locations along the tube and recorded simultaneously on an oscillograph paper. Using these data, the statistical properties such as the standard deviation, frequency function and spectral density of pulsating pressure are calculated. These are discussed with the particular emphasis on the relationship with the flow rates of both phases and the flow patterns.

1. Introduction

In general two-phase gas-liquid flow presents drastic pulsation characteristics, which cannot be seen in single-phase flow. For instance, void fraction, velocities of both phases and static pressure in two-phase flow locally fluctuate violently even in a steady flow.

On the other hand, the observed flow patterns in two-phase flow are also multifarious (i.e. bubbly, slug, froth, annular etc.) as compared with single-phase flow, and the transition zones from one flow pattern to the other one are very obscure. As would be expected, however, there must be close inter-relationships between flow pattern and pulsation characteristics of two-phase flow.

Hence, the investigation of these fluctuation phenomena peculiar to two-phase flow must greatly contribute to enriching fundamental knowledge of the flow mechanism and setting up a more appropriate flow model. In recent years a few attempts to study the two-phase flow phenomena have been made from some particular viewpoints with emphasis laid on these points, and some of the results thereby have been reported^{(1) (2)}.

In this report we would discuss how the statistical characteristics of pressure pulsation change with

the increase or decrease of flow rates and with the transition of the flow pattern from one to another.

2. Nomenclature

- $C_a(\tau, L)$: general space-time correlation function, $(\text{kg/m}^2)^2$
 f : frequency, 1/sec
 L : axial distance, m or mm
 $P(t, L)$: static pressure, kg/m^2
 $\overline{P(L)}$: time average component of static pressure, kg/m^2
 $p(t, L)$: time varying component of static pressure, kg/m^2
 $p'(L)$: intensity of static pressure pulsation defined in Eq. (2), kg/m^2
 $p_{\text{loc}}'(0), p_{\text{sys}}'(0)$: local and system elements of pressure pulsation intensities, kg/m^2
 Q_{g0} : volumetric gas flow rate, m^3/sec
 Q_{l0} : volumetric liquid flow rate, m^3/sec
 $R_a(\tau, L)$: general space-time auto-correlation coefficient, dimensionless
 $R_c(\tau, L)$: general space-time cross-correlation coefficient, dimensionless
 $R_{a\text{loc}}(\tau, L)$: space-time auto-correlation coefficient of local element, dimensionless
 $R_{c\text{loc}}^*(\tau, L)$: space-time cross-correlation coefficient of local element, dimensionless
 $R_{\text{sys}}^*(\tau)$: time correlation coefficient of system element, dimensionless
 $S(f)$: density of power spectrum, sec
 t : time, sec
 τ : lag time, sec
 w_{g0} : superficial gas velocity: $4Q_{g0}/\pi D^2$, m/sec

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w_{i0} : superficial liquid velocity: $4Q_{i0}/\pi D^2$,
m/sec

3. Experimental apparatus and procedure

The equipment used in this study is shown schematically in Fig. 1. Air and water are brought together in a mixer to form a froth, and pass through a test section into an air-water separator. The end of the tube is not bent, jutting out into the atmosphere. The water introduced into the separator is sent back to the water storage tank, while the air is released into the atmosphere through the separator. The water is pumped up to the constant head tank from the water storage tank and thence sent to the air-water mixer through the flow control valve and the flow meter. The air supplied from a compressor is also introduced into the air-water mixer through the flow control valve and the flow meter.

The air-water mixer consists of two concentric tubes as shown in Fig. 2. The air is led to the air-water mixer through a copper tube of 13 mm inner diameter and blown into the water, flowing in the annular space, through 15 injection holes (1.0 mm in dia.) on the periphery of the inner tube.

The test section of 26.0 mm in inner diameter and 5.2 m in length is made of acrylic resin to observe the flow patterns. This is smooth enough for the friction factor to be represented with the Blasius formula. Pressure taps are inserted in the test section over 0.1, 0.25, 0.5 and 1 m intervals, and the most upstream pressure tap is at 2.25 m from the lowest end of the vertical tube. In order to avoid the attenuation of pressure pulsation in pressure leads from pressure taps to the corresponding

strain gage pressure transducers, the pressure leads are made of hard vinyl resin, slanted downward at the taps and filled up with water.

