621. 822. 033 : 534. 113

On the Allowable Unbalance of a Rigid Rotor Supported in Externally Pressurized Gas Journal Bearings*

By Kyosuke ONO**

A rigid rotor which is supported in externally pressurized gas journal bearings has resonant speeds. When the width of the bearing is small and the bearing clearance is large, the damping force of the bearing is very small at resonance. Then the rotor can not be run up over the resonant speeds, unless an unbalance of the rotor is substantially removed. This paper, therefore, investigates the resonant characteristics of the gas bearing and the allowable unbalance of the rotor theoretically and experimentally in relation to the dimensional and operational conditions of the bearing. Theoretical results are found in good agreement with experimental ones for the radial clearances over 0.03 mm. An available design chart with which the allowable unbalance of rotor could be known is presented.

1. Introduction

Whirl instability of externally pressurized gas journal bearing has been studied by several researchers^{(1)~(3)} and some design methods have been already presented from a viewpoint of dynamical characteristics⁽⁴⁾.

If we want to know the dynamical problems to be investigated, we have only to run a rotor supported in externally pressurized gas journal bearings. Figure 1 shows the amplitudes of the whirling motion of a rotor versus the rotating speed at both ends of the rotor. It is seen from this figure that the rotor has two resonant speeds. Although the

behavior of resonant whirl depends on the bearing conditions and the distribution of the unbalance mass of the rotor, observation makes us notice a fact that the rotor whirls in different modes of motion, namely in conical mode and parallel mode. In this test apparatus the first resonance is in conical mode and the second is in parallel respectively.

If the bearing clearance is large and the width of the bearing is small, the amplitude of the whirling motion of the journal becomes very large at resonance, and, therefore, even such a difficulty as that the rotor speed can not be raised over the resonant speeds takes place. Accordingly, it will be of great value and necessary to make clear the relation between the whirl amplitude of the journal at resonance and the bearing conditions.

If the rotor speed is increased further over the two resonances, the rotor begins to exhibit unstable whirl at last. The threshold to this unstable whirl has been already investigated and agreement between theory and experiment has been obtained by the author and other researchers. Then this paper undertakes to clarify the resonant characteristics of a rotor supported in externally pressurized gas journal bearings and presents a method to calculate the allowable unbalance of a rotor for the safe acceleration of the rotor speed through resonance.

Fig. 1 Amplitude characteristics of vibration of journal

* Received 25th March, 1969.

** Research Fellow, Electric Communication Laboratory, Musashino.

2. Nomenclature

- A : rate of amplitude of the whirling motion of journal to radial clearance of bearing
 $A_{\max}$: maximum value of A at resonant speed of rotor
 C_r : radial clearance of bearing mm
 F_x, F_y : reacting forces of air film in x and y directions respectively kg
 J : moment of inertia of rotor kg·mm·sec²
 L : width of bearing mm
 N : number of feeding holes of supply gas
 P_a : ambient pressure kg/mm²
 P_s : supply pressure kg/mm²
 R : bearing radius mm
 R_0 : gas constant kg·mm/°K/kg
 T_i : absolute temperature in bearing °K
 C_0 : flow coefficient of feeding hole
 d : diameter of feeding hole mm
 g : acceleration of gravity mm/sec²
 l : distance between two bearings mm
 m : additional unbalance mass kg·sec²/mm
 p_s : nondimensional supply pressure ($=P_s/P_a$)
 p_{0i} : nondimensional pressure in feeding plane of bearing
 p_m : nondimensional mean pressure in bearing
 t : time sec
 $O-xy$: nondimensional fixed coordinates
 $\Gamma = \frac{6\mu C_0 d N}{P_a C_r^2} \left(\frac{L}{2R} \right) \sqrt{g R_0 T_i}$
 Ω : angular natural frequency of rotor at non-rotating speed rad/sec
 Ω_r : angular resonant speed of rotor rad/sec
 δ : phase shift of unbalance force to displacement of journal
 ϵ : rate of eccentricity of center of gravity of

- journal to radial clearance of bearing
 ϵ_a : nondimensional allowable eccentricity of center of gravity
 μ : coefficient of viscosity of gas kg·sec/mm²
 $\lambda = 6\mu \left(\frac{R}{C_r} \right)^2 \frac{\omega}{P_a}$
 ω : angular rotational speed of rotor rad/sec

3. Analysis

3.1 Method of analysis

This study treats an externally pressurized gas journal bearing which has a peripheral row of feeding holes in its center plane.

