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On the Mechanism of Contact between Solid Surfaces*

(2nd Report, The Real Area of Contact, the Separation and the Penetrating Depth)

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Assuming that the asperities on normally distributed surface are cones of the same slope that depends on the surface roughness, it is deduced theoretically by the slip-line theory that the real area of contact in the case of contact between a soft conical asperity and a hard flat surface or between a hard conical asperity and a soft flat surface depends on the slopes of the conical asperities, the coefficient of friction at the interface of contact asperities, and the flow pressure. If the real area of contact is corresponding to the geometrical area obtained immediately from the bearing-area curve, the relation between the real area of contact and the separation or the penetrating depth is also obtained theoretically. A comparison of calculated values based on this theory with experimental data shows good agreement.

1. Introduction

It is necessary for the analyses of the contact phenomena between two solid surfaces to estimate quantitatively the real area of contact, the separation and the penetrating depth between metal surfaces in contact under applied load. For example, the cardinal fact determining the nature of the mechanism of friction and wear between nominally mating surfaces under moderate load is that the real area of contact is in general only a small fraction of the apparent contact area. Then, some informations on the mechanism of contact between metal surfaces have been derived experimentally and theoretically from contact resistance measurements, the observation of the points of real contact, the Hertzian equations and the statistical analysis of the surface roughness^{(1)~(4)}. As a result of them, when the local pressures suffice to cause plastic flow, an approximate value of the total area of real contact may be obtained by dividing the applied load by the flow pressure of softer material. But relatively little is known about the estimation of the flow pressure for the work-hardened surfaces, the penetrating depth, which has an effect on the static stiffness of machine tool joints⁽⁵⁾, the separation between metal surfaces under the applied load.

In this paper, the effects of the slopes of the asperities and of the coefficient of static friction on

the real area of contact are examined theoretically for the normally distributed surfaces by the use of Hill's⁽⁶⁾ and Grunzweig's equation⁽⁷⁾. Moreover, the measurement of the real area of contact between

Nomenclature

A : real area of contact
c_1 : constant
$F(u) = \int_u^{m\sigma} f(u) du$
$f(u) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(1/2)(u/\sigma)^2}$
$g(w) = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_1^2 + \sigma_2^2}} e^{-(1/2)(w/\sqrt{\sigma_1^2 + \sigma_2^2})^2}$
H : maximum height of asperities
H_V : Vickers hardness number (microhardness number)
k : shear yield stress
$L_a L_y$: apparent contact area
m : constant (m is dependent on H and type of machining operation)
M : number of intersecting points in gratings covered with individual areas of contact
N_0 : number of observations under microscope
P : total number of intersecting points in gratings
p_m : flow pressure
r_1 : radius of indentation
t : dimensionless separation $= u/\sigma$
u : separation between a smooth surface and a rough surface
U : penetrating depth
w : separation between two rough surfaces in contact
$\alpha = \lambda + \eta - (\pi/4)$
2β : apical angle of conical asperities
η : angle in slip-line field
λ : angle between α -slip line and surface of wedge
μ : coefficient of friction
σ : standard deviation
σ_0 : tensile yield stress
σ_{c2} : normal pressure at interface
τ : shear stress at interface
$\varphi(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}, \quad \Phi(t) = \int_t^m \varphi(t) dt$

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metal surfaces by visual means is also carried out in order to check the validity of the theoretical consideration and find the reasonable method of estimation of flow pressure. A comparison between the theoretical and the experimental results is also made to obtain the effects of surface roughness, material and type of finish on the penetrating depth and the separation between two metal surfaces.

2. Theory

When two normally flat metal surfaces are placed together under an applied load, the plastic deformation and flow of the tips of the contact asperities occur under the intense pressure (flow pressure) exhibited by the localized points of contact until the real area of contact is sufficiently large to support the load. In this case, an approximate value of the total area of real contact A may be usually obtained by dividing the applied load W by the flow pressure of the softer metal p_m as follows:

$$A = \frac{W}{p_m} \quad \text{.....(1)}$$

In this relation, it was suggested that the value of p_m is determined directly from hardness measurements. But there was no evidence that the slopes of the asperities, the coefficient of friction at the

Fig. 1 Conical asperity pressed by hard flat surface

Fig. 2 Relation between ratio of flow pressure to micro-hardness number p_m/H_v and semi-apical angle β

interface in the real contact and the work-hardened layers of surfaces have no effect on estimation of the real area of contact. Therefore, it seems to be necessary to consider the effects of them by the use of Hill's⁽⁶⁾ and Grunzweig's theory⁽⁷⁾.

