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Transient Comparison Methods of Simultaneous Measurement of Thermal Properties*

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Transient comparison methods of simultaneous determination of thermal conductivity, thermal diffusivity and heat capacity with a plane-shaped standard sample are investigated.

Series method and parallel method are proposed concerning the arrangement of the standard sample and the sample to be measured. Measuring method and necessary conditions of measurement are discussed, and the merits of the above two methods are revealed: Series comparison method is a very simple measuring method applicable to solids, powders and liquids, and the parallel comparison method is suitable for the precise measurement of relative properties among solids of resembling properties.

Experimental results for plastics and spindle oil showed that the same accuracy with the transient absolute measurement could be obtained by these transient comparison methods.

1. Introduction

Several methods of simultaneous measurements of thermal conductivity, thermal diffusivity and heat capacity by transient heating have been investigated such as the methods of constant stepwise heating with a thin resistor foil by O. Krischer et al.⁽¹⁾, W. Knappe et al.⁽²⁾ and the authors⁽³⁾, or the methods of radiation heating by Ogawa⁽⁴⁾ and Kobayashi⁽⁵⁾. These measuring methods are designated as the absolute measuring methods by transient heating.

In this report, new methods, designated as the transient comparison methods of simultaneous measurements of thermal conductivity, thermal diffusivity and heat capacity with a plane-shaped standard specimen, developed from the above mentioned absolute measuring method by the authors, are investigated.

Transient comparison methods, which measure relative properties of specimens with the standard specimens, have the following merits:

(a) Able to measure thermal conductivity, thermal diffusivity and heat capacity simultaneously with a single equipment in a short time.

(b) Able to measure relative values of the above quantities accurately.

(c) Able to handle liquids and powders besides solids.

(d) Equipments are simple and the measuring

procedures are very easy.

In contrast to the researches of the comparison methods in the steady-state condition, few researches have been made on the transient comparison methods except the measurement of temperature dependency of thermal conductivity by K. Eiermann using a copper standard specimen⁽⁶⁾.

Two comparison methods, designated as the parallel method and the series method, of plane-shaped standard specimen heated by a thin resistor foil at a constant stepwise heat flux are developed and the principle and the conditions of measurement

Nomenclatures

Nomenclature	Quantity	Dimension
a	thermal diffusivity	m ² /hr
c	specific heat	kcal/kg°C
λ	thermal conductivity	kcal/mhr°C
ρ	specific weight	kg/m ³
$1/k^2 = a_2/a_1$	the ratio of thermal diffusivity	
σ	$\sigma = \sqrt{(\lambda c \rho)_2} / \sqrt{(\lambda c \rho)_1}$	
ω	$\omega = (\sigma - 1) / (\sigma + 1)$	
θ	temperature	°C
t	time	hr
S	thickness of the specimen	m
L	longitudinal length of the specimen	m
$2R$	transverse length of the specimen	m
$2q$	heat flux	kcal/m ² hr
$\xi = S / (2\sqrt{at})$	non-dimensional number	

$$\operatorname{ierfc} x = \int_x^\infty \operatorname{erfc} u du$$

$$\operatorname{erfc} x = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-u^2) du$$

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are discussed.

That the parallel method is an useful method to obtain either the temperature dependency of thermal properties of solid specimens or the relative differences of thermal properties of same kind of materials, and that the series method is a convenient method to measure the thermal properties especially of powders and liquids, are verified.

2. Comparison methods

For the comparison measuring method, three kinds of arrangements of the standard specimen and the test specimen can be considered in the direction of heat flow as follows: (1) parallel, (2) perpendicular, and (3) combined the above. They are

Table 1
(a) Classification of the series methods

No.		No.	
(1)		(6)	
(2)		(7)	
(3)		(8)	
(4)		(9)	
(5)			A constant stepwise-heating source

(b) Characteristic of the series methods

No*	Obtainable thermal properties	Thermal properties of standard specimen already known	Kinds of specimen	Characteristic**
				(a) (b) (c) (d) (e) (f)
(1)	λ_2	$\sqrt{(\lambda c \rho)_1}$	Solid	○ △ ○ × × ×
(2)		$\sqrt{(\lambda c \rho)_1}$	Solid Powder	○ △ ○ × × ×
(3)		$\sqrt{(\lambda c \rho)_1}$	Solid Liquid Powder	○ △ ○ ○ ○ ×
(4)	a_2	$\lambda_1, a_1, (c \rho)_1$	Solid	× △ ○ × × ×
(5)		$\sqrt{(\lambda c \rho)_1}$	Solid Powder	○ △ ○ × × ×
(6)	$(c \rho)_2$	$\lambda_1, a_1, (c \rho)_1$	Solid Powder	× ○ × ○ ○ ×
(7)		$\lambda_1, a_1, (c \rho)_1$	Solid Powder	× × ○ × ○ ×
(8)		$\lambda_1, a_1, (c \rho)_1$	Solid Powder	× ○ × ○ ○ ○
(9)		$\lambda_1, a_1, (c \rho)_1$	Solid Powder	× × ○ × × ○

* These numbers coincide with numbers of Table (a).

** Characteristic

(a) Able to measure easily with a simple apparatus.

(b) Able to get the initial condition within a short time.

(c) Able to measure within a short heating time.

(d) Able to get thickness of specimens for powders and liquids.

(e) Not necessary to attach thermocouples to the test specimen.