Five output signals from the pressure transducers were recorded simultaneously on an oscillograph paper. Data of 540 pressures obtained by digitizing each trace successively at a small time interval were analyzed by means of the method to be described later. Consequently the total number of data for a test run is

(number of data from one signal)

$\times$ (number of signals) = $540 \times 5 = 2700$

Table 1 shows the range of experiments and the flow patterns observed in this study. B, S, F and A in the table respectively correspond to bubbly, slug, froth and annular flow. The root mean square error values, i.e. standard deviations, of pulsating pressures are evaluated for all the test runs, and also the auto- and cross-correlation functions are examined for test runs marked by asterisks in the table.

4. Treatment of data

Static pressure in two-phase air-water flow pulsates in a random manner, but if the flow conditions remain steady its statistical characteristics will remain unchanged with time. When the phenomenon is a stationary random process the origin of the time to be measured can be taken arbitrarily, provided that the record obtained is long enough. In the present paper, analyses are performed by assuming

Table 1 Range of experiments and flow patterns

w_{i0} m/sec	w_{g0} m/sec	0.01	0.02	0.04	0.1	0.2	0.4	1.0	2.0	3.0
0.03	S	S*	S	S*	S	B-S*	B-S*	B	B*	
0.08	S	S*	S	S*	S	B-S*	B-S*	B	B*	
0.2	S	S	S	S	S	S	S	B	B	
0.4	—	S*	S	S*	S	S*	S*	B	B*	
1.0	—	S	S	S	S	S	F	B-F	B	
2.0	—	S-F*	S-F	S-F*	F	F*	F*	B-F	B-F*	
4.0	—	F	F	F	F	F	F	B-F	B-F	
8.0	A	A*	F-A	F-A*	F-A	F-A*	F*	B-F	B-F*	

A: annular flow, B: bubbly flow, F: froth flow,
S: slug flow

B-F, F-A, B-S and S-F designate the transition zone from one to another.

Superscript * designates the test run for which auto- and cross-correlation functions are examined.

Fig. 1 Schematic diagram of experimental apparatus

Fig. 2 Details of air-water mixer

that the phenomenon is stationary and the ergodic property holds good. Here will be introduced various relations necessary for the analyses of the static pressure pulsation.

4.1 Frequency distribution of static pressure pulsation

It is unavoidable from the nature of a random quantity that its value at any time t is not predictable in advance. If the actual value of a randomly varying quantity is a subject of interest, use is often made of distribution function directly to define the probability that the value of the quantity lies between two limits. Hence, the frequency distribution of pulsating pressure will be discussed in this report.

4.2 Intensity of static pressure pulsation

The static pressure in the cross section at distance L from a reference cross section at time t , $P(t, L)$, can be expressed as the sum of its time average component $\bar{P}(L)$ and time varying component $p(t, L)$ as follows.

$$P(t, L) = \bar{P}(L) + p(t, L) \quad (1)$$

where t is the time and L is the distance from the reference cross section. $p(t, L)$ can be determined from the above equation if $\bar{P}(L)$ is evaluated from the record of $P(t, L)$. The standard deviation of $p(t, L)$ is expressed by

$$p'(L) = \sqrt{\overline{p^2(t, L)}} = \sqrt{\overline{\{P(t, L) - \bar{P}(L)\}^2}} \quad (2)$$

where $p'(L)$ will be referred to as "intensity of static pressure pulsation". This term may be available to estimate the degree of disturbance of two-phase flow.