Now let us consider the method of analysis so as to know the allowable unbalance of a rigid rotor for safe acceleration of the rotor speed through resonance in relation with the bearing conditions. The unbalance force applied to the bearing can be considered to be caused by an equivalent mass attached at the journal of the rotor. When we balance a rotor, on the other hand, we are used to choosing journal position as a measuring plane and to removing the equivalent unbalance mass at the journal until the unbalance force at the journal reduces to an allowable limit. For this reason, it will be reasonable that the resonant characteristics of the journal supported in an externally pressurized gas journal bearing is made clear in relation to an equivalent mass of the journal. Thus, analysis is made about an unbalance vibration of the rotor-gas bearing system which has one degree of freedom as shown in Fig. 2.

3.2 Analysis of unbalance vibration

As shown in Fig. 2, consider a balanced rigid rotor, one end of which is supported freely and the other end of which is supported in an externally pressurized gas journal bearing. Denote the moment of inertia of the rotor about the supporting position II by J and an unbalance mass attached to the point at an axial distance l_1 from the supporting position II and a radial distance r from the center of the rotor by m .

The reacting force of gas film has already been solved in the previous paper⁽²⁾. If we denote the reacting force components in the $-x$ and $-y$ directions by F_x and F_y respectively, they are expressed as

$$\left. \begin{aligned} F_x &= 2\pi R^2 P_a p_m \left\{ \left(\frac{\chi}{p_m} a_1 + b_1 \right) x + \left(\frac{\chi}{p_m} a_2 + b_2 \right) y + \frac{2b_2}{\omega} \dot{x} - \frac{2b_1}{\omega} \dot{y} \right\} \\ F_y &= 2\pi R^2 P_a p_m \left\{ \left(\frac{\chi}{p_m} a_1 + b_1 \right) y - \left(\frac{\chi}{p_m} a_2 + b_2 \right) x + \frac{2b_2}{\omega} \dot{y} + \frac{2b_1}{\omega} \dot{x} \right\} \end{aligned} \right\} \dots\dots\dots (1)$$

where

Fig. 2

$$\begin{aligned}
 a_1 &= \frac{(e^{L/2R} + 1) \left(\cosh \frac{\alpha L}{R} + \cos \frac{\beta L}{R} - 2 \cosh \frac{\alpha L}{2R} \cos \frac{\beta L}{2R} + 2 \frac{\lambda}{p_m} \sinh \frac{\alpha L}{2R} \sin \frac{\beta L}{2R} \right)}{(e^{L/2R} - 1)^2 (1 + \lambda^2/p_m^2) \left(\cosh \frac{\alpha L}{R} + \cos \frac{\beta L}{R} \right)} \\
 a_2 &= \frac{(e^{L/2R} + 1) \left\{ 2 \sinh \frac{\alpha L}{2R} \sin \frac{\beta L}{2R} - \frac{\lambda}{p_m} \left(\cosh \frac{\alpha L}{R} + \cos \frac{\beta L}{R} - 2 \cosh \frac{\alpha L}{2R} \cos \frac{\beta L}{2R} \right) \right\}}{(e^{L/2R} - 1)^2 (1 + \lambda^2/p_m^2) \left(\cosh \frac{\alpha L}{R} + \cos \frac{\beta L}{R} \right)} \\
 b_1 &= \frac{\lambda/p_m}{(1 + \lambda^2/p_m^2)} \left\{ \frac{L}{2R} \frac{\lambda}{p_m} - \frac{(\alpha \lambda/p_m + \beta) \sinh \frac{\alpha L}{R} + (\beta \lambda/p_m - \alpha) \sin \frac{\beta L}{R}}{(1 + \lambda^2/p_m^2)^{1/2} \left(\cosh \frac{\alpha L}{R} + \cos \frac{\beta L}{R} \right)} \right\} \\
 b_2 &= \frac{\lambda/p_m}{(1 + \lambda^2/p_m^2)} \left\{ \frac{L}{2R} - \frac{(\alpha \lambda/p_m + \beta) \sin \frac{\beta L}{R} - (\beta \lambda/p_m - \alpha) \sinh \frac{\alpha L}{R}}{(1 + \lambda^2/p_m^2)^{1/2} \left(\cosh \frac{\alpha L}{R} + \cos \frac{\beta L}{R} \right)} \right\} \\
 \chi &= \frac{(e^{L/2R} - 1)^2 \Gamma p_s \phi}{p_m \left\{ (1 + e^{L/2R}) \frac{L}{2R} + \phi \Gamma p_s \phi (1 - e^{L/2R}) \right\}} \\
 \alpha &= \sqrt{\frac{\sqrt{1 + \lambda^2/p_m^2} + 1}{2}} \quad \beta = \sqrt{\frac{\sqrt{1 + \lambda^2/p_m^2} - 1}{2}} \\
 p_m &= \frac{2(p_{0i}^2 + p_{0i} + 1)}{3(p_{0i} + 1)}
 \end{aligned} \tag{2}$$