(f) Not necessary to measure the heat flux.

designated as (1) the series method, (2) the parallel method, (3) the combined method, as shown in Fig. 1. From the combinations of specimen arrangements under the three boundary conditions: (i) zero temperature at the infinite distance, (ii) zero temperature maintained at the surface, and

Fig. 1 The classification of the comparison methods of measurements

Table 2
(a) Classifications of the parallel methods

No.	
(1)	
(2)	
(3)	

(b) Characteristic of the parallel methods

No*	Obtainable thermal properties	Kinds of specimen	Characteristic*
			(a) (b) (c) (d) (e)
(1)	$\lambda_1, a_1, (c \rho)_1, \lambda_2, a_2, (c \rho)_2, \lambda_1', a_1', (c \rho)_1$	Solid	○ △ ○ ○ ○
(2)			× ○ × ○ ○
(3)			× △ ○ ○ ○

* These numbers coincide with numbers of Table (a).

** Characteristic

(a) Able to measure easily with a simple apparatus.

(b) Able to get the initial condition within a short time.

(c) Able to measure within a short heating time.

(d) Able to determine the relative values precisely.

(e) Not necessary to measure the heat flux for determining the relative values.

(iii) no heat flux across the surface, nine series methods and three parallel methods can be considered as listed in Tables 1-(a), 1-(b), 2-(a), and 2-(b).

Absolute or relative values of thermal conductivity, thermal diffusivity and heat capacity can be determined simultaneously by all the methods in Tables 1 and 2.

The method (3) of the series method, in Table 1, is capable of measuring not only solids but also liquids or powders, and needs only one plate specimen.

The three parallel methods, in Table 2, are fit

The fundamental equation,

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{a_j} \frac{\partial \theta}{\partial t} \quad \left\{ \begin{array}{l} j=1 \text{ in the regions 1, } x < 0 \text{ and } x > S. \\ j=2 \text{ in the region 2, } 0 \leq x \leq S. \end{array} \right\} \quad (1)$$

The initial condition, $t=0$, $\theta=0$, $-\infty < x < \infty$ (2)

the boundary conditions,

$$\left\{ \begin{array}{l} \text{(i)} \quad x \rightarrow \pm \infty, \quad \theta = 0 \\ \text{(ii)} \quad x = 0, \quad \left[\begin{array}{l} \theta_{0-0} = \theta_{0+0} \\ \lambda_1 \left(\frac{\partial \theta}{\partial x} \right)_{0-0} - \lambda_2 \left(\frac{\partial \theta}{\partial x} \right)_{0+0} = 2q \quad (t \geq 0) \end{array} \right. \\ \text{(iii)} \quad x = S, \quad \left[\begin{array}{l} \theta_{S-0} = \theta_{S+0} \\ -\lambda_2 \left(\frac{\partial \theta}{\partial x} \right)_{S-0} = -\lambda_1 \left(\frac{\partial \theta}{\partial x} \right)_{S+0} \end{array} \right. \end{array} \right\} \quad (3)$$

solutions,

$$\theta = \frac{2q}{\sqrt{(\lambda c \rho)_1}} (1-\omega) \sqrt{t} \sum_{n=0}^{\infty} \omega^{2n} \left\{ \text{ierfc} \frac{-x+2nSk}{2\sqrt{a_1 t}} + \omega \text{ierfc} \frac{-x+2(n+1)Sk}{2\sqrt{a_1 t}} \right\} \quad (x < 0) \quad (4)$$

$$\theta = \frac{2q}{\sqrt{(\lambda c \rho)_1}} (1-\omega) \sqrt{t} \sum_{n=0}^{\infty} \omega^{2n} \left\{ \text{ierfc} \frac{x+2nS}{2\sqrt{a_2 t}} + \omega \text{ierfc} \frac{2(n+1)S-x}{2\sqrt{a_2 t}} \right\} \quad (0 \leq x \leq S) \quad (5)$$

$$\theta = \frac{2q}{\sqrt{(\lambda c \rho)_1}} (1-\omega^2) \sqrt{t} \sum_{n=0}^{\infty} \omega^{2n} \text{ierfc} \frac{x-S+(2n+1)Sk}{2\sqrt{a_1 t}} \quad (x > S) \quad (6)$$

in which

$$\omega = (\sigma - 1)/(\sigma + 1) \quad (7)$$

$$k = (a_1/a_2)^{1/2} \quad (8)$$

$$\sigma = \sqrt{(\lambda c \rho)_2} / \sqrt{(\lambda c \rho)_1} \quad (9)$$

Let the temperatures be θ_0 and θ_S at $x=0$ and $x=S$. Then

$$\theta_0 = \frac{2q}{\sqrt{(\lambda c \rho)_1}} (1-\omega) \sqrt{t} \sum_{n=0}^{\infty} \left\{ \text{ierfc} 2n\xi_2 + \omega \text{ierfc} 2(n+1)\xi_2 \right\} \quad (10)$$

$$\theta_S = \frac{2q}{\sqrt{(\lambda c \rho)_1}} (1-\omega^2) \sqrt{t} \sum_{n=0}^{\infty} \omega^{2n} \text{ierfc} (2n+1)\xi_2 \quad (11)$$

Heat source
① : Standard sample
② : Test specimen

Fig. 2 Co-ordinate system for the series method

for the measurement of the relative values of properties because the measurements errors can be cancelled. Particularly, the method (1) is a simple one. In this paper, the method (3) of the series methods in Table 1, and the method (1) of the parallel methods in Table 2 are described in the following chapters.

