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The Acoustic Resonance in a Tube Bank*

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In this investigation experimental studies on the acoustic resonance are undertaken.

In the first place the effect of the pitch of tubes on Strouhal number or the nondimensional acoustic resonance frequency is studied in a wind tunnel test using many model banks. According to the result the Strouhal number increases with a decreasing pitch of tubes.

In the second place a two-dimensional model basin test is undertaken and putting together the theory on the single tube in a flow previously reported and the observation result on the model basin test, a self-induced vibration theory for the acoustic resonance is proposed.

1. Introduction

In a tube bank which is located normal to the direction of the fluid flow some times a violent noise and vibration break out and frequently fatigue failures occur at various parts⁽¹⁾.

It is admitted that the cause of these phenomena is the vortex shedding in wakes behind tubes but the detail of the mechanism of the occurrence and the Strouhal number which determine the frequency of vortex shedding are not yet clear.

In this study the noise in the tube bank is investigated by a wind tunnel test with a model of the tube bank and the relationship between the Strouhal number and the pitch of tube is established and the self-induced vibration theory is suggested on the basis of the consideration about the vibration of a circular cylinder in a flow which was described previously⁽²⁾.

2. The experimental apparatus and the experimental method

2.1 The wind tunnel and the model of the duct

The wind tunnel and the model of the duct are shown in Fig. 1. Dimensions of the wind tunnel are as follows.

Delivery flow	710 m ³ /min
Delivery pressure	204 mm Aq.
Blower speed	1 750 rpm.
Output of the motor	60 PS
Sectional area of the duct	720 mm × 300 mm
Length of the duct	3 000 mm

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The model tube bank is located at the middle point of the duct.

2.2 The model of the tube bank

An example of the model tube bank is shown in Fig. 2. It is constructed by a steel pipe with the outside diameter of 25 mm. The values of d/t and d/l (d : diameter of tube, t : the pitch perpendicular to the flow, l : the pitch in direction of the flow) of the model bank are given in Table 1. Five tube banks with the values of d/t which are 0.6, 0.5, 0.4, 0.3, 0.2 are prepared and each bank is

Fig. 1 Experimental apparatus

Fig. 2 Model banks

constructed to have values of d/l of 1.0, 0.75, 0.5, 0.25 respectively using 3 kinds of spacers. At d/l of 1.0 no spacers are used and in this case only the staggered bank is tested.

2.3 Measuring apparatus

Measuring apparatuses are shown in Fig. 1. The velocity of flow is measured by a Pitot tube located at front of model banks. The frequency of the acoustic resonance and its noise level are measured by the sound level meter, the frequency analyser and the high speed level recorder. When no resonance occurs the vortex shedding frequency is measured by the hot wire anemometer at the mid-span of several tubes at centre of the duct width.

2.4 The experimental method

With each model bank set in the duct, the wind tunnel is started and then the velocity of flow is increased gradually. Then the acoustic resonance is watched carefully.

In these resonance conditions at each speed of flow the flow velocity, the sound level and the frequency are measured. In the early stage of the experiment as the resonance was very weak at d/t of 0.2, the hot wire anemometer was used at the same time.

3. The experimental results

3.1 The mode of acoustic resonance

As shown in Fig. 3 the acoustic resonance appears in the direction perpendicular to the axis of the tube and the fluid flow, and in the experiment three kinds of mode as shown in Fig. 3 are distinguished. The frequency of the acoustic resonance f_a is estimated by $f_a = nc/L$ (where n : number of a standing wave in the duct width, c : the velocity of sound, L : the duct width). With the increase of the flow velocity the resonance shifts to higher mode discontinuously. Measured modes in the

Table 1 Dia. to pitch ratio of model banks

d/t	0.6	0.5	0.4	0.3	0.2	5 banks for each kind of bank
d/l	0.75	0.5	0.25	4 spacers are used for each kind of spacers

Fig. 3 Modes of the acoustic resonance

experiment are shown in Table 2. As shown in this table in all cases of staggered banks acoustic resonance is detected always but in the case of lined banks the resonance does not occur so easily.

