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Analysis on the Pressure Distribution around the Impeller of Volute Pumps*

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When a volute pump is operated under various off-design conditions, the impeller suffers radial thrust due to the pressure varying around the impeller. The author analyzes the pressure distribution around the impeller under the assumption that the total pressure and the velocity of the fluid leaving the impeller depends upon the local static pressure. In the analysis the impeller periphery is replaced by a straight line, the volute by a wedge shape space, and the flow in the wedge shape space is disintegrated into potential flows and a flow with vortices so that the combined flow may satisfy the boundary conditions along the impeller periphery as well as along the volute wall.

The theory predict that the pressure near the tang is constant regardless of the flow rate of pump and that the pressure variation along the periphery is little near the tang and is significant near the exit of the volute. These predictions agree well with experimental facts quoted in the literature.

1. Introduction

When a volute pump is operated under various off-design conditions, the impeller suffers radial thrust due to the pressure varying around the impeller. Since pressure varies along the impeller periphery, the total pressure and the local flow rate at the impeller exit must also vary along the periphery. In the literature, however, some⁽¹⁾ have assumed that the exit velocity from the impeller is uniform and others⁽²⁾ that the total pressure of flow from the impeller is uniform around the

impeller.

In order to understand correctly the flow inside the impeller as well as the flow in the volute casing, one has to make analysis under a more realistic assumption that the local flow rate from the impeller depends upon the local static pressure around the impeller. The static pressure distribution as well as the condition of flow in the volute casing, in turn, entirely depends upon the distribution of the flow rate at which the impeller discharges. That is, the local flow rate from the impeller and the static pressure distribution around the impeller-exit influence each other.

Mixing takes place inside the volute casing after the fluid is discharged from the impeller. Since mixing process in the volute casing is not fully understood, it is extremely difficult to predict the accurate pressure distribution around the impeller. However, if an analysis is made on two extreme cases in one of which complete mixing takes place immediately after the fluid has left the impeller and in the other, no mixing occurs in the volute casing, we may be able to understand the realistic flow as an intermediate state between these two extreme cases. In this paper an analysis is made under the assumption that no mixing occurs in the volute casing. Another analysis based on the other assumption will be presented in a separate paper.

Fig. 1 A volute casing and an impeller

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2. The condition of flow at the exit of impeller

When the pressure is not constant along the impeller periphery, the flow rate through each channel of the impeller varies periodically with the rotation of the impeller. Accordingly, the instantaneous relationship between the flow rate and the static pressure rise in the impeller channel may be different from that in steady state. In the present analysis, the author will simply assume this relation of steady state as applicable to unsteady cases, since the relationship in unsteady state is not well known.

The direction and the magnitude of velocity vary between two adjacent blades at the exit of the impeller, and the direction of the mean relative velocity is not identical with the direction of blades. In this analysis the deviation of flow from the blades at the exit of the impeller is considered, though the non-uniformity of flow between the two adjacent blades is disregarded.

The velocity triangle at the exit of the impeller is shown in Fig. 2, where β_b is the blade angle at the exit, W and V are respectively relative and absolute velocities, and subscript ∞ refers to the condition where the fluid flows in the direction of blades. The real exit absolute velocity is V_1 instead of V_∞ due to the slip or deviation of the relative flow from the impeller blades.

For pumps with backward leaning blades, the theoretical head, or the work input per unit mass of fluid, decreases linearly with the increase of the flow rate. This fact implies that the tip of absolute velocity vector, when the flow rate varies, moves on a straight line which is shown as a chain line in Fig. 2. According to the literature⁽³⁾ the angle β between the chain line and the circumferential direction is nearly equal to the blade exit angle

Fig. 2 The velocity triangle at the exit of an impeller

Fig. 3 The wedge shape space which represents the volute casing

β_b . Consequently, the absolute exit velocity-vector V at any flow rate is composed of the velocity vector V_1 and a velocity vector V_2 which makes an angle β with the circumferential direction. In the shut-off condition the tip of the absolute velocity vector is at the junction of the peripheral velocity vector U and the chain line. $(1-\delta)U$ indicates the magnitude of the velocity and δ is called the slip coefficient.