3 Principles of measurement

3.1 Principle of the series method

The arrangement of the specimens ① and ② is assumed as shown in Fig. 2. The regions $x < 0$ and $x > S$ are of the specimen ①, and the region $0 \leq x \leq S$ is of specimen ②. Subscripts 1 and 2 denote the properties of specimens ① and ② respectively.

in which $\xi_2 = S/(2\sqrt{a_2 t})$.

(a) Measurement of the quantity $\sqrt{(\lambda c \rho)_2}$.

In the process soon after the starting of heating, Eq. (10) can be approximated by

$$\theta_0 \approx \theta_{0i} = \frac{2q}{\sqrt{\pi} \sqrt{(\lambda c \rho)_1}} (1-\omega) \sqrt{t} \quad (12)$$

From Eq. (12), σ and $\sqrt{(\lambda c \rho)_2}$ become as

$$\sigma = \frac{4q}{\sqrt{\pi} \sqrt{(\lambda c \rho)_1} (d\theta_{0i}/d\sqrt{t})} - 1 \quad (13)$$

$$\sqrt{(\lambda c \rho)_2} = \sigma \sqrt{(\lambda c \rho)_1} \quad (14)$$

When the value of the quantity $\sqrt{(\lambda c \rho)_1}$ of specimen ① is known, the quantity $\sqrt{(\lambda c \rho)_2}$ of

specimen ② can be determined from Eqs. (13) and (14) by the measurements of the heat flux q and the rate of temperature rise $(d\theta_{01}/d\sqrt{t})$.

(b) Measurement of thermal diffusivity a_2 .

$$\text{Let } \phi(\omega^2, \xi_2) = \sum_{n=0}^{\infty} \omega^{2n} \text{ierfc}(2n+1)\xi_2 \quad \dots\dots\dots(15)$$

From Eq. (11),

$$\phi(\omega^2, \xi_2) = \frac{\sqrt{(\lambda c \rho)_1} \theta_s}{2q(1-\omega^2)\sqrt{t}} \quad \dots\dots\dots(16)$$

From the value of ϕ obtained from Eq. (16) and the graph $\phi-\xi_2$ (Fig. 3) obtained from Eq. (15), the value of ξ_2 can be found.

From Eq. (17),

$$a_2 = S_2^2 / (4\xi_2^2 t) \quad \dots\dots\dots(17)$$

the thermal diffusivity of specimen ② can be determined.

(c) Measurement of thermal conductivity λ_2 and heat capacity $(c\rho)_2$.

These are calculated from $\sqrt{(\lambda c \rho)_2}$ and a_2 as follows,

$$\lambda_2 = \sqrt{(\lambda c \rho)_2} \sqrt{a_2} \quad \dots\dots\dots(18)$$

$$(c\rho)_2 = \sqrt{(\lambda c \rho)_2} / \sqrt{a_2} \quad \dots\dots\dots(19)$$

3.2 Principles of the parallel method

The arrangement of the specimens ① and ② in this case is assumed as shown in Fig. 4.

The fundamental equation,

Fig. 3 $\phi(\omega^2, \xi_2) - \xi_2$ relations

Fig. 4 Co-ordinate system for the parallel method obtaining the relative values of specimens ① and ②

$$\frac{\partial^2 \theta_j}{\partial x^2} = \frac{1}{a_j} \frac{\partial \theta_j}{\partial t} \quad (x \geq 0) \quad \dots\dots\dots(20)$$

the initial condition is

$$x \geq 0, \theta_j = 0 \quad \dots\dots\dots(21)$$

the boundary conditions are

$$\begin{cases} x=0, & -\lambda_j \frac{\partial \theta_j}{\partial x} = q_j, \quad (t \geq 0) \\ x \rightarrow \infty, & \theta_j = 0 \end{cases} \quad \dots\dots\dots(22)$$

$$\text{and the solution is } \theta_j = \frac{2q_j}{\sqrt{(\lambda c \rho)_j}} \sqrt{t} \text{ierfc} \frac{x}{2\sqrt{a_j t}} \quad (x \geq 0) \quad \dots\dots\dots(23)$$

where $j=1$ for specimen ①, $j=2$ for specimen ②.

$$\text{When } x=0, \begin{cases} \theta_1 = \frac{2q_1}{\sqrt{\pi} \sqrt{(\lambda c \rho)_1}} \sqrt{t} \quad \dots\dots\dots(24) \\ \theta_2 = \frac{2q_2}{\sqrt{\pi} \sqrt{(\lambda c \rho)_2}} \frac{1}{\sigma'} \sqrt{t} \quad \dots\dots\dots(25) \end{cases}$$

$$\text{when } x=S, \begin{cases} s\theta_1 = \frac{2q_1}{\sqrt{(\lambda c \rho)_1}} \sqrt{t} \text{ierfc} \xi_1 \quad \dots\dots\dots(26) \\ s\theta_2 = \frac{2q_2}{\sqrt{(\lambda c \rho)_2}} \frac{1}{\sigma'} \sqrt{t} \text{ierfc} k'\xi_1 \quad \dots\dots\dots(27) \end{cases}$$

$$\text{where } \frac{1}{\sigma'} = 1 + \Delta \frac{1}{\sigma'} = \frac{q_2}{q_1} \frac{1}{\sigma}, \quad k' = 1 + \Delta k' = \frac{S_2}{S_1} k \quad \dots\dots\dots(28)$$