3.2 The nondimensional frequency of acoustic resonance

The Strouhal number is defined by $f_v d/V$ (where f_v : vortex shedding frequency, d : diameter of tube, V : velocity of flow). In the case of tube bank the Strouhal number is affected by d/t and d/l unlike the case of single tube and the vortex shedding frequency is controlled by the resonance in the duct, so that the vortex shedding frequency is locked in the resonance frequency. In this paper these frequencies are defined as the nondimensional acoustic frequency $f_a d/V$ (where f_a : acoustic frequency)

In Fig. 4 to Fig. 9 the nondimensional acoustic frequencies are shown against Reynolds number Vd/ν (ν : kinematic coefficient of viscosity) at various values of d/t and d/l .

Figure 4 to Fig. 8 correspond with d/t of 0.2 to 0.6 in the staggered banks respectively and Fig. 9 is that of inlined bank.

Table 2 Wind tunnel test results
○ resonance × no resonance

Type	d/t	d/l	1.0	0.75	0.5	0.25
Stagger	0.2		○ 1/2	○ 1/2	○ 1/2	○ 1/2
	0.3		○ 1/2, 1	○ 1/2, 1	○ 1/2	○ 1/2
	0.4		○ 1/2, 1, 3/2	○ 1, 2	○ 1/2, 1	○ 1/2
	0.5		○ 1/2, 1, 3/2	○ 1, 3/2	○ 1/2	○ 1/2
	0.6		○ 1, 3/2, 2	○ 3/2, 2	○ 1/2	○ 1/2 (3 banks)
Inline	0.4		×	×	×	○ 1/2
	0.5		×	×	×	○ 1/2
	0.6		×	×	○ 1/2	○ 1/2 (3 banks)

Fig. 4 Test result

Fig. 5 Test result

In Fig. 4 on account of wide pitch of tube bank acoustical resonances are rather weak so the frequency is measured by the hot wire anemometer. In Fig. 5 at d/l of 0.75 and 1.0 serious resonance noise is present and these modes are that of half wave length or that of one wave length and the non-dimensional frequencies are larger than 0.2. In Fig. 6, at d/t of 0.4 modes are that of half wave length to two wave lengths and with an increasing d/l the nondimensional frequency becomes larger and a discontinuous variation of modes is shown distinctly. In Fig. 7 and Fig. 8 similar tendencies are shown also.

Figure 9 showed the result of the inlined bank test and all modes of acoustic resonance are half wave length and their noise level is lower than that of the staggered bank tests but the velocity range of resonance is the same as in staggered bank tests.

All of the results mentioned above are rearranged collectively in Fig. 10. In this figure the velocity range of acoustical resonance is shown at every

value of d/t and d/l and it is seen that the non-dimensional frequency increases with d/t and d/l considerably.

4. The two-dimensional model basin test

4.1 The purpose of the investigation and the experimental apparatus

As stated in the previous paragraph on the wind

Fig. 6 Test result

Fig. 7 Test result

Fig. 8 Test result

Fig. 9 Test result

Fig. 10 Final test results

tunnel test the condition of the occurrence of the acoustic resonance has been revealed but the detail of the mechanism of the resonance is not yet clear.

For a single tube in a flow, details of the flow around the tube are known considerably from a very large number of literatures and the previous report but the phenomena in the tube bank may be very complex on account of the interaction of tubes. So in this study for recognition of the mechanism the experimental investigation using a two-dimensional model basin is undertaken.

In Fig. 11 the experimental apparatus is shown. The width of the basin is 185 mm and its depth is about 50 mm. The velocity of water is adjusted by the water supply valve and the weir at the basin end. The velocity of water is measured by the stop watch that measures the time during which a small paper chip on the water surface passes two indexes separated by 300 mm distance the flow.

The observation of flow is undertaken by the dispersion of aluminum powder on the water surface and its photography by the 16 mm cine camera and the 35 mm camera.