3. Flow in a wedge shape space

In this analysis a volute casing is replaced by a wedge shape space of angle α . That is, in Fig. 3 wall OA of the wedge shape space represents the wall of a volute casing, while line OX represents the circumference of the impeller. If the fluid flows parallel to wall OA into the wedge shape space uniformly through line OX, the velocity and static pressure at all points will become constant. This condition represents the normal flow rate. In order to obtain the condition of flow at a flow rate not normal, a few special types of flow patterns are added to the normal uniform flow. Obviously the resultant flow ought to satisfy the conditions at the impeller exit as well as the boundary condition of wall OA.

Before analysis, a potential flow in a wedge shape passage is reviewed.

The complex velocity of flow in a wedge shape passage AOB with an apex angle π/m is

$$u-iv=mV_0r^{m-1}\exp\{i(m-1)\theta\} \quad (1)$$

where r and θ are the radius and the argument of a polar coordinate system with the origin at O and OB as the base-line in Fig. 4. Line OB is taken as the ξ -axis and the η -axis is normal to it. u and v are the velocity components in ξ and η directions respectively. V_0 is a parameter indicating the magnitude of velocity.

OC is a line passing the origin and making an angle θ_0 with the ξ -axis. It is obvious from Eq. (1) that the direction of streamline is constant on line OC and that streamlines make an angle $-m\theta_0$

Fig. 4 Streamlines of a potential flow in a wedge shape space

with line OC. If OC is the bisector of wedge angle AOB, all streamlines meet at right angles with line OC.

If m and θ_0 are decided by

$$m = (\pi - \beta)/\alpha \text{ and } \theta_0 = \beta/m \quad \dots\dots\dots(2)$$

so that $m\theta_0 = \beta$ and angle $AOC = \pi/m - \theta_0 = \alpha$ in Fig. 4, then the wedge shape AOC, a part of the wedge shape passage AOB, in Fig. 4 corresponds to the wedge shape AOX in Fig. 3 and the direction of flow relative to line OC becomes β at all points on line OC. As indicated by a velocity vector V_2 in Fig. 2, this condition is the one required of a flow which is to be superposed on the uniform flow V_1 .

If OC is considered the X -axis, the complex velocity of the flow is

$$u_2 - iv_2 = mV_{20}r^{(m-1)} \exp\{i(m-1)\theta + im\theta_0\} \quad \dots\dots\dots(3)$$

where V_{20} is a parameter indicating the magnitude of velocity. Then the magnitude of velocity along the X -axis is

$$V_2(X) = mV_{20}X^{m-1} \quad \dots\dots\dots(3)'$$

where X is the distance from the origin along the X -axis or $X=r$ at $\theta=0$, and $V(X)$ indicates the velocity V on the X -axis.

4. Distribution of the total head along the periphery of the impeller

If there is no preswirl at the entrance of an impeller, the total head H transferred to the fluid by the impeller will be represented by

$$gH = UV_u \quad \dots\dots\dots(4)$$

where U is the peripheral velocity of the impeller and V_u is the circumferential component of the velocity of fluid at the exit of the impeller.

Now, it is assumed that the velocity is composed of V_1 and V_2 . As V_2 varies with X as indicated by Eq. (3)', V_u and the total head H also vary along X .

In a flow with total pressure gradient, there is a distribution of vortices. There the gradient of total head can be related to the vorticity ζ and the local velocity V by Crocco's equation⁽⁴⁾

$$g \frac{\partial H}{\partial n} = V(2\zeta) \quad \dots\dots\dots(5)$$

where n is the distance normal to the streamline. The vorticity is indicated by the gradient of velocity V_1 which is parallel to wall OA. If s is the