2.1 Real area of contact

2.1.1 Contact between a soft conical asperity and a hard flat surface

If a soft conical asperity is pressed by a hard flat surface under a load as shown in Fig. 1, it will deform plastically according to the following equations:

$$\begin{aligned} \sigma_{c2} &= k(1 + 2\eta + \sin 2\lambda) \\ &= k \left\{ 1 + \frac{\pi}{2} - \cos^{-1} \left(\frac{\tau}{k} \right) + \sqrt{1 - \left(\frac{\tau}{k} \right)^2} + 2\alpha \right\} \\ \sigma_{c1} &= k \left\{ 1 + \frac{\pi}{2} - \cos^{-1} \left(\frac{\tau}{k} \right) - \sqrt{1 - \left(\frac{\tau}{k} \right)^2} + 2\alpha \right\} \\ \tau &= k \cos 2\lambda = \mu \sigma_{c2} \end{aligned} \quad \text{.....(2)}$$

$$\tan \beta = \frac{\left\{ 1 + \sqrt{1 + \left(\frac{\tau}{k} \right)^2} \sin \alpha \right\}^2}{\sqrt{1 + \left(\frac{\tau}{k} \right)^2} \cos \alpha \left\{ 2 + \sqrt{1 + \left(\frac{\tau}{k} \right)^2} \sin \alpha \right\}}$$

$$\mu = \frac{\left(\frac{\tau}{k} \right)}{\left\{ 1 + \frac{\pi}{2} - \cos^{-1} \left(\frac{\tau}{k} \right) + \sqrt{1 - \left(\frac{\tau}{k} \right)^2} + 2\alpha \right\}} \quad \text{.....(3)}$$

where

$$\alpha = \lambda + \eta - \frac{\pi}{4}, \quad k = \frac{\sigma_0}{\sqrt{3}} \quad \text{.....(4)}$$

k : shear yield stress

σ_0 : tensile yield stress

σ_{c2} : normal pressure at the interface

τ : shear stress at the interface

λ : angle between α -slip line and surface of wedge (this angle depends on semi-angle of wedge and coefficient of friction)

β : semi-angle of wedge

μ : coefficient of friction

The value of σ_0 for the highly work-hardened materials is written according to Tabor's theory as follows:

$$\sigma_0 = \frac{H_v}{c_1} \quad \text{.....(5)}$$

where c_1 : constant = 2.9 ~ 3.0

H_v : Vickers hardness number

When the values of μ and β are determined, the flow pressure, $p_m = \sigma_{c2}$ may be estimated from Eqs. (2), (4), and (5). Then, Fig. 2 shows the theoretical p_m/H_v versus β plot for two values of μ .

2.1.2 Contact between a hard conical asperity and a soft flat surface

If a soft flat surface is pressed by a hard conical asperity under a load as shown in Fig. 3, the

contact pressure in the plastic indentation by a hard asperity is presented by Eq. (2), where the relations between μ and β are written as

$$\left. \begin{aligned} & \{\cos \beta - \sqrt{2} \cos \lambda \sin(\beta - \alpha)\} \\ & \times \{\sin \beta + \sqrt{2} \cos \lambda \cos(\beta - \alpha)\} \\ & = \sqrt{2} \cos \lambda \cos \alpha \\ & \mu = \frac{\cos 2\lambda}{1 + 2\eta + \sin 2\lambda} \end{aligned} \right\} \dots\dots\dots (6)$$

where, $\alpha = \lambda + \eta - \frac{\pi}{4}$

On the contrary, the normal load W on the conical asperity is written as

$$W = \int_0^{r_1} 2\sigma_{c2} \sin \beta \frac{\pi r dr}{\sin \beta} + \int_0^{r_1} 2\mu \sigma_{c2} \cos \beta \frac{\pi r dr}{\sin \beta} \\ = \pi r_1^2 (1 + \mu \cot \beta) \sigma_{c2} \dots\dots\dots (7)$$

where, r_1 : radius of indentation produced on the softer flat surface

Hence, the flow pressure is given as follows:

$$p_m = W / \pi r_1^2 \\ = (1 + \mu \cot \beta) \sigma_{c2} \dots\dots\dots (8)$$

Then, the theoretical p_m/H_V versus β plot estimated from Eqs. (2), (4), (5) and (8) for two values of μ is shown in Fig. 4*.