Considering that the relative values of similar specimens are obtained by the parallel method, the values of $\sigma' = \{\sqrt{(\lambda c \rho)_2} / \sqrt{(\lambda c \rho)_1}\} (q_1/q_2)$

$$\text{and } k'^2 = (a_1/a_2) \times (S_2/S_1)^2 \text{ are decided as follows,} \quad 0.7 \leq \sigma' \leq 1.3, \quad 0.7 \leq k'^2 \leq 1.3 \quad \dots\dots\dots(29)$$

The temperature ratios θ_0 and θ_s are defined as Eqs. (30) and (31).

$$\theta_0 = \left(\frac{d(\theta_1 - \theta_2)}{d\sqrt{t}} \right) / \left(\frac{d\theta_1}{d\sqrt{t}} \right) \quad \dots\dots\dots(30)$$

$$\theta_s = (s\theta_1 - s\theta_2) / s\theta_1 \quad \dots\dots\dots(31)$$

From Eqs. (24)~(28), θ_0 and θ_s become

$$\theta_0 = -\Delta \frac{1}{\sigma'} \quad \dots\dots\dots(32)$$

$$\theta_s = 1 - \frac{1}{\sigma'} \frac{\text{ierfc} k'\xi_1}{\text{ierfc} \xi_1} \quad \dots\dots\dots(33)$$

$$\text{Let } g(\xi_1, \Delta k') = 1 - \text{ierfc}(1 + \Delta k')\xi_1 / \text{ierfc} \xi_1 \quad \dots\dots\dots(34)$$

From Eq. (33), this becomes

$$g = (\theta_s - \theta_0) / (1 - \theta_0) = \sigma' (\theta_s - 1) + 1 \quad \dots\dots\dots(35)$$

From Eqs. (28), (29) and (34), $\Delta k'$ is written approximately as

$$\Delta k' = \alpha g + \beta g^2 \quad \dots\dots\dots(36)$$

where α and β are functions of ξ_1 as

$$\alpha = \frac{\text{ierfc} \xi_1}{\xi_1 \text{erfc} \xi_1}, \quad \beta = \frac{\exp(-\xi_1^2) (\text{ierfc} \xi_1)^2}{\sqrt{\pi} \xi_1 (\text{erfc} \xi_1)^3} \quad \dots\dots\dots(37)$$

The properties of specimen ① are found using the transient absolute measuring method by the authors⁽³⁾.

(a) Measurement of the quantity $\sqrt{(\lambda c \rho)_1}$ of specimen ①.

From Eq. (24),

$$\sqrt{(\lambda c \rho)_1} = \frac{2q_1}{\sqrt{\pi} (d\theta_1/d\sqrt{t})} \quad \dots\dots\dots(38)$$

(b) Measurement of thermal diffusivity a_1 of specimen ①.

From Eqs. (24) and (26),

$$s\theta_1/\theta_1 = \sqrt{\pi} \operatorname{ierfc} \xi_1, \dots\dots\dots(39)$$

Eq. (39) $\rightarrow \xi_1 \rightarrow a_1 = S_1^2/(4\xi_1^2 t)$.

(c) Measurement of the ratio

$$\sigma = \sqrt{(\lambda c \rho)_2} / \sqrt{(\lambda c \rho)_1},$$

and the quantity $\sqrt{(\lambda c \rho)_2}$ of specimen ②.

The ratio σ and the quantity $\sqrt{(\lambda c \rho)_2}$, after measuring the values of $d(\theta_1 - \theta_2)/d\sqrt{t}$, $d\theta_1/d\sqrt{t}$ and q_2/q_1 , are determined by the following equations obtained from Eqs. (30) and (32),

$$\frac{\sqrt{(\lambda c \rho)_2}}{\sqrt{(\lambda c \rho)_1}} = \sigma = \frac{q_2}{q_1} \sigma' = \frac{q_2}{q_1} \frac{1}{1 + \Delta(1/\sigma')} \dots\dots\dots(40)$$

$$\sqrt{(\lambda c \rho)_2} = \sigma \sqrt{(\lambda c \rho)_1} \dots\dots\dots(41)$$

(d) Measurement of the ratio (a_2/a_1) and thermal diffusivity a_2 of specimen ②.

θ_s is calculated by Eq. (31) after measuring the temperatures of $(s\theta_1 - s\theta_2)$ and $s\theta_1$. Substituting θ_s and θ_0 into Eq. (35), the value of g can be found. Substituting the value of ξ_1 obtained in (6) into Eq. (37), the values of α and β can be found. $\Delta k'$ can be determined from Eq. (36). The thermal diffusivity ratio (a_2/a_1) and the thermal diffusivity a_2 of specimen ② are calculated from the following relations:

$$a_2/a_1 = 1/k^2 = (S_2/S_1)^2 / (1 + \Delta k')^2 \dots\dots\dots(42)$$

$$a_2 = a_1/k^2 \dots\dots\dots(43)$$

(e) Measurement of thermal conductivity λ_2 and heat capacity $(c\rho)_2$ of specimen ②.

The thermal conductivity and the heat capacity are calculated from the following relations:

$$\lambda_2 = \sqrt{(\lambda c \rho)_2} \sqrt{a_2}, \quad (c\rho)_2 = \sqrt{(\lambda c \rho)_2} / \sqrt{a_2} \dots\dots\dots(44)$$

If $q_2/q_1 = 1$, the relative values

$$\sigma = \{\sqrt{(\lambda c \rho)_2} / \sqrt{(\lambda c \rho)_1}\}$$

and (a_2/a_1) can be determined without measuring the heat flux q_1 .