Fig. 5 Composition of an exit velocity vector

distance from wall OA

$$2\zeta = dV_1/ds \quad \dots\dots\dots(6)$$

Although vortices are transported by the flow, here it is assumed to simplify the theory that vortices are transported only by a parallel flow with velocity V_1 . Then,

$$2\zeta = dV_1/ds = \frac{dV_1}{dX} \frac{dX}{ds} = \frac{dV_1}{dX} / \sin \alpha \quad \dots\dots\dots(6)'$$

Using Eqs. (5) and (6)'

$$g \frac{dH}{dX} = g \frac{dH}{dn} \frac{dn}{dX} = V \left(\frac{dV_1}{dX} \right) \frac{dn}{dX} \frac{1}{\sin \alpha}$$

Differentiating Eq. (4) with respect to X

$$U \frac{dV_u}{dX} = g \frac{dH}{dX} = V \frac{dV_1}{dX} \frac{dn}{dX} \frac{1}{\sin \alpha} \quad \dots\dots\dots(7)$$

In this equation, dn/dX is the sine of the angle cut by the velocity vector V and the X -axis. V_u is expressed as

$$V_u = V_1 \cos \alpha + V_2 \cos \beta \quad \dots\dots\dots(8)$$

$$dV_u/dX = \cos \alpha (dV_1/dX) + \cos \beta (dV_2/dX) \quad \dots\dots\dots(8)'$$

5. Adjustment of velocity triangle

If a potential flow given by Eq. (3) is superimposed on a uniform flow parallel to wall OA, the resultant velocity vector is represented by the vector sum of V_1 and V_2 . It ought to satisfy the condition described in the section 3, that is, at any point on the X -axis the tip of velocity vector is to be on the chain line in Fig. 2. However, since the magnitude of velocity V_1 which is parallel to wall OA varies along the X -axis, as

$$V_1(X) = V_1(0) + \Delta V_1(X)$$

the tip of velocity vector $V_1(X) + V_2(X)$ is not on the chain line as indicated in Fig. 5.

Another flow with velocity V_3 may be superimposed on the combined flow so that on the X -axis the tip of the combined velocity vector

$$V_1(X) + V_2(X) + V_3(X)$$

may be on the chain line. The third flow V_3 is to remain irrotational while the velocity on the X -axis or $V_3(X)$ is to be kept free from a velocity component in the X -direction lest the distribution of total head should be disturbed by the flow. The potential flows (a), (b) and (c) of Fig. 6 are a few of the ones which satisfy the conditions mentioned above as well as the boundary condition on wall OA provided that the wedge angle is 2α . The potential flow (a) is a special case of Fig. 4, that is, OC is the bisector of the apex angle AOB, which is equal to 2α , and OC is chosen as the X -axis. Therefore, p must be given by

$$p = \pi/2\alpha \quad \dots\dots\dots(9)$$

and the complex velocity is

$$u_3 - iv_3 = pV_{30}r^{p-1} \exp\{i\theta(p-1) + i\pi/2\} \quad \dots\dots\dots(10)$$

where V_{30} is a parameter to indicate the magnitude

of velocity. Then the magnitude of velocity on the X -axis is

$$V_3(X) = pV_{30}X^{p-1} \dots (10)'$$

If it is positive, the fluid flows downward and perpendicular to the X -axis.

The potential flows (b) and (c) of Fig. 6 are derived by replacing p of Eq. (10) with $3p$ and $5p$ respectively, and the complex velocities are

$$u_4 - iv_4 = 3pV_{40}X^{3p-1} \exp\{i\theta(3p-1) + i\pi/2\} \dots (11)$$

$$u_5 - iv_5 = 5pV_{50}X^{5p-1} \exp\{i\theta(5p-1) + i\pi/2\} \dots (12)$$

The magnitude of velocity on the X -axis is then

$$V_4(X) = 3pV_{40}X^{3p-1} \dots (11)'$$

$$V_5(X) = 5pV_{50}X^{5p-1} \dots (12)'$$

The variation of $V_3(X)$, $V_4(X)$ and $V_5(X)$ along the X -axis is respectively different from the variation of $V_1(X)$ along the X -axis. It is therefore impossible to satisfy at all points of the X -axis the condition in section 3, that the tip of combined velocity vector should be on the chain line, even though the magnitudes of V_{30} , V_{40} and V_{50} are respectively adjusted and these three potential flows V_3 , V_4 and V_5 are superposed on the combined flow $V_1 + V_2$. However, everytime a potential flow is superimposed, the number of points on the X -axis where the condition written in section 3 is satisfied, increases by one. This condition is also satisfied at $X=0$. Therefore, superimposition of three