and there is the following relation between Γ , p_s and p_{0i} .

$$\frac{p_{0i}^2 - 1}{\phi \cdot p_s} = \Gamma \tag{3}$$

Furthermore, ψ and ϕ are written as

$$\begin{aligned}
 \psi &= \frac{\left\{ \frac{2}{\kappa} (p_{0i}/p_s)^{2/\kappa} - \frac{(1+\kappa)}{\kappa} (p_{0i}/p_s)^{(1+\kappa)/\kappa} \right\}}{4p_{0i}^2 \left\{ (p_{0i}/p_s)^{2/\kappa} - (p_{0i}/p_s)^{(1+\kappa)/\kappa} \right\}} \\
 \phi &= \sqrt{\frac{2g\kappa}{(\kappa-1)} \left\{ \left(\frac{p_{0i}}{p_s} \right)^{2/\kappa} - \left(\frac{p_{0i}}{p_s} \right)^{(1+\kappa)/\kappa} \right\}} \quad p_{0i}/p_s \geq \left(\frac{2}{1+\kappa} \right)^{\kappa/(\kappa-1)} \\
 \phi &= 0 \quad p_{0i}/p_s < \left(\frac{2}{1+\kappa} \right)^{\kappa/(\kappa-1)} \\
 \phi &= \sqrt{g\kappa \left(\frac{2}{1+\kappa} \right)^{(\kappa+1)/(\kappa-1)}}
 \end{aligned} \tag{4}$$

By means of Eq. (1) we obtain the following solution of unbalance vibration of such a rotor as shown in Fig. 2.

$$\begin{aligned}
 x &= A \cos(\omega t - \delta) \\
 y &= A \sin(\omega t - \delta)
 \end{aligned}$$

where

$$\begin{aligned}
 A &= \frac{\varepsilon \omega^2}{\left[\left\{ \omega^2 - \Omega^2 (a_1 - p_m b_1/\chi) \right\}^2 + \Omega^4 (a_2 - p_m b_2/\chi)^2 \right]^{1/2}} \\
 \Omega &= \sqrt{\frac{2\pi R^2 P_a \chi l^2}{J C_r}} \quad \varepsilon = \frac{l_1}{l} \frac{mr}{C_r} \frac{l^2}{J} \\
 \delta &= \arctan \left[\frac{(a_2 - p_m b_2/\chi)}{\left\{ \left(\frac{\omega}{\Omega} \right)^2 - (a_1 - b_1 p_m/\chi) \right\}} \right]
 \end{aligned} \tag{5}$$

From Eq. (5) we can calculate the amplitude and the phase shift of the unbalance vibration of journal.

Figure 3 illustrates the amplitude and the phase shift versus the rotor speed choosing the radial clearance as a parameter in two cases where $R=15\text{mm}$, $L/2R=1.0$, $\Omega=2\pi \times 150\text{ rad/sec}$ and $2\pi \times 300\text{ rad/sec}$. Although p_m varies with C_r and Ω , its value is

assumed to be 1.0. It can be said from Fig. 3 that A/ε takes a maximum value when $\omega=\Omega$ if both C_r and Ω are not so small. It will be also seen that the maximum amplitude at resonance increases considerably with increase of C_r and slightly with increase of Ω . As for the phase shift δ , it takes 90° when ω is slightly smaller than Ω , and this tendency becomes more remarkable as the bearing clearance decreases.

The amplitude of the unbalance vibration of the journal calculated from Eq. (5) can be regarded as a displacement response of the journal to the rotating or alternating load whose amplitude is $l_1 mr\omega^2/l$ and whose angular frequency is ω .