It is evident from Figs. 2 and 4 that the value of p_m/H_V decreases with the decrease of the apical angle of the conical asperity 2β , i.e., the real area of contact increases and on the other hand it

Fig. 3 Flat surface pressed by hard conical asperity

Fig. 4 Relation between ratio of flow pressure to micro-hardness number p_m/H_V and semi-apical angle β

* The theoretical values in Figs. 2 and 4 were obtained for $c_1 = 2.9$ in Eq. (5).

decreases with the increase of the coefficient of friction.

2.2 Separation and penetrating depth

2.2.1 Contact between an ideal flat and a rough surface

When an ideal flat surface is brought close to a rough surface, the initial separation, i.e., the distance between two median planes at the beginning of contact is equal to or less than m . Then consider the real area of contact A resulting from the movement of the ideal flat surface through a distance $U = m\sigma - u$ under an applied load. In other words A is the summation of the individual areas of asperities crossed by the ideal flat surface at given separation u . For the normally distributed rough surface which has the same profile curve in any section perpendicular to y -axis in Fig. 5, A is given statistically by⁽⁸⁾

$$A = \int_0^{L_y} L_x \frac{F(u)}{F(-m\sigma)} dy \\ = L_x L_y \frac{F(u)}{F(-m\sigma)} = L_x L_y \frac{\phi(t)}{\phi(-m)} \dots\dots\dots (9)$$

where

$$F(u) = \int_u^{m\sigma} f(u) du$$

$$f(u) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(1/2)(u/\sigma)^2}$$

$$\phi(t) = \int_t^m \phi(t) dt$$

$$\phi(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$$

σ : standard deviation

m : constant depends on the surface roughness and the type of finish

$L_x L_y$: apparent area of contact between two surfaces

Since we may assume perfectly plastic deformation of the metal occurring at the contact, the flow pressure should be constant under any load. In this case the real area of contact A under the applied load W can be written as Eq. (1). Combining Eq. (1) with Eq. (9) we obtain

$$L_x L_y \frac{\phi(t)}{\phi(-m)} = \frac{W}{p_m} \dots\dots\dots (10)$$

Fig. 5 Abbott's bearing curves on L_x and L_y

Table 2 Types of finish and degrees of surface roughness of specimens for various combinations of metals
(a) Contact between smooth surface and rough surface

Specimen Properties	Smooth surface	Rough surface		
Material	SKS 3	AIB 3	CuB 2	CuB 2
Hardness $H_V(0.01\sim0.05)$	924	29.6	140	140
Finish	Grinding	Sandpaper-finishing	Sandblasting	Grinding
Surface roughness H_μ	0.4	32.1 14.7 7.8	19.0 10.9 4.2	6.0 1.6

(b) Contact between two rough surfaces

Specimen Properties	Upper specimen	Lower specimen
Material	AIB 3	AIB 3
Hardness $H_V(0.01\sim0.05)$	29.6	29.6
Finish	Sandblasting	Sandblasting
Surface roughness H_μ	32.1 25.3 15.5	32.1 25.3 15.5

Table 3 Types of finish and degrees of surface roughness of specimens for various combinations of metals

Specimen Properties	Smooth surface	Rough surface		
Material	SKS 3	SS 41	CuB 2	AIB 3
Hardness $H_V(0.01)$	946	234	134	34.3
Finish	Grinding	Sandblasting	Sandblasting	Sandblasting
Surface roughness H_μ	0.4	20.6 13.8 3.5	35.4	39.7 28.3 20.5

Table 4 Types of finish and degrees of surface roughness of specimens for various combinations of metals
(a) Contact between smooth surface (SKS 3) and rough surface

Specimen Properties	Smooth surface	Rough surface								
Material	SKS 3	SS 41	S 25 C		CuB 2			AIB 3		
Hardness $H_V(0.01)$	946	219	255	255	56.4	134	134	29.6	34.3	34.3
Finish	Grinding	Sandpaper-finishing	Sandblasting	Grinding	Sandpaper-finishing	Sandblasting	Grinding	Sandpaper-finishing	Sandblasting	Grinding
Surface roughness H_μ	0.4	11.6 7.2 3.4	18.2 14.3 6.4	7.2 4.8 4.7	16.0 6.7 2.6	16.8	4.5	24.7 11.1 3.3	17.0	8.3