Model tube banks tested in the experiment are shown in Table 3. The diameter of model tubes is 15 mm.

At the model basin having a free surface the wave height corresponds to the acoustic pressure in the compressible fluid so that the state of the flow is equal to each other. The nondimensional frequency when the resonance happens in the basin is the same as in the wind tunnel test.

4.2 The observation of the flow by 16 mm cine camera

Figure 12 shows the record of the test on the staggered bank model with d/t and d/l of 0.5. The frame speed in photography is 8 frames per second

Fig. 11 Two-dimensional model basin test apparatus

Stagger $d/t=0.5$, $d/l=0.5$
Fig. 12 Test record

and the exposure time is about 1/15 second. The time in the photograph flows from right to left.

The right end frame in these continuous photographs shows that the deviating velocity (velocity component perpendicular to the main flow) is nearly zero and the main flow between tubes is straight. In the following frame the deviation turns to right side and the third frame also shows the right side deviation; then the maximum deviation may be between the second and the third frame. In the 4th frame the deviation turns again to the left side and there is the maximum left side deviation between 4th and 5th frame. In the 6th frame there is no deviation just like as the first frame.

The above mentioned alternation of flow corresponds to one cycle of the resonance, and as seen in these photographs stream lines between tubes in each frame contain a synchronous deviating flow.

4.3 The observation of the flow by the 35 mm camera

As the observation by the 16 mm cine camera did not show details of stream line, the observation by 35 mm camera was further undertaken. As the recording in this method is not continuous typical examples are selected from among many photographs taken arbitrarily.

In Fig. 13 photographs at d/t and d/l of 0.5 and lined model bank are shown. The record at d/t and d/l of 0.5 and staggered bank shows stronger deviation than Fig. 13 but it is omitted as it was reported in other papers already.

In every picture of Fig. 13 the main fluid flows from left to right; (1) and (3) show the minimum deviation, but (2) shows the upward deviation and (4) the downward deviation.

In photograph (1) the flow between tubes runs through straightly to the next bank and two vortices behind each tube have the same strength and there is no deviation. In photograph (2) there are upward deviating flows between a small vortex in

Table 3 dia. to pitch ratio of model banks

Type	Stag.	Stag.	Stag.	Stag.	Inl.	Inl.
d/t	0.5	0.4	0.5	0.6	0.5	0.5
d/l	0.5	0.25	0.25	0.25	0.5	0.25

anticlockwise direction under side in the wake and a large vortex in clockwise direction upper side. As there is a difference between two vortices behind the tube it is considered that a circulation around each tube may be induced. In photograph (3) the flow becomes straight again just like (1) but in photograph (4) there is a deviating flow downward. The period mentioned above is a cycle of the resonance and it is observed that the flow is perfectly synchronized at any time in the period.

$d/t=0.5$, $d/l=0.5$ in line
Fig. 13 Test record

5. Consideration of the resonance mechanism

5.1 The thinking model of the resonance mechanism

Applying the excitation mechanism for single tube in a flow which was previously reported to the experimental observation mentioned above, the following self-induced vibration theory may be introduced.

In the acoustic resonance as shown in Fig. 3 there is an alternating deviating velocity $\dot{y}$ in direction perpendicular to the main flow. Namely if the deviation of the fluid particle is indicated by y the distribution of the deviation velocity $\dot{y}$ is shown as Fig. 3 so that the deviation velocity $\dot{y}$ and the sound pressure come out periodically as shown in Fig. 14 (a) and (b).

Now the consideration mentioned in the previous report is applied to these phenomena. The natural frequency f_v of the angular oscillation of the dead stream region behind the tube about the axis of it may be proportional to the flow velocity just like that of single tube. Accordingly when the flow velocity increases at special velocity the natural frequency of the dead stream region coincides with the natural frequency of the acoustic resonance of the fluid in the duct. At this velocity the vibration of the dead stream region becomes the resonance state under the alternating pressure gradient dp/dy so that the angular displacement of dead stream α

Fig. 14 One cycle in resonance

is delayed by about $\pi/2$ with the pressure gradient and the amplitude of the angular vibration of the dead stream region is magnified.

The angular displacement of the dead stream region is larger than the attack angle α_0 consisting of $\dot{y}$ and the main flow velocity V . The variation of the flow direction $(\alpha - \alpha_0)$ induces a vortex clockwise behind the tube and a circulation anticlockwise

Fig. 15 Flow around tubes

Fig. 16 Model of dead stream

around the tube. So the tube is affected by the hydraulic lift force downward and in consequence of the reaction of the lift force the force upward is applied to the fluid between tubes.

These forces are in-phase with the deviation velocity in the flow as shown in Fig. 14 (d) and then the vibration energy is supplied from the flow itself as shown in (e).

5.2 The forced vibration of the dead stream region induced by the acoustic resonance

The forced vibration of the dead stream region behind the tube in unit length induced by the alternating pressure gradient of the acoustic resonance is shown as follows.

$$I \frac{d^2 \alpha}{dt^2} + c \frac{d\alpha}{dt} + k(\alpha - \alpha_0) = -\Delta p \sin \omega_a t \quad \dots (1)$$

where notations are as follows and as shown in Fig. 16.

I : the moment of inertia of the dead stream

k : the coefficient of the restoring moment acting on the dead stream

c : the coefficient of damping of the dead stream

$-\Delta p$: the forcing moment amplitude induced by the pressure difference between both sides of the dead stream and this is negative when the pressure gradient dp/dy is positive.

α : the angular displacement of the dead stream, anticlockwise is positive

α_0 : the attack angle consisting of the main flow velocity V and the deviating velocity

ω_a : the frequency of the acoustic resonance.

Eq. (1) is rewritten as follows

$$I \frac{d^2 \alpha}{dt^2} + c \frac{d\alpha}{dt} + k\alpha = -\Delta p \sin \omega_a t + k \frac{\bar{y}}{V} \left[\sin \left(\omega_a t + \frac{\pi}{2} \right) \right] = -\sqrt{\Delta p^2 + \left(k \frac{\bar{y}}{V} \right)^2} \sin \left(\omega_a t - \tan^{-1} \frac{k \frac{\bar{y}}{V}}{\Delta p} \right) \quad \dots (2)$$

because

$$\alpha_0 = \frac{\bar{y}}{V} \left[\sin \left(\omega_a t + \frac{\pi}{2} \right) \right] \quad \dots (3)$$

so that

$$\alpha = -\frac{\sqrt{\Delta p^2 + \left(k \frac{\bar{y}}{V} \right)^2}}{k} \frac{1}{\sqrt{\left\{ 1 - \left(\frac{\omega_a}{\omega_v} \right)^2 \right\}^2 + \left(\frac{c}{c_c} \frac{\omega_a}{\omega_v} \right)^2}} \sin \left\{ \omega_a t - \tan^{-1} \frac{k \frac{\bar{y}}{V}}{\Delta p} - \tan^{-1} \frac{c}{c_c} \frac{\frac{\omega_a}{\omega_v}}{1 - \left(\frac{\omega_a}{\omega_v} \right)^2} \right\} \quad \dots (4)$$

where

$$c_c = \sqrt{kI} \quad \dots (5)$$

$$\omega_v = \sqrt{\frac{k}{I}} \quad \dots (6)$$

5.3 The exciting force acting on the fluid between tubes

The angular displacement of the dead stream α is in-phase with the attack angle α_0 in the resonance state, and as shown in Fig. 15 α is greater than α_0 and when α is positive there is a clockwise vortex in the dead stream so the anticlockwise circulation around the tube is presented and the downward lift force occurs just like the Magnus effect. The lift force may be proportional to the effective angular displacement $(\alpha - \alpha_0)$ so that if the proportional coefficient is f_m , the lift force F_m is shown as follows

$$F_m = -f_m(\alpha - \alpha_0) \quad \dots (7)$$

Substituting Eq. (3) and (4) into Eq. (7) gives;

$$F_m = -f_m \left[-\frac{\sqrt{\Delta p^2 + \left(k \frac{\ddot{y}}{V}\right)^2}}{k} \frac{1}{\sqrt{\left\{1 - \left(\frac{\omega_a}{\omega_v}\right)^2\right\}^2 + \left(\frac{c}{c_c} \frac{\omega_a}{\omega_v}\right)^2}} \sin \left\{ \omega_a t - \tan^{-1} \frac{k \frac{\ddot{y}}{V}}{\Delta p} - \tan^{-1} \frac{c}{c_c} \frac{\frac{\omega_a}{\omega_v}}{1 - \left(\frac{\omega_a}{\omega_v}\right)^2} \right\} - \frac{\dot{y}}{V} \right] \quad \dots\dots\dots(8)$$

As the exciting force is $-\Delta p \sin \omega_a t$, $\dot{y}$ is $\ddot{y} \cos \omega_a t$ as shown in Fig. 14, so that Eq. (8) is rewritten as follows divided into the term of $\sin \omega_a t$ and that of $\cos \omega_a t$

$$F_m = f_m \left[-\frac{\sqrt{\Delta p^2 + \left(k \frac{\ddot{y}}{V}\right)^2}}{k} \frac{1}{\sqrt{\left\{1 - \left(\frac{\omega_a}{\omega_v}\right)^2\right\}^2 + \left(\frac{c}{c_c} \frac{\omega_a}{\omega_v}\right)^2}} \cos \delta + \frac{\ddot{y}}{V} \right] \cos \omega_a t \\ + f_m \frac{\sqrt{\Delta p^2 + \left(k \frac{\ddot{y}}{V}\right)^2}}{k} \frac{1}{\sqrt{\left\{1 - \left(\frac{\omega_a}{\omega_v}\right)^2\right\}^2 + \left(\frac{c}{c_c} \frac{\omega_a}{\omega_v}\right)^2}} \cos \delta \sin \omega_a t \quad \dots\dots\dots(9)$$

where

$$\delta = \tan^{-1} \frac{k \frac{\ddot{y}}{V}}{\Delta p} + \tan^{-1} \frac{c}{c_c} \frac{\frac{\omega_a}{\omega_v}}{1 - \left(\frac{\omega_a}{\omega_v}\right)^2} \quad \dots\dots\dots(10)$$

As it is defined that

$$y = \ddot{y} \sin \omega_a t \quad \dots\dots\dots(11)$$

$$\dot{y} = \ddot{y} \cos \omega_a t = \omega_a \ddot{y} \cos \omega_a t \quad \dots\dots\dots(12)$$

Substituting Eq. (11) and (12) to Eq. (9) gives;

$$F_m = -f_m \left[\frac{\sqrt{\Delta p^2 + \left(k \frac{\ddot{y}}{V}\right)^2}}{k} \frac{1}{\sqrt{\left\{1 - \left(\frac{\omega_a}{\omega_v}\right)^2\right\}^2 + \left(\frac{c}{c_c} \frac{\omega_a}{\omega_v}\right)^2}} \sin \delta - \frac{\ddot{y}}{V} \right] \frac{1}{\omega_a \ddot{y}} \dot{y} \\ + f_m \frac{\sqrt{\Delta p^2 + \left(k \frac{\ddot{y}}{V}\right)^2}}{k} \frac{1}{\sqrt{\left\{1 - \left(\frac{\omega_a}{\omega_v}\right)^2\right\}^2 + \left(\frac{c}{c_c} \frac{\omega_a}{\omega_v}\right)^2}} \cos \delta \frac{1}{\ddot{y}} y \quad \dots\dots\dots(13)$$