3.3 Analysis of allowable unbalance

If the rotor is to be operated at high speed, it must be run up through the resonant speeds. At that time such a trouble as that the rotor speed can not be raised easily over the resonant speeds because of a large resonant whirl of the journal causing contact between the rotor and the bearing would

take place if the bearing clearance is large and the width of the bearing is small. In order to avoid this trouble, it will be necessary to reduce the resonant amplitude of the journal to a certain limit by removing the unbalance mass of the rotor. Many practical experiences tell us that safe acceleration of the rotor speed is possible if the maximum amplitude remains within a half bearing clearance. Hence, we define here an unbalance which gives the amplitude of the resonant whirl with magnitude of a half bearing clearance as the allowable unbalance

Fig. 3 Amplitude and phase shift versus rotor speed

Fig. 4 Nondimensional allowable eccentricity of center of gravity

and discuss the amount of the nondimensional allowable eccentricity of the center of gravity ϵ_a which gives the allowable unbalance of the journal.

Since the condition described above is expressed as

$$A_{\max} = 0.5 \dots \dots \dots (6)$$

substituting Eq. (6) into Eq. (5), we obtain

$$\epsilon_a = 0.5 \times \left\{ \left[1 - \left(\frac{\Omega}{\omega} \right)^2 (a_1 - b_1 p_m / \chi) \right]^2 + \left(\frac{\Omega}{\omega} \right)^4 (a_2 - b_2 p_m / \chi)^2 \right\}_{\min}^{1/2} \dots \dots \dots (7)$$

Figure 4 shows ϵ_a as a function of the angular resonant speed Ω_r choosing C_r as a parameter. Eccentricity ϵ_a is calculated for the case where $R = 15$ mm and $L/2R = 1.0$ with the assumption that $p_m = 1.0$. It is seen from Fig. 4 that ϵ_a decreases with increase of Ω_r in general, but that it strangely begins to decrease again slightly with increase of Ω_r if C_r is very large.

Now we consider how to calculate ϵ_a of the rotor supported in externally pressurized gas journal bearings in practice. Properly speaking, we should calculate Ω_r from the bearing conditions such as supply pressure, number of feeding holes, diameter of feeding holes and so on. When a rotor-gas bearing system is given in practice, however, we can easily know Ω_r approximately by measuring experimentally the natural frequency of the rotor. Thus it will be better to use the experimental value of Ω_r for the calculation of ϵ_a . Nondimensional mean pressure in the bearing p_m , on the other hand, is calculated from Eqs. (2) and (3) and illustrated in Fig. 5 as a function of P_s and Γ . Eccentricity ϵ_a calculated in this procedure is compared with experimental results in the following.

4. Experiment and consideration

4.1 Apparatus and method of experiment

The experimental apparatus is shown in Fig. 6. Two equivalent bearings are made of brass as one

Fig. 5 Nondimensional mean pressure in bearing

body for the purpose of complete alignment. The distance between them is 115 mm. Nominal size of the air bearing is 30 mm in diameter and 25 mm in width. Pressurized air is fed to the bearing through a peripheral row of feeding holes on the central plane of the bearing. The number of feeding holes is twelve in all and their diameter is 0.8 mm. The pressure of plenum chamber in the bearing shell is measured and defined as the supply pressure. The measurement of the vibration of the rotor is carried out under the conditions in which the supply pressure of one side of the bearing is varied from 0.5 kg/cm² to 2.5 kg/cm² (gauge pressure) whereas that of the other side of the bearing is held constant at 6.0 kg/cm². In such a condition the test apparatus can be symbolized by a single degree of freedom vibrational system as shown in Fig. 2. The moment

of inertia of the rotor about one side of the bearing is 0.566 kg·mm·sec².

The measurement of the displacement of the rotor is made at both ends of the rotor by means of electrostatic capacity type displacement meter and the displacement of the journal is calculated.

In order to make clear the effect of the bearing clearance, four rotors are fabricated which have slightly different diameters of journal from each other as shown in table 1. Rotor had been balanced well before tests, but some residual unbalance of from 0.3 μ to 0.4 μ seems to remain. Unbalance mass is applied at the end of the rotor on the side of the test bearing where $l_1=172$ mm and $r=8.5$ mm.