(b) Contact between two rough surfaces

Specimen Properties	Rough surface						
Material	S 25 C			CuB 2		AIB 3	
Hardness $H_V(0.01)$	255	255	255	134	134	34.3	34.3
Finish	Sandblasting	Grinding*	Grinding**	Sandblasting	Grinding	Sandblasting	Grinding
Surface roughness H_μ	17.4 11.7 7.5	12.5 4.5 2.9	4.3	15.6	5.2	17.0	8.0

* One lay perpendicular to the other

** One lay parallel to the other

where, M : number of intersection points in the gratings covered with the individual areas of contact

N_0 : number of observations under the microscope

P : total number of intersecting points in the gratings

Table 2 shows the type of finish and the surface roughness of the specimens for various metals. The surfaces were prepared by sandpaper-finishing, sandblasting and grinding. Care was taken to avoid the introduction of any defect in flatness during

① Upper specimen ② Lower specimen
③ Proving ring ④ Electric micrometer
Fig. 8 Apparatus for measuring penetrating depth

finishing. The surface roughness H which was measured on the profile curves of the surface according to JIS B0601 represents, respectively, the mean value of 10 times measurements.

3.2 Separation and penetrating depth

3.2.1 Separation

The distance $u = t\sigma$ between two median planes of the surfaces in contact under the applied load W , the separation, was measured directly on the profile curves of metal surfaces pressed by a smooth and harder metal surface. Each value of separation for each specimen was read from four profile curves of the surface and each surface roughness was the mean of 10 times measurements. Table 3 shows the type of finish and the surface roughness of the specimens for various metals.

3.2.2 Penetrating depth

The device employed for measuring the penetrating depth is shown in Fig. 8. The penetrating depth between metal surfaces in contact under the applied load was measured by the four electric micrometers ④ fixed on the lower specimen ③. In this case, after the asperities of the soft metal were deformed plastically under a given load and the load was then decreased to the initial load (27 kg), each difference of penetrating depth between the former initial load and the latter was determined. The type of finish and the surface roughness of the specimens of various metals are summarized in Table 4.

4. Experimental results and consideration

4.1 Real area of contact

Smooth surface (SKS3) on sandpaper-finished surface (AIB3)
Fig. 9 Dimensionless real contact area $A/L_x L_y$ versus dimensionless apparent pressure $(W/L_x L_y)/p_m$

Measured and theoretical real areas of contact are compared in dimensionless form in Figs. 9, 10, 11 and 12. The mean values of three times measurements and their range are also shown in respective figures. The agreement between theory and experiment is good within the range of these applied loads. It is seen from these experimental results that the real area of contact increases proportionally with the increase of the applied load. The effect of the surface roughness on the real area of contact is illustrated in Figs. 10 and 11, as predicted by the theory. The real area of contact increases a little with an increase of the surface roughness

Smooth surface (SKS3) on sandblasted surface (CuB2)
Fig. 10 Dimensionless real contact area $A/L_x L_y$ versus dimensionless apparent pressure $(W/L_x L_y)/p_m$

Smooth surface (SKS3) on ground surface (CuB2)
Fig. 11 Dimensionless real contact area $A/L_x L_y$ versus dimensionless apparent pressure $(W/L_x L_y)/p_m$

because the greater the surface roughness, the smaller the apical angles of the asperities 2β and the flow pressure become as predicted by the theoretical curves in Figs. 2 and 4. The variance of the experimental data in Fig. 12 may be due to the fact that it is difficult to spread the coloring

Sandblasted surface (AIB3) on sandblasted surface (AIB3)
Fig. 12 Dimensionless real contact area $A/L_x L_y$ versus dimensionless apparent pressure $(W/L_x L_y)/p_m$

Fig. 13 Microhardness number versus projected area of indentation under various measuring loads

Smooth surface (SKS3) on sandblasted surfaces
Fig. 14 Dimensionless separation u/σ versus dimensionless apparent pressure $(W/L_x L_y)/p_m$

fluid for aluminum over the clearance between two mating surfaces.