potential flows V_3 , V_4 and V_5 is sufficient in order to satisfy the condition at four points on the X -axis, that is, at both ends $X=0$, $X=1$ and at two points X_1 and X_2 in between.

Parameters V_{30} , V_{40} and V_{50} should be decided such that the following equation may be satisfied at $X=X_1$, X_2 and 1.0

$$\begin{aligned} (V_1 - V_1(0))(\sin \alpha + \cos \alpha \tan \beta) \\ = pV_{30}X^{p-1} + 3pV_{40}X^{3p-1} + 5pV_{50}X^{5p-1} \dots (13) \end{aligned}$$

At the present stage the relationship between V_1 and X is not known. Therefore, V_{30} , V_{40} and V_{50} are not completely decided yet.

6. Determination of flow

On the X -axis the velocity vectors V_1 , V_2 , V_3 , V_4 and V_5 have fixed directions respectively. Therefore, these symbols are used as scalar quantities in the following sections, and they are positive when the direction of flow is indicated by arrows in Fig. 5.

At the end of section 4, it has been explained that dn/dX is the sine of the angle cut by the velocity vector V and the X -axis. On the X -axis it is

$$\frac{dn}{dX} = \frac{V_1 \sin \alpha - V_2 \sin \beta - V_3 - V_4 - V_5}{V} \dots (14)$$

Substituting Eqs. (14) and (8)' into Eq. (7) and arranging the terms with respect to dV_1/dX

$$\begin{aligned} \frac{dV_1}{dX} = \frac{m(m-1)UV_{20}\sin \alpha \cos \beta X^{m-2}}{V_1 \sin \alpha - mV_{20} \sin \beta X^{m-1} - pV_{30}X^{p-1} - 3pV_{40}X^{3p-1} - 5pV_{50}X^{5p-1} - U \sin \alpha \cos \beta} \dots (15) \end{aligned}$$

This equation may be solved as follows. At first the velocity $V_1(1)$, i.e. the velocity V_1 at $X=1.0$, is fixed and the parameter V_{20} is assumed.

In addition, variation of $V_1(X) - V_1(0)$ with respect to X is assumed, then values of V_1 at $X=X_1$ and X_2 are calculated. By applying these assumed values to Eq. (13) at $X=X_1$, X_2 and 1.0, three equations are established. Then the three equations are solved for V_{30} , V_{40} and V_{50} . Using these values we integrate Eq. (15) from $X=0$ to 1.0 and the velocity V_1 at $X=1.0$, or $V_1(1)$, is obtained. If the integrated value $V_1(1)$ is not equal to the originally fixed value $V_1(1)$, the assumed value V_{20} is then adjusted, and the calculation is repeated. The velocity V_1 at $X=X_1$ and X_2 is calculated by integrating Eq. (15) from $X=0$ to X_1 , and X_2 respectively. If these values are considerably different from the assumed values, it is desirable to adjust the assumed values and recalculate V_{30} , V_{40} and V_{50} .

Fig. 6 Examples of potential flow streamlines which meet at right angles with the X -axis

Fig. 7 Comparison of the results of calculation with the assumed values of V_1

Integration of Eq. (15) is considerably simplified if V_1 on the right hand side of the equation is properly assumed. Figure 7 demonstrates the relationship between $V_1 - V_1(0)$ and X for $\alpha = \beta = 30^\circ$ and slip coefficient $\delta = 0.22$. The parameter Q/Q_0 is the ratio of the flow rate to the normal flow rate. Though the relation varies depending upon the flow rate ratio Q/Q_0 , it is always very close to that of $V_1 - V_1(0) = X^{m-1}[V_1(1) - V_1(0)]$ as indicated by the broken line in Fig. 7.