4.3 System and local element of pressure pulsation intensities

It has been already made clear that the time varying component of the pulsating pressure $p(t, L)$ in Eq. (1) consists of the local element whose spacial cross-correlation function abruptly tends to zero and of the system element which prevails over a fairly wide range of the tube lengths (3). Then, since the system element is independent of distance L , $p(t, L)$ is expressed as follows.

$$p(t, L) = p_{loc}(t, L) + p_{sys}(t) \quad (3)$$

Using this equation, the cross-correlation coefficient between $p(t, 0)$ and $p(t, L)$ becomes

$$R_c(\tau, L) = R_{c_{loc}}^*(\tau, L) + R_{sys}^*(\tau) \quad (4)$$

where

$$R_{c_{loc}}^*(\tau, L) = \overline{p_{loc}(t, 0)p_{loc}(t+\tau, L)} / p'^2(L) \quad (5)$$

$$R_{sys}^*(\tau) = \overline{p_{sys}(t)p_{sys}(t+\tau)} / p'^2(L) \quad (6)$$

Substituting $\tau=0$ and $L=0$ into Eqs. (4), (5) and (6), we get the following equations

$$R_c(0, 0) = R_a(0, 0) = R_{c_{loc}}^*(0, 0) + R_{sys}^*(0) \quad (7)$$

$$R_{a_{loc}}^*(0, 0) = \overline{p_{loc}^2(t, 0)} / p'^2(0) \quad (8)$$

$$R_{sys}^*(0) = \overline{p_{sys}^2(t)} / p'^2(0) \quad (9)$$

Then the local and system element of pressure pulsation intensities become as follows.

$$p'_{loc}(0) = \{R_{a_{loc}}^*(0, 0)\}^{1/2} p'(0) \\ = \{1 - R_{sys}^*(0)\}^{1/2} p'(0) \quad (10)$$

$$p'_{sys}(0) = \{R_{sys}^*(0)\}^{1/2} p'(0) \quad (11)$$

4.4 Characteristic length of local element of pulsating pressure

Substitution of $\tau=0$ into Eq. (4) results in

$$R_{c_{loc}}^*(0, L) = R_c(0, L) - R_{sys}^*(0) \quad (12)$$

Cross-correlation coefficient of local element $R_{c_{loc}}^*(0, L)$ in Eq. (12) tends to zero when L becomes sufficiently large. Auto-correlation coefficient of system element $R_{sys}^*(0)$, therefore, can be evaluated from the cross-correlation coefficient $R_c(0, L)$ in the region where L is sufficiently large. And $R_{c_{loc}}^*(0, L)$ in Eq. (12) also can be determined.

If $R_{c_{loc}}^*(0, L)$ gradually tends to zero as distance L becomes large, the local element of the pulsating pressure at a cross section may be considered to prevail over the wider range of the flow tube. Reversely, if $R_{c_{loc}}^*(0, L)$ abruptly tends to zero with L , it may exert an influence on the relatively narrow range. To estimate the magnitude of these ranges, defined herein is the length $L_{1/2R_{c_{loc}}^*(0, 0)}$, at which $R_{c_{loc}}^*(0, L)$ becomes a half of $R_{c_{loc}}^*(0, 0)$. This is named "characteristic length of local element of pulsating pressure".

4.5 Power spectral density of static pressure pulsation

The auto-correlation function of the time varying component of static pressure $p(t, L)$ in Eq. (1) is defined as

$$C_a(\tau, L) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T p(t, L)p(t+\tau, L)dt \quad (13)$$

Substitution of $\tau=0$ into Eq. (13) results in

$$C_a(0, L) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T p^2(t, L)dt \quad (14)$$

This value is the variance of $p(t, L)$. Normalized by Eq. (14), Eq. (13) is expressed as

$$R_a(\tau, L) = \frac{C_a(\tau, L)}{C_a(0, L)} \\ = \lim_{T \rightarrow \infty} \frac{\int_0^T p(t, L)p(t+\tau, L)dt}{\int_0^T p^2(t, L)dt} \quad (15)$$

The spectral density of static pressure pulsation is defined as Fourier transform of the correlation function $R_a(\tau, L)$, i. e.

$$S(f) = \int_{-\infty}^{\infty} R_a(\tau, L)e^{-2\pi f\tau}d\tau \quad (16)$$

In practice records of signals for infinitely long time could not be used in calculating $R_a(\tau, L)$ from Eq.

Fig. 3 Frequency distribution of static pressure pulsation

(15). Furthermore the truncation of the Fourier integral in Eq. (16) smeared the spectral density. So the smearing effect of the estimated power spectral density must be minimized by introducing an appropriate lag window. In this report the power spectral density estimates with the Hamming window⁽⁴⁾ are made.