Eq. (13) is simplified as follows

$$F_m = -F_1 \frac{dy}{dt} + F_2 y \quad \dots\dots\dots(14)$$

where

$$F_1 = f_m \left[\frac{\sqrt{\Delta p^2 + \left(k \frac{\ddot{y}}{V}\right)^2}}{k} \frac{1}{\sqrt{\left\{1 - \left(\frac{\omega_a}{\omega_v}\right)^2\right\}^2 + \left(\frac{c}{c_c} \frac{\omega_a}{\omega_v}\right)^2}} \sin \delta - \frac{\ddot{y}}{V} \right] \frac{1}{\omega_a \ddot{y}} \quad \dots\dots\dots(15)$$

$$F_2 = f_m \frac{\sqrt{\Delta p^2 + \left(k \frac{\ddot{y}}{V}\right)^2}}{k} \frac{1}{\sqrt{\left\{1 - \left(\frac{\omega_a}{\omega_v}\right)^2\right\}^2 + \left(\frac{c}{c_c} \frac{\omega_a}{\omega_v}\right)^2}} \cos \delta \frac{1}{\ddot{y}} \quad \dots\dots\dots(16)$$

As the reaction of F_m is applied to the fluid $F_1 \frac{dy}{dt}$ corresponds to the negative damping.

5.4 The effect of the drag force

The drag force acts in the direction of the attack angle as shown in Fig. 15. The reaction of the drag force is applied to the fluid then when the deviation is too large the component of the drag force F_D in the direction of y axis, F_{dy} is shown as follows.

$$F_{dy} = \frac{1}{2} \rho (V^2 + \dot{y}^2) C_D d \frac{\dot{y}^2}{\sqrt{V^2 + \dot{y}^2}} \quad \dots\dots\dots(17)$$

For the explanation of the limit cycle of the resonance the term of $\dot{y}^3$ is retained, then

$$F_{dy} = \frac{1}{2} \rho C_D d \left\{ V \dot{y} + \frac{1}{2V} \dot{y}^3 \right\} \equiv F_3 \dot{y} + F_4 \dot{y}^3 \quad \dots\dots\dots(18)$$

where

$$F_3 = \frac{1}{2} \rho C_d d V \dots\dots\dots (19)$$

$$F_4 = \frac{1}{4} \rho C_d d \frac{1}{V} \dots\dots\dots (20)$$

These forces act on the tube so that their reactions are applied on the fluid as the damping force.

5.5 The equation of the acoustic resonance

Above mentioned is the consideration of force acting on the tube of unit length. For the resonance in the duct these reactions have to be integrated along the duct width.

Strictly speaking the analysis must be made on the equation of the wave propagation, but for the sake of simplicity if the equivalent mode of half wave resonance is considered, namely if M is the mass of the fluid at middle part of the duct, if K is the spring constant of both sides of the duct where the deviation is small and if C is the damping coefficient based on the absorption of the sound and if $\bar{F}_1$ to $\bar{F}_4$ are integrations of F_1 to F_4 mentioned above along the width corresponding to M , the equation of the resonance is shown as follows.

$$M \frac{d^2 y}{dt^2} + C \frac{dy}{dt} + Ky = (\bar{F}_1 - \bar{F}_3) \frac{dy}{dt} - \bar{F}_4 \left(\frac{dy}{dt} \right)^3 - \bar{F}_2 y \dots\dots\dots (21)$$

In the equation, $\bar{F}_1$ becomes a large positive value near $\omega_a = \omega_v$ and when $\bar{F}_1 > \bar{F}_3 + C$ the self-

induced vibration will happen. Moreover as the effect of $\bar{F}_2$ is small the acoustic frequency $\omega_a/2\pi$ is determined only by M and K so that the frequency is kept constant in spite of the variation of the fluid velocity.

6. Conclusion and acknowledgement

In the investigation as mentioned in previous paragraphs the effect of the pitch on the nondimensional frequency and the resonance mechanism are studied.

According to the results, considerably the non-dimensional frequency becomes larger with a decrease in the pitch of tubes and the aeroelastic self-induced vibration on the fluid in the duct is proposed by the analogy to the theory for single tube previously reported.

For the numerical treatment of the phenomenon further studies must be carried out, say the measurement of the pressure variation on the tube surface during the resonance.

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