In the present theory, the variation of V_1 with respect to X is included in expressing the distribution of vortices or in expressing the distribution of total head along the X -axis, while the variation of work delivered by the impeller along the periphery is expressed with V_u . Since V_u is related to only V_1 and V_2 as Eq. (8), there is an intimate relationship between V_1 and V_2 . It is reasonable to assume that $V_1(X) - V_1(0)$ is approximately proportional to X^{m-1} or V_2 .

As $V_1(X) - V_1(0)$ is approximately proportional to X^{m-1} , V_1 on the right hand side of Eq. (15) may be approximated as

$$V_1 = V_1(0) + \frac{(pV_{30} + 3pV_{40} + 5pV_{50})X^{m-1}}{\sin \alpha + \cos \alpha \tan \beta} \quad \text{.....(16)}$$

This equation obviously satisfies Eq. (13) at $X = 1.0$. In several numerical examples it is demonstrated that the error due to the approximation of Eq. (16) is negligibly small.

Since parameters V_{20} , V_{30} , V_{40} and V_{50} are already established, V_1 is calculated as a function of X by integration of Eq. (15). When these values are used, the complex velocity at any point in the wedge shape space is calculated as the sum of the five complex velocities, and using the velocity distribution, streamlines can be drawn.

7. Working equations of velocity, pressure and streamlines

The velocity V_1 at a point $X + iY$ in the wedge shape space is identical to the velocity V at the point $X - Y \cot \alpha$ on the X -axis. This velocity is indicated as $V_1(X - Y \cot \alpha)$.

The X -component of the combined velocity V at a point $X + iY$ is

$$u = V_1(X - Y \cot \alpha) \cos \alpha + u_2 + u_3 + u_4 + u_5 \quad \text{.....(17)}$$

The Y -component of the combined velocity V is given by

$$v = V_1(X - Y \cot \alpha) \sin \alpha + v_2 + v_3 + v_4 + v_5 \quad \text{.....(18)}$$

where $u_2, v_2; u_3, v_3; u_4, v_4; u_5, v_5$ are the X - and the Y - components of complex velocities of Eqs. (3), (10), (11) and (12) respectively. The symbols r and θ in these equations are

$$r = \sqrt{X^2 + Y^2}, \quad \theta = \tan^{-1} Y/X \quad \text{.....(19)}$$

We are most concerned with the velocity and

the pressure on the X -axis, i.e. along the impeller periphery. If we accept the approximation of Eq. (16) for V_1 instead of the integrated value of Eq. (15), the velocity component normal to the X -axis V_m will be

$$V_m = V_1(0) \sin \alpha + (G \tan \alpha - mV_{20} \sin \beta) X^{m-1} - pV_{30} X^{p-1} - 3pV_{40} X^{3p-1} - 5pV_{50} X^{5p-1} \quad \text{.....(20)}$$

where

$$G = p(V_{30} + 3V_{40} + 5V_{50}) / (\tan \alpha + \tan \beta) \quad \text{.....(21)}$$

The flow rate from the impeller, or the mean value of V_m is obtained by integration of Eq. (20) from $X = 0$ to $X = 1.0$, that is

$$\bar{V}_m = V_1(0) \sin \alpha + (G/m) \tan \alpha - V_{20} \sin \beta - V_{30} - V_{40} - V_{50} \quad \text{.....(22)}$$

Similarly the velocity component parallel to the X -axis V_u is

$$V_u = V_1 \cos \alpha + V_2 \cos \beta = V_1(0) \cos \alpha + (G + mV_{20} \cos \beta) X^{m-1} \quad \text{.....(23)}$$