5. Results and discussion

5.1 Frequency distribution of static pressure pulsation

Some calculation results of the frequency distribution are shown in Figs. 3 (a) through (c). The ordinate is the frequency of static pressure. The abscissa is pressure and its origin indicates average static pressure $\bar{P}(L)$.

Generally speaking, these figures can be divided into two categories. One comes close to normal distribution and the other is a distribution with two peaks, which is called "two-peaks distribution" in this report. Now, attention is focused on the tendency of the shape of frequency curve changing with an increasing air flow at constant water flow.

As an example, the case of Fig. 3 (b) ($w_{l0} = 0.4 \text{ m/sec}$) is considered. When w_{g0} is less than 0.2 m/sec , the length of air slug is less than about 0.3 m , and the air-water interface is very smooth. In such a case, the static pressure is normally distributed around the average value of pulsating pressure $\bar{P}(L)$. As air flow increases, the length of air slug increases, and the air-water interface becomes rough. In such a case, the frequency curve presents two-peaks distribution. As flow pattern approaches annular flow regime from froth flow, the peak of the higher pressure side on a two-peaks distribution curve gradually diminishes, and then the shape of the distribution becomes that of normal distribution.

The two categories of frequency

distribution may be considered in connection with flow pattern as follows:

Bubbly flow regime: static pressure presents normal distribution.

Slug flow regime: the frequency distributes almost normally if the length of air slug is less than about 0.3 m; however, its distribution comes to present two peaks as longer air slug appears due to the increase of air flow.

Froth flow regime: almost all frequency curves belong to two-peaks distribution.

Annular flow regime: frequency curve becomes nearly normal distribution.

5.2 Intensity of static pressure pulsation

Figure 4 shows the intensity of static pressure pulsation $[p'(0)]$ as a function of the superficial average velocity of two-phase flow $(w_{g0}+w_{l0})$ for various superficial water velocities. The letters affixed to the data points indicate the flow patterns observed. In bubbly and slug flow, $p'(0)$ increases with the air flow at constant water flow and decreases with the water flow at constant superficial average velocity of two-phase flow. However, in froth and annular flow the latter tendency is not

Fig. 4 Intensity of static pressure pulsation vs. superficial mean velocity of two-phase flow

Fig. 5 System and local elements of pressure pulsation intensities vs. superficial mean velocity of two-phase flow

observed.

In the lower water flow ($w_{l0}=0.01\sim0.2$ m/sec) the curve of $p'(0)$ at constant water flow presents the maximum in the transition from slug to froth flow.

Next, Fig. 5 shows the system and local elements of pressure pulsation intensities $[p_{sys}'(0)$ and $p_{loc}'(0)]$ as a function of $(w_{g0}+w_{l0})$ for various superficial water velocities. In the higher water flow ($w_{l0}=1.0\sim2.7$ m/sec), both the values of $p_{sys}'(0)$ and $p_{loc}'(0)$ become nearly the same. In the lower water flow, most data show that $p_{loc}'(0)$ is larger than $p_{sys}'(0)$.

5.3 Characteristic length of local element of pulsating pressure

Figure 6 presents the characteristic length $L_{1/2R_{cloc}*(0,0)}$ introduced in section 4.4, as a function of the superficial air velocity w_{g0} with the superficial water velocity w_{l0} as a parameter. In the lower water flow ($w_{l0}=0.02\sim0.1$ m/sec), the characteristic length increases with air flow and then begins to decrease, while in the higher water flow it monotonically increases. The magnitude of the characteristic length obtained covers the range from 30 mm to 600 mm.

Fig. 6 Characteristic length of local element of pulsating pressure vs. superficial air velocity

Fig. 7 Characteristic length vs. intensity of local pulsation

Figure 7 presents the characteristic length as a function of the intensity of local pulsation $[p_{loc}'(0)]$. From this figure, there seems to be such a relationship that $L_{1/2R_{cloc}}(0,0)$ is roughly proportional to $p_{loc}'(0)$.