Figure 13 shows the relation between the microhardness number H_v and the projected area of the indentation for the work-hardened layers of the sandblasted and the ground surface. The average areas of contact points obtained experimentally were $(3\sim 40) \times 10^{-5} \text{ mm}^2$ for a smooth part on a rough surface prepared by sandblasting and $(2\sim 10) \times 10^{-4} \text{ mm}^2$ for a smooth part on a rough surface prepared by grinding (see Figs. 5 and 6 in the third report). Therefore, it is evident from the comparison between

(a) SKS3 on SS41

(b) SKS3 on CuB2

(c) SKS3 on AIB3

Fig. 15 Dimensionless penetrating depth in the case of contact between smooth surface (SKS3) and sand-paper-finished surface as a function of dimensionless apparent pressure

the flow pressure in Figs. 10 and 11, and the microhardness number in Fig. 13, that the flow pressure is nearly equal to the microhardness number under measuring load of 5~100 g, in which the projected area of the indentation is nearly equal to the average area of contact points.

4.2 Separation and penetrating depth

4.2.1 Separation

Measured and theoretical separations are compared in dimensionless form in Fig. 14. The experimental values in this figure are the mean of 5 times measurements and their ranges are also shown. Though the scatter of the experimental results is large, the general trend of the experimental u/σ versus $(W/L_x L_y)/p_m$ plot for the various metals agreed with the full line predicted by the theory⁽¹⁰⁾ as expressed by Eqs. (10) and (11). Thus it is suggested from the above-mentioned results that the separation of two metal surfaces in contact is inversely proportional to one tenth power of the

applied load over the wide range of $(W/L_x L_y)/p_m = 10^{-4} \sim 10^{-2}$.

4.2.2 Penetrating depth

Comparisons between the predicted and the experimental results of the penetrating depth, whose values were the mean for five times experiments, as a function of the applied load are presented in nondimensional form in Figs. 15~19. The ordinate $(U-U_0)/\sigma$ is the ratio of the difference between the penetrating depth U under a given load and U_0 under the initial load to the standard deviation of the profile curve σ . Abscissa is the apparent contact pressure $W/L_x L_y$ expressed as a ratio of the flow pressure p_m of the materials in contact. Though the scatters of the experimental results are large, the general trend of the experimental $(U-U_0)/\sigma$ versus $(W/L_x L_y)/p_m$ plots for the three kinds of metals agreed with the full lines predicted by the theory as expressed by Eqs. (10) and (12) or by Eqs. (10) and (16). The discrepancy between the experimental and the theoretical results in Figs.

Fig. 16 Dimensionless penetrating depth in the case of contact between smooth surface (SKS3) and sandblasted surface and between two sandblasted surfaces as a function of dimensionless apparent pressure

Fig. 17 Dimensionless penetrating depth in the case of contact between smooth surface (SKS3) and ground surface and between two ground surfaces as a function of dimensionless apparent pressure

17 and 19 seems to be due to the deviated distribution curves of surfaces prepared by grinding from a normal distribution curve. The larger experimental values than the theoretical under the heavy load in Fig. 19 suggest the plastic deformation of metals below the region of contact.

5. Conclusions

From the comparisons between the theoretical and the experimental results and the considerations made previously, some conclusions can be drawn with regard to the real area of contact, the separation and the penetrating depth between two metal surfaces in contact as follows:

Fig. 18 Dimensionless penetrating depth in the case of contact between smooth (SKS3) and rough surface (CuB2) and between two rough surfaces (CuB2) as a function of dimensionless apparent pressure

Fig. 19 Dimensionless penetrating depth in the case of contact between smooth surface (SKS3) and rough surface (AIB3) and between two rough surfaces (AIB3) as a function of dimensionless apparent pressure

(1) The real area of contact between two metal surfaces is proportional to the applied load and inversely proportional to the flow pressure of the softer metal.

(2) The real area of contact is affected theoretically by the roughness of the surfaces in contact and the coefficient of friction at the interface of contact asperities, i.e., the real area of contact increases with the increase of the surface roughness and decreases with the increase of the coefficient of friction. But it was seen from the experimental results that the effects of them on the real area of contact are small.

(3) The flow pressure for estimation of the real area of contact between the highly work-hardened metals is given by the microhardness number in measuring load of 5~100 g, in which the projected area of the indentation is nearly equal to the average area of contact points.

(4) The separation between two metal surfaces u is inversely proportional to one tenth power of the applied load over a fairly wide range of $(W/L_x L_y)/p_m = 10^{-4} \sim 10^{-2}$ and the separation in nondimensional form $t = u/\sigma$ is not influenced by the degree of roughness.

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