If M is the amount of momentum in the X -direction which flows into the wedge shape space passing through the unit length of X

$$M = \rho V_u V_m \quad \text{.....(24)}$$

Integration of Eq. (24) from $X = 0$ to 1.0 gives the total amount of momentum which flows into the wedge shape space per unit time. That is,

$$\begin{aligned} \int_0^1 (M/\rho) dX &= V_1(0)^2 \sin \alpha \cos \alpha \\ &- V_1(0) (V_{30} + V_{40} + V_{50}) \cos \alpha \\ &+ \frac{2}{m} V_1(0) G \sin \alpha + V_1(0) V_{20} (\sin \alpha \cos \beta \\ &- \cos \alpha \sin \beta) + (G + mV_{20} \cos \beta) \\ &\times \left\{ \frac{1}{2m-1} (G \tan \alpha - mV_{20} \sin \beta) \right. \\ &\left. - \frac{pV_{30}}{m-1+p} - \frac{3pV_{40}}{m-1+3p} - \frac{5pV_{50}}{m-1+5p} \right\} \quad \text{.....(25)} \end{aligned}$$

The mass average value of the X -component velocity $\bar{V}_u$ is expressed by the ratio of Eq. (25) and Eq. (22), that is

$$\bar{V}_u = \int_0^1 (M/\rho) dX / \bar{V}_m \quad \text{.....(26)}$$

If the total pressure at the pump inlet is assumed as zero, the local pressure P on the X -axis will be given by

$$P = \rho U V_u - \rho V^2 / 2 \quad \text{.....(27)}$$

Streamlines in the wedge shape space may be drawn by the following process. The flow rate between the wall and a point in the wedge shape space is calculated. The calculation is repeated for many points and the points where the flow rate is identical are connected by a line which is now a streamline.

Although the velocity components in the wedge shape space are given as Eqs. (17) and (18), it is simpler to calculate the flow rate between the wall and a point $re^{i\theta}$ for V_1, V_2, V_3, V_4 and V_5

individually and finally sum them up to calculate the flow rate q between the wall and the point. Symbol q_i indicates the flow rate between the wall and the point $re^{i\theta}$ due to the flow V_i .

$$q_1 = \int_0^{X_0} V_1 dX \sin \alpha \quad (28)$$

where

$$X_0 = r \sin(\alpha - \theta) / \sin \alpha = r \cos \theta - r(\sin \theta / \tan \alpha) \quad (29)$$

If V_1 is expressed by Eq. (16)

$$\begin{aligned} q_1 &= V_1(0) X_0 \sin \alpha + \frac{p(V_{30} + 3V_{40} + 5V_{50})}{1 + \cot \alpha \tan \beta} \\ &\times \int_0^{X_0} X^{m-1} dX \\ &= V_1(0) r \sin \alpha (\cos \theta - \sin \theta \cot \alpha) \\ &+ \frac{p}{m} \frac{V_{30} + 3V_{40} + 5V_{50}}{1 + \cot \alpha \tan \beta} r^m (\cos \theta - \sin \theta \cot \alpha)^m \end{aligned} \quad (28)'$$

According to Eq. (3) the angle between the velocity vector V_2 at a point $re^{i\theta}$ and the X -direction is given by $-(m-1)\theta - m\theta_0$, and the angle between the vector V_2 and the radial direction θ from the X -axis is

$$\theta + (m\theta - \theta) + m\theta_0 = m(\theta + \theta_0)$$

Consequently, the component of the velocity vector V_2 in the radial direction, with the angle of θ from the X -axis, is

$$mV_{20}r^{(m-1)} \cos m(\theta + \theta_0)$$

This velocity component is multiplied by r and it is integrated from $\theta = \theta_0$ to α . The result is the flow rate q_2 between the wall and the point $re^{i\theta}$ due to the flow V_2 , that is

$$q_2 = V_{20}r^m \{ \sin m(\alpha + \theta_0) - \sin m(\theta + \theta_0) \} \quad (30)$$