5.4 Spectral density of static pressure pulsation

Figures 8 (a) through (e) show the calculated results of spectral density for various flow patterns observed in this study. The spectral density of pressure pulsation for each flow condition was

calculated over a frequency range between 0 and 50 c/sec. Since the values of the spectral density were nearly zero at higher frequencies than 15 c/sec, the spectral density curves were drawn only for the frequency range below 15 c/sec in the figures. In general, the curves with the highest peak at zero frequency suggest that the static pressure pulsates in a highly random manner.

Closer inspection of these figures in connection with flow patterns reveals the following results.

Bubbly flow regime: It should be noted that

there exist some peaks of spectral density and hence the static pressure pulsates in a relatively periodic manner, even though a large number of bubbles seem to be dispersed without regularity to the naked eye. All the frequencies corresponding to these spectral density peaks are too low in comparison with the order of frequency, which will be expected if each bubble passing through the tube cross section is separately relevant to the generation of pressure pulsation. Therefore, it can be considered that the pressure pulsation with rather low frequency is not attributed to individual bubbles, but to a series of dense and sparse bubble clouds along

Fig. 8 Spectral density of static pressure pulsation

the tube.

Slug flow regime: Most results present the single peak of spectral density being at lower frequencies than 1 c/sec. These may be arranged under two categories. The one is the case that the peak takes place at zero frequency; i.e., there does not appear any distinguished pressure variation with a particular frequency. This will be due to a variety of lengths of gas and liquid slugs occurring at random. The other is the case that the peak takes place at a certain frequency except zero. In general, this is for large gas slugs to pass in a fairly regular period. Considering the connections of the above two categories with another flow regime, the former borders on the bubbly flow regime, and the latter on the froth flow regime.

Froth flow regime: The pressure pulsation in this flow regime generally presents obvious periodicity. The spectral density curves with single peak are also seen. These are almost the same in quality as ones in the slug flow regime. However, the number of peaks increases with an increasing liquid flow rate at constant gas flow rate. As a result, the pattern of the spectral density distribution has close resemblance to that of the bubbly flow regime.

Annular flow regime: The increase of gas flow rate at constant liquid flow rate makes the flow pattern change from froth flow to annular flow, then the pulsation characteristic are also changed from periodic pulsation to random one. However, this random pulsation is considered to occur only in the transition zone from froth flow to annular flow. When the flow pattern enters the disturbance wave region with more increase of gas flow rate, periodic pressure pulsations will be generated owing to the large disturbance waves flowing with fairly regular periods.

6. Conclusions

The pulsating pressure in upward air-water two-phase flow was measured using an acrylic resin tube of 26.0 mm I.D. and 5.2 m long. From these data, the standard deviation (i.e. intensity), characteristic length (i.e. magnitude), frequency function and spectral density function of static pressure pulsation were calculated. Discussion was made with the particular emphasis on the relationship of the above factors with flow patterns.

The following results are obtained.

(1) In bubbly flow, the frequency curve

roughly presents a normal distribution. Both the values of the intensity and magnitude of pressure pulsation are small in this flow regime.

(2) In the most cases of slug and froth flow, the static pressure pulsates periodically with low frequency, and the frequency function of pulsation had two peaks. Furthermore, the values of the intensity and magnitude of pressure pulsation become large.

(3) In annular flow, the static pressure pulsates in random manner and its frequency function roughly presents the normal distribution as in the bubbly flow. The values of the intensity and magnitude of pressure pulsation are also small. In some cases of slug flow, the pressure pulsation has the same characteristics as in this annular flow.

(4) A flow pattern distinguishable under visual observations does not always fall into the category of spectral density distribution, which can be defined by the feature of distribution and order of magnitude in spectral densities. As to slug flow, for instance, its spectra seem to belong to two categories. This fact implies that if these characteristics of spectra are applied to make quantitative determination of flow regimes, borderlines between thus determined flow regimes will probably lie widely shifting from those of visual determination. In spite of such wide shifts, however, the quantitative flow regime detection must be one of justifiable means. For this reason, it is necessary that more efforts be made to collect and analyze the information on two-phase flow pulsation characteristics.

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