4. The conditions of measurements

The conditions for the measuring time, intensity of heat flux and the sizes of the specimen, concerning the series method and the parallel method are discussed. In both of these methods, the same initial and boundary conditions are used as in our former report⁽³⁾ about transient absolute measuring method, and those values can be decided in the same ways. These conditions are applicable to any kind of specimen, but non-metals are mainly discussed in this report.

4.1 The conditions of measurement of series method

4.1.1 The range of the ratio

$$\sigma = \{\sqrt{(\lambda c \rho)_2} / \sqrt{(\lambda c \rho)_1}\}$$

The values of $\sqrt{(\lambda c \rho)}$ for most non-metals are

5 ~ 30 kcal/m² hr^{1/2}°C. The ranges of σ and ω are assumed to be

$$1/3 \leq \sigma \leq 3 \text{ and } -1/2 \leq \omega \leq 1/2 \dots\dots\dots(45)$$

4.1.2 The range of ξ_2

As the value of ξ_2 is read from $\phi(\omega^2, \xi_2) - \xi_2$ diagram, the value of ϕ is desirable to depend scarcely on ω^2 . Moreover, θ_s/θ_0 should be taken to be large, so that the temperature gradient in the specimen can be small. Then the range of values of ξ_2 is chosen as $1/3 \leq \xi_2 \leq 2/3$. For this range of ξ_2 , the specimen of 5 ~ 15 mm thickness can be measured within 5 ~ 15 minutes, for the values of thermal diffusivities of most non-metals are (3 ~ 50) $\times 10^{-4}$ m²/hr.

4.1.3 The range of the process approximated by Eq. (12)

The value of ξ_2 should be decided so that θ_0 satisfies the condition $|(\theta_0 - \theta_{0i})/\theta_{0i}| \leq 0.5\%$, where θ_0 can be approximated by Eq. (12). When $\omega = 0$, the range is $\xi_2 \geq 0$, and even when $\omega = \pm 1/2$, the range $\xi_2 \geq 0.85$ is sufficient.

4.1.4 The intensity of heat flux

The intensity of heat flux can be decided from the following equations, by assuming the maximum temperature rise and the thermal conductivity of the specimen,

$$q = (\lambda_2 \theta_{0\max}) / \{S f(\omega)\} \dots\dots\dots(46)$$

where

$$f(\omega) = 3(1 + \omega) \sum_{n=0}^{\infty} \omega^{2n} [\operatorname{ierfc}(2n/3) + \omega \operatorname{ierfc}\{2(n+1)/3\}] \dots\dots\dots(47)$$

4.1.5 The sizes of the specimen

In Fig. 5, the sizes of the specimen, L_1 , L_2 , and $2R$ are decided as follows.

(a) Lengths L_1 , L_2 in the direction of heat flow

The values of L_1/S and L_2/S should be decided so that the temperatures θ_{-L_1} , θ_{L_2} at $x = -L_1$, $x = L_2$ may satisfy $\theta_{-L_1}/\theta_0 \leq 0.5\%$, $\theta_{L_2}/\theta_0 \leq 0.5\%$ and $\theta_{L_2}/\theta_s \leq 0.5\%$ within the maximum time $\xi_2 = 1/3$, then the lengths L_1 , L_2 are considered approximately infinite.

Then $L_1/S \geq 5.4k$, $L_2/S \geq 4.7k + 1$.

(b) Length $2R$ in the direction perpendicular to heat flow.

Fig. 5 Model for the determination of the sizes of the specimens of the series method

Assuming that the specimen is an infinite cylinder having uniform thermal properties, $2R/S$ is decided from the following solutions.

The fundamental equation,

$$\frac{\partial^2 \theta'}{\partial x^2} + \frac{\partial^2 \theta'}{\partial r^2} + \frac{1}{r} \frac{\partial \theta'}{\partial r} = \frac{1}{a} \frac{\partial \theta'}{\partial t} \quad (x \geq 0, 0 \leq r \leq R) \quad (48)$$

the initial condition,

$$t=0, \quad \theta'=0 \quad (49)$$

and the boundary conditions,

$$t > 0, \quad \left. \begin{array}{l} r=R, \quad \theta'=0 \\ x=0, \quad -\lambda(\partial \theta' / \partial x)=q \\ x \rightarrow \infty, \quad \theta'=0 \end{array} \right\} \quad (50)$$

Let the temperatures at $r=0$ and $x=S$ be θ_s' , and $\theta_s = \lim_{R \rightarrow \infty} \theta_s'$. The heat loss from the side of the specimen can be ignored when $\theta_s' / \theta_s \geq 0.995$.

The range of $2R/S$ is decided as follows. When $a_2 \geq a_1$, the range $2R/S \geq 12$ can be obtained for $\xi=1/3$ from Fig. 6, $\theta_s' / \theta_s - 2R/S$ diagram, considering that θ_s' / θ_s satisfies $\theta_s' / \theta_s \geq 0.995$. When $a_2 < a_1$, the range of $2R/S$ is decided, putting $\xi=1/3k$.

4.2 The conditions of measurement by the parallel method

4.2.1 The range of ξ_1

$$1/3 \leq \xi_1 \leq 2/3^{(3)}$$

Fig. 6 $\theta_s' / \theta_s - 2R/S$ relation

Fig. 7 Model for the determination of the sizes of specimens of the parallel method

4.2.2 The intensity of heat flux

$$q_1 = \frac{\sqrt{\pi}}{3} \frac{\lambda_1}{S_1} \theta_{1max}^{(3)} \quad (51)$$

4.2.3 The sizes of the specimen

In Fig. 7, the sizes of the specimen, L_1 , L_2 , $2R_1$, and $2R_2$ are decided as follows.