According to Eq. (10) the angle between the velocity vector V_3 at a point $re^{i\theta}$ and the X -axis is $\theta - p(\theta + \alpha)$ and the angle between the velocity vector and the radial direction θ is

$$\theta - \{ \theta - p(\theta + \alpha) \} = p(\theta + \alpha)$$

Consequently, the velocity component of V_3 in the radial direction of θ is

$$pV_{30}r^{(p-1)} \cos p(\theta + \alpha)$$

The flow rate q_3 is obtained by multiplying this velocity component by r and integrating it from $\theta = \theta_0$ to α , that is,

$$q_3 = V_{30}r^p \{ \sin 2p\alpha - \sin p(\theta + \alpha) \} \quad (31)$$

According to Eq. (11) the angle between the velocity vector V_4 at a point $re^{i\theta}$ and the X -axis is $\theta - p(3\theta + \alpha)$ and the angle between the velocity vector and the radial direction θ is

$$3pV_{40}r^{(3p-1)} \cos p(3\theta + \alpha)$$

The flow rate q_4 is obtained by multiplying this velocity component with r and integrating it from $\theta = \theta_0$ to α , that is,

$$q_4 = V_{40}r^{3p} \{ \sin 4p\alpha - \sin p(3\theta + \alpha) \} \quad (32)$$

Similarly,

$$q_5 = V_{50}r^{5p} \{ \sin 6p\alpha - \sin p(5\theta + \alpha) \} \quad (33)$$

Finally,

$$q = q_1 + q_2 + q_3 + q_4 + q_5 \quad (34)$$

Streamline is a line which passes through the points where the flow rate q is identical.

8. Numerical example

Numerical calculations have been made for two impellers with a different blade exit angle β_b and for two wedge shape spaces with a different wedge angle α . The pressure and velocity distributions around the impeller are calculated for several flow rates. Also presented are streamlines in the wedge shape space for some of these cases.

Fig. 8 The velocity vectors at various points on the X -axis

In all of these calculations the slip coefficient δ is assumed as $\delta=0.22$. For $\alpha=30^\circ$ and $\beta=\beta_b=30^\circ$ this slip coefficient corresponds to the work coefficient $H/H_\infty=0.64$ at the normal flow rate, where H is the theoretical pump head and H_∞ is the theoretical pump head with an infinite number of blades.

Exit velocity vectors on the X -axis are shown in Fig. 8, where the abscissa corresponds to the peripheral velocity vector of the impeller. The chain line in the diagram is the same as that indicated in Fig. 2. The tips of all velocity vectors should be on the chain line and the broken line is

the locus of the tips of the calculated velocity vectors.

It is noticed that the velocity at $X=0$ is constant regardless of the flow rate of the pump. This fact may be explained as follows. At $X=0$ the fluid flows along the wall or the absolute velocity vector must be oriented to α direction. Also, the tip of the velocity vector must be on the chain line. The velocity at $X=0$ is then fixed by these two conditions and therefore it is constant.

In the cases of non-normal flow rate, the tip of velocity vector locally deviates from the chain

Fig. 9 The patterns of streamlines at various flow rates

Fig. 10 The patterns of streamlines with different blade exit angle β at $\alpha=30^\circ$

line, but as a whole, it can be said that the tips of velocity vectors are reasonably close to the chain line. When the flow rate is close to the normal flow rate, the tips of velocity vectors at any value

of X are almost on the chain line.

It is also noticed that the variation of velocity vector is small between $X=0$ and $X=0.6$, but that the variation is conspicuous near the end $X=1.0$.

Fig. 11 The patterns of streamlines with different blade exit angle β at $\alpha=20^\circ$

Fig. 12 Variation of the pressure distribution due to the flow rate

The tip of the mass averaged velocity vector is shown with a small open circle in the diagram and it is located close to the tip of the velocity vector at $X=0.65$ for these flow rates. Figure 9 shows the streamlines in the wedge shape space. The streamlines are obviously parallel to the wall at the normal flow rate, although this condition is not presented in Fig. 9. When the flow rate is larger than the normal, a reverse flow is recognized near the wall at the exit of the wedge shape space. When the flow rate is smaller than the normal, a reverse flow zone occurs in the middle of the passage near the exit, and this zone moves upstream as the flow rate decreases.