4.2 Results of experiment

First of all, the amplitude components of the journal motion in the horizontal and the vertical directions were compared with each other in order to check the assumption that the reacting forces of air film are linear and homogeneous to the displacements of the journal in the horizontal and the vertical directions. From the test results it was found that the difference of the amplitudes of the resonant whirl between the horizontal and the vertical directions lies within 10 percent. Then it can be seen that the journal exhibits nearly circular motion at resonance. Therefore, the experimental data only in the horizontal direction are presented in the

①: Test bearing ②: Test rotor ③: Thrust bearing
④: Poles for the measurement of vibration
⑤: Sealing bolt ⑥: Micrometer
Fig. 6 Experimental apparatus

Table 1 Radial clearance mm

		Number of rotor			
		1	2	3	4
Number of bearing	1	0.027	0.037	0.046	0.057
	11	0.026	0.037	0.045	0.058

Fig. 7 Amplitude and phase shift

following.

Figure 7 shows general aspects of the amplitude and phase characteristics of the unbalance motion of the journal. As seen in Fig. 7 (a), it is found that

Fig. 8

(a)

Fig. 9

if the clearance is as small as 0.026 mm, the amplitude becomes maximum when the phase shift is over 90° . This trend is consistent with the analytical result shown in Fig. 3. The reason why the phase shift does not always approach the values of 0° or 180° at the speeds far from resonance seems to be that the center of the cross circle of the rotor in measuring plane displaces slightly from the rotating axis.

Figure 8 shows the amplitude versus the rotor speed taking supply pressure as a parameter when $C_r = 0.027$ mm and 0.057 mm. The same unbalance mass is applied to both the rotors and its equivalent eccentricity of the center of gravity of the journal is 2.25 μ . From this figure it is seen that although the maximum amplitude increases with increase of

(a)

Fig. 10

resonant speed or supply pressure when the bearing clearance is small, it has a tendency to decrease again with increase of resonant speed when the bearing clearance is large. Now, let us illustrate the maximum amplitude versus the resonant speed in Fig. 9 taking the eccentricity of center of gravity as a parameter. The tendencies described above will be noticed more clearly in Fig. 9. The fact that the amplitude takes still a value of 2 or 3 μ even when the additional mass is zero seems to be due to the residual unbalance and the fabrication error of the rotor.

To know the relation between the maximum amplitude and the unbalance mass, maximum amplitude versus eccentricity is shown in Fig. 10 taking the resonant speed as a parameter. It is found from Fig. 10 that when the bearing clearance is small, the increase of maximum amplitude with increase of eccentricity is not linear, but that the relation curves can be approximated by two straight lines. When the bearing clearance is large, on the other hand, the increase of maximum amplitude is proportional to that of eccentricity of the center of gravity.

Then, we illustrate the rate of maximum amplitude to eccentricity of the center of gravity ($=A_{\max}/\epsilon$) as a function of the resonant speed taking the bearing clearance as a parameter in Fig. 11. The reason why $A_{\max}/\epsilon$ is shown by certain domain in Fig. 11 is that it varies with eccentricity when the bearing clearance is not large. The boundary lines between the upper and the lower limit in Fig. 11 corresponds to the solid line and the dotted line in Fig. 10 respectively. It is seen that the calculated results agree well with the experimental ones unless the bearing clearance is very small.

Figure 12 shows the nondimensional allowable eccentricity of the center of gravity ϵ_a which gives the resonant amplitude of 0.5 C_r . When $A_{\max}/\epsilon$

varies with ϵ , the experimental value of ϵ_a is plotted taking the average between the upper and the lower values of $A_{\max}/\epsilon$. It is found that agreement between theory and experiment is good when the radial clearance is larger than 0.03 mm. Since precise balancing would be necessary for the radial clearance larger than 0.03 mm in practice, this analysis is of great use to estimate beforehand the allowable unbalance of rotor. Although ϵ_a decreases generally with increase of the resonant speed, it hardly decreases when C_r is large. Hence, even when the resonant speed of the rotor is varied with the supply pressure, it will be sufficient if ϵ of the journal is reduced below ϵ_a calculated for the case where $\Omega=2\pi \times 300$ rad/sec.

5. Design chart for allowable unbalance of rotor

Although the allowable unbalance of a rigid rotor supported in externally pressurized gas journal bearings can be calculated from Eq. (7), we seek further to make a simple design chart which is applicable to various kinds of the rotor-gas bearings system.

It may be said from Figs. 3 and 7 that if the bearing clearance is not so small, the amplitude of the vibration of the journal becomes maximum when $\delta=90^\circ$ or

$$\omega = \Omega(a_1 - p_m b_1 / \chi) \dots \dots \dots (8)$$

Then, substituting Eq. (8) into Eq. (5), we get

$$A_{\delta=90^\circ} = \frac{\epsilon(a_1