(a) Thicknesses of the specimens L_1 , L_2 .

The thickness of the specimens should be determined such that the specimens can be considered approximately infinite solids. Let θ_s be the temperature solution under the boundary conditions either of no heat flux at $x=L_2$ or of a constant temperature at $x=L_2$.

L_1/S_1 and L_2/S_2 are determined such that the difference $|(\theta_s' - \theta_s) / \theta_s| \leq 0.5\%$ is satisfied. Considering the condition of properties in Eq. (29), the thicknesses of the specimens are $L_1/S_1 = L_2/S_2 \geq 4$.

(b) Dimensions $2R_1$ and $2R_2$ of the specimens in the direction perpendicular to heat flow.

The range $2R_1/S_1 \geq 12$ is obtained in the same manner as by the series method. The range of $2R_2/S_2$ is obtained from Fig. 6, putting $\xi=1/3k$.

4.2.4 The values of (S_2/S_1) and (q_2/q_1)

(S_2/S_1) , $(q_2/q_1) \approx 1$, are sufficient conditions to satisfy the conditions of Eq. (29).

Quantities to measure θ_s , θ_0 , current and voltage

①: Standard specimen

②: Test specimen

Fig. 8 Measuring apparatus by the series method

Quantities to measure: θ_1 , $s\theta_1$, $(\theta_1 - \theta_2)$, $(s\theta_1 - s\theta_2)$, current and voltages V_1 and V_2 .

Fig. 9 A measuring apparatus by the parallel method

5. Experiments

Measuring apparatus for the parallel and the series methods following the above mentioned principles and conditions are described, and, the results of the measurements by these apparatus are shown.

The standard specimen ① was polymethylmethacrylate (PMMA), and the specimen ② was polyvinylchloride (PVC) for both methods, and spindle oil for the series method.

5.1 Measuring apparatus and the procedures of measurement

Measuring apparatus for the parallel and the series method are illustrated in Figs. 8 and 9 respectively. The dimensions of the samples were determined following the conditions in 4.1.5 and 4.2.3. The heat flux was determined from Eq. (46) and (51) to keep the maximum rise of temperature in at $3 \sim 5^\circ\text{C}$ in order to consider the dependency of the measuring results on the temperature. A stepwise constant heat flux was achieved with a nichrome foil electric heater of a thickness 0.03mm heated by direct current, and the heat quantity was calculated by the measurements of voltage and current.

The surfaces of specimens were finished smooth and flat and were set with silicone grease in order to minimize the heat resistances at the surfaces.

The apparatus was kept in a constant temperature bath, and the measurement was started after the temperature distributions in the specimens were indicated to be constant. Temperatures were measured with Copper-Constantan thermocouples of 0.1 mm ϕ and the changes were recorded with a pen-recorder after compensating the initial temperature by potentiometers as shown in Fig. 8.

The detailed procedures were the same as that of the author's former report⁽³⁾ about the absolute method.

5.2 Measuring results

Table 3 One of the experimental results by the series method

Standard sample ①: PMMA.		
$\sqrt{(\lambda c \rho)_1} = 8.27 \text{ kcal/m}^2 \text{ hr}^{1/2} \text{ } ^\circ\text{C}$		
Test specimen ②: PVC.		
Dimensions: $S=4.60 \text{ mm}$, $2R=60 \text{ mm}$, 65 mm		
Specific weight: $\rho_2=1.435 \text{ kg/m}^3$		
Initial temperature	T_0	30.2°C
Maximum temperature rise	$\theta_{0\text{max}}$	3.8°C
Heat flux	$2q$	$134 \text{ kcal/m}^2 \text{ hr}$
σ		0.810
$\sqrt{(\lambda c \rho)_2}$		$6.70 \text{ kcal/m}^2 \text{ hr}^{1/2} \text{ } ^\circ\text{C}$
$\omega = (\sigma - 1)/(\sigma + 1)$		-0.105
Thermal diffusivity	a_2	$4.22 \times 10^{-4} \text{ m}^2/\text{hr}$
Thermal conductivity	λ_2	$0.138 \text{ kcal/mhr } ^\circ\text{C}$
Heat capacity	$(c\rho)_2$	$326 \text{ kcal/m}^3 \text{ } ^\circ\text{C}$
Specific heat	c_2	$0.227 \text{ kcal/kg } ^\circ\text{C}$

5.2.1 Measuring results by the series method

The value of $\sqrt{(\lambda c \rho)_1}$ of standard specimen (P MMA) was determined to be $\sqrt{(\lambda c \rho)_1} = 8.27 \text{ kcal/m}^2 \text{ hr}^{1/2} \text{ } ^\circ\text{C}$ from our many experimental results by the author's absolute method.

One of the experimental results for PVC by the series comparison method is shown in Table 3, and Figs. 10 and 11. The average measuring results and the average deviation of the results among different measurements for the same PVC specimen of different thickness are shown in Table 5, and those for spindle oil are shown in Table 6. The data for spindle oil were obtained from the measurements of early stages of heating, before the effect of natural convection of oil began, and the specimen oil was

Specimen ①: PMMA. *: The series method
Specimen ②: PVC **: The parallel method

Fig. 10 One of the experimental data by the series and the parallel method

*: The series method, $a_2 = \left\{ \frac{S}{2} \frac{1}{d \epsilon_2 / d(1/\sqrt{t})} \right\}^2 = 4.22 \times 10^{-4}$
**: The parallel method,
 $a_1 = \left\{ \frac{S_2}{2} \frac{1}{d \epsilon_1 / d(1/\sqrt{t})} \right\}^2 = 3.80 \times 10^{-4}$

Fig. 11 $\epsilon \sim 1/\sqrt{t}$ diagram from data of Fig. 10

heated downwards to decrease the free convection.

5.2.2 Measuring results by the parallel method

One of the experimental results for PVC by the parallel comparison method is shown in Table 4 and Figs. 10 and 11. The average measuring results and the average deviation of results among different measurements are shown in Table 7.

The results of the relative properties of the same materials were tested more than ten times by changing the heat flux and the thickness of the specimens using the same PMMA for the specimens ① and ②, and all the results coincided with each other within $\pm 1 \sim 2\%$ errors.

5.3 Discussion of the experimental results

The data shown in Figs. 10 and 11 correspond to straight lines and the obtained ratios of thermal diffusivity (a_1/a_2) are constant throughout the measurement. These facts mean that the apparatus and the procedures of measurement satisfy the necessary conditions determined in 4.1 and 4.2 both for the parallel and for the series method.

The results of the series method in Fig. 5 show that the same measuring results can be obtained by

changing the thickness of the specimens. This means that the effects of the surface heat resistance and the thickness of the specimens can be avoided

Table 6 Results of measurement of liquid by the series method

Standard sample ①: PMMA.

$$\sqrt{(\lambda c \rho)_1} = 8.27 \text{ kcal/m}^2\text{hr}^{1/2}\text{°C}$$

Test specimen ②: Spindle oil

$$S = 5.01 \text{ mm}, \rho_2 = 886 \text{ kg/m}^3$$

Initial temperature: $30.2 \sim 30.9^\circ\text{C}$,

Maximum temperature rise: $4 \sim 10^\circ\text{C}$,

Heat flux: $130 \sim 300 \text{ kcal/m}^2\text{hr}$

Properties	Average values	Average deviation of test results
σ	0.812	+2.4% -2.4
$\sqrt{(\lambda c \rho)_2} \text{ kcal/m}^2\text{hr}^{1/2}\text{°C}$	6.72	+2.4 -2.4
$a_2 \text{ m}^2/\text{hr}$	2.67×10^{-4}	+4.9 -5.2
$\lambda_2 \text{ kcal/mhr}^\circ\text{C}$	0.110	+2.7 -2.7
$(c\rho)_2 \text{ kcal/m}^3\text{°C}$	411	+3.9 -1.7
$c_2 \text{ kcal/kg}^\circ\text{C}$	0.464	+3.9 -1.7

$$\text{Average values: } \bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

$$\text{Average deviation of test results: } \frac{\max}{\min} \left(\frac{X_i - \bar{X}}{\bar{X}} \right)$$

Table 4 One of the experimental results by the parallel method

Standard specimen ①: PMMA.

Dimensions: $S_1 = 5.25 \text{ mm}$, $2R_1 = 60 \text{ mm}$, 65 mm ; Specific weight: $\rho_1 = 1170 \text{ kg/m}^3$

Test specimen ②: PVC.

Dimensions: $S_2 = 5.40 \text{ mm}$, $2R_2 = 60 \text{ mm}$, 65 mm ; Specific weight: $\rho_2 = 1435 \text{ kg/m}^3$

Initial temperature: $T_0 = 30.5^\circ\text{C}$, Heat flux: $2q_1 = 131 \text{ kcal/m}^2\text{hr}$, Maximum temperature rise:

$$\theta_{1\max} = 3.4^\circ\text{C}, q_2/q_1 = 1.00, S_2/S_1 = 1.03$$

Standard specimen ①: PMMA		Properties ratio		Test specimen ②: PVC	
$\sqrt{(\lambda c \rho)_1} \text{ kcal/m}^2\text{hr}^{1/2}\text{°C}$	8.20	$\sqrt{(\lambda c \rho)_2}/\sqrt{(\lambda c \rho)_1}$	0.833	$\sqrt{(\lambda c \rho)_2} \text{ kcal/m}^2\text{hr}^{1/2}\text{°C}$	6.83
$a_1 \text{ m}^2/\text{hr}$	3.80×10^{-4}	a_2/a_1	1.08	$a_2 \text{ m}^2/\text{hr}$	4.12×10^{-4}
$\lambda_1 \text{ kcal/mhr}^\circ\text{C}$	0.160			$\lambda_2 \text{ kcal/mhr}^\circ\text{C}$	0.139
$(c\rho)_1 \text{ kcal/m}^3\text{°C}$	421			$(c\rho)_2 \text{ kcal/m}^3\text{°C}$	336
$c_1 \text{ kcal/kg}^\circ\text{C}$	0.360			$c_2 \text{ kcal/kg}^\circ\text{C}$	0.234

Table 5 The experimental results by the series method

Standard sample ①: PMMA. $\sqrt{(\lambda c \rho)_1} = 8.27 \text{ kcal/m}^2\text{hr}^{1/2}\text{°C}$

Test specimen ②: PVC.

Dimensions: (a) $S = 4.60 \text{ mm}$, $2R = 60 \text{ mm}$, 65 mm

(b) $S = 5.42 \text{ mm}$, $2R = 60 \text{ mm}$, 65 mm

Specific weight: $\rho_2 = 1435 \text{ kg/m}^3$

Initial temperature: $30.1 \sim 30.3^\circ\text{C}$ Maximum temperature rise: $2 \sim 4^\circ\text{C}$ Heat flux: $2q = 70 \sim 140 \text{ kcal/m}^2\text{hr}$

Properties	(a) $S = 4.60 \text{ mm}$		(b) $S = 5.42 \text{ mm}$	
	Average values	Average deviation of test results	Average values	Average deviation of test results
σ	0.814	+2.2% -2.1	0.813	+1.1% -1.4
$\sqrt{(\lambda c \rho)_2} \text{ kcal/m}^2\text{hr}^{1/2}\text{°C}$	6.73	+2.2 -2.1	6.72	+1.1 -1.4
$a_2 \text{ m}^2/\text{hr}$	4.25×10^{-4}	+1.4 -2.6	4.18×10^{-4}	+3.4 -3.8
$\lambda_2 \text{ kcal/mhr}^\circ\text{C}$	0.139	+2.3 -1.3	0.137	+0.5 -0.9
$(c\rho)_2 \text{ kcal/m}^3\text{°C}$	327	+2.0 -2.6	327	+4.6 -4.3
$c_2 \text{ kcal/kg}^\circ\text{C}$	0.228	+2.0 -2.6	0.228	+4.6 -4.3

$$\text{Average value: } \frac{1}{n} \sum_{i=1}^n X_i = \bar{X}, \quad \text{Average deviation of test results: } \frac{\max}{\min} \left(\frac{X_i - \bar{X}}{\bar{X}} \right)$$

Table 7 The experimental results by the parallel method

Standard specimen ①: PMMA.

Dimensions: $S_1=5.25$ mm, $2R_1=60$ mm, 65 mm; Specific weight: $\rho_1=1.170$ kg/m³

Test specimen ②: PVC.

Dimensions: $S_2=5.40$ mm, $2R_2=60$ mm, 65 mm; Specific weight: $\rho_2=1.435$ kg/m³Initial temperature: $T_0=30.2\sim 30.5^\circ\text{C}$, Maximum temperature rise: $\theta_{1\max}=3\sim 5^\circ\text{C}$,Heat flux: $2q_1=100\sim 200$ kcal/m²hr, $q_2/q_1=1.00$

Standard specimen ① PMMA	Properties	$\sqrt{(\lambda c \rho)_1}$ kcal/m ² hr ^{1/2} °C	a_1 m ² /hr	λ_1 kcal/mhr°C	$(c\rho)_1$ kcal/m ³ °C	c_1 kcal/kg°C
	Average value	8.21	3.76×10^{-4}	0.159	424	0.362
	Average deviation of test results	+0.7, -0.7%	+1.3, -1.3%	+0.5, -1.3%	+1.2, -0.6%	+1.2, -0.6%
Properties ratio	Properties ratio	$\sqrt{(\lambda c \rho)_2}/\sqrt{(\lambda c \rho)_1}$	a_2/a_1			
	Average value	0.829	1.08			
	Average deviation of test results	+0.5, -0.6%	+1.6, -1.2%			
Test specimen ② PVC	Properties	$\sqrt{(\lambda c \rho)_2}$ kcal/m ² hr ^{1/2} °C	a_2 m ² hr	λ_2 kcal/mhr°C	$(c\rho)_2$ kcal/m ³ °C	c_2 kcal/kg°C
	Average value	6.81	4.07×10^{-4}	0.137	337	0.235
	Average deviation of test results	+0.3, -0.9%	+1.2, -1.8%	+1.1, -1.8%	+1.4, -0.7%	+1.4, -0.7%

Average value: $\frac{1}{n} \sum_{i=1}^n X_i = \bar{X}$, Average deviation of test results: $\max_{\min} \left(\frac{X_i - \bar{X}}{\bar{X}} \right)$

in these experiments.

The test results of spindle oil by the series method, shown in Table 6, coincided with our test results $\lambda=0.114$ kcal/mhr°C of the same oil by the other transient line source method investigated by H. Pfriem⁽⁷⁾, and our series method can be applied also to the measurements of liquids.

In the average deviations of measuring results shown in Tables 5 and 7, the deviations by the series method in Table 7 are larger than those of the parallel method. The reason is that the results by the series method are directly dependent on the errors of each measurement, especially when σ is small. But the average values of the test results are the same by these two methods.

The accuracy of the measurements by these methods under the same measuring conditions is the same and an accuracy of more than 95% could be obtained by these methods, and the accuracy of measurement of the relative values by parallel method is more than 98%, as shown in 5.2.2.

6. Conclusions

Transient comparison methods of simultaneous determination of thermal conductivity, thermal diffusivity and heat capacity with a plane-shaped standard specimen are investigated. These methods are classified into (1) parallel method, (2) series method, (3) combined method by the arrangement of the standard and the specimen, and the most useful parallel and series methods are chosen. The principles and the procedures of measurements are described, and the conditions of measurement and the dimensions of specimen are discussed. The results of measurements by the apparatus, following

the above discussion, are shown.

The merits of the transient comparison measuring methods, described in this paper, are shown to be:

(1) By either the parallel method or the series method

i) able to measure either absolute values or relative values of thermal conductivity, thermal diffusivity and heat capacity simultaneously in a short time to within a few degrees.

ii) able to measure easily with a very simple apparatus.

(2) The series method has the advantage of being able to measure the properties of liquids and powders easily besides solids specimen, with a standard plate specimen.

(3) The parallel method has the advantage of obtaining the relative properties of solid samples with similar properties precisely without measuring the heat flux. This method can be applied to obtain temperature dependency of thermal properties of solid by measuring relative properties of two specimens, keeping one at a constant temperature as a standard, and heating the other to the desired temperature.

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