It is assumed in this analysis that vortices produced by the impeller are carried by the flow V_1 parallel to the wall. When the flow rate is considerably different from the normal flow rate, the reverse flow zone shifts the streamlines. However, since the streamlines are almost parallel to the wall near the X -axis, the accuracy of this theory may not be poor for estimation of pressure and velocity at the exit of the impeller.

Streamlines for different blade angle β and for different wedge angle α are shown in Figs. 10 and 11. It can be recognized from these figures that the influence of flow rate on the flow pattern in the wedge shape space is larger when the blade angle is smaller.

The pressure distribution along the X -axis is shown in Fig. 12. The parameter in the figure is the flow rate ratio. The calculation did not converge at the flow rates outside of the indicated parameter range. It becomes plain that the pressure at $X=0$ is constant regardless of the flow rate and that the pressure variation along X is little near $X=0$ while it is considerably large near the exit of the wedge shape space.

Comparing Fig. 12 (a) with Fig. 12 (c) or Fig. 12 (b) with Fig. 12 (d), it is recognized that for a given value of β the variation of pressure at $X=1.0$ due to the change of flow rate ratio Q/Q_0 becomes more conspicuous as the wedge angle α increases. Also it is noticed that as the wedge angle increases the normal flow rate Q_0 increases and the normal pressure rise decreases. Therefore, the ratio of the variation of pressure to the normal pressure rise becomes much larger as the wedge angle increases.

Similarly comparing Fig. 12 (a) with Fig. 12 (b) or Fig. 12 (c) with Fig. 12 (d), it is recognized that for a given value of α the variation of pressure at $X=1.0$ due to change of flow rate ratio Q/Q_0 becomes somewhat larger as the blade exit angle β decreases. Also it is noticed that as the

blade exit angle decreases the normal flow rate as well as the normal pressure rise decreases. Therefore, the ratio of the variation of pressure to the normal pressure-rise becomes larger as the blade exit angle β decreases.

A simple analysis based upon a one-dimensional flow model demonstrates that the variation of volute exit pressure due to change of flow rate depends upon the blade exit angle β as well as the volute size or the wedge angle α . The relationship qualitatively agrees with the conclusions based upon Fig. 12.

9. Comparison with an experiment

Figure 13 depicts a figure in reference 5. In the reference, the figure is presented as a typical example of pressure distribution at the impeller exit of a volute pump. Although dimensions and shape of both the impeller and the volute casing are not indicated, we can understand the trend of pressure distribution and the relationship between the pressure variation and the flow rate. In this figure, the pressure at the impeller exit is the ordinate, the flow rate is the abscissa, and the parameter is the circumferential location.

For the range of flow rates between 0.8 and 1.1, which corresponds to the range analyzed in this theory, the pressure at stations 2 and 4, which are located near the beginning of the volute casing, is not influenced by the flow rate and the pressure varies mostly near the exit of the volute casing that is shown between stations 8 and 12. The trend agrees well with the theoretical prediction.

A real volute casing has a three-dimensional geometry and mixing takes place in it. Besides, the clearance between the impeller periphery and the tang may influence the pressure distribution. Though the present theory does not include these effects, the theory qualitatively agrees with the experimental results reported in reference 5.

Fig. 13 Relationship between the pressure distribution and the flow rate of a volute pump

10. Conclusion

An analysis is made under the assumption that the local exit velocity and the local total pressure delivered by an impeller depend upon the local static pressure at the impeller periphery, and theoretically examined is the variation of pressure and velocity distributions around the impeller due to flow rate. In the analysis, the impeller periphery is replaced by a straight line, the volute casing by a two-dimensional wedge shape space, and four types of potential flow and one flow containing vortices are superimposed in order to satisfy the condition at the impeller periphery as well as the wall boundary condition.

The results of calculation show that the pressure near the tang is almost constant regardless of the flow rate, and that variation of pressure along the periphery is small near the tang but it is large near the exit of the volute casing at any other flow rate than normal. These facts qualitatively agree

well with experimental results reported in the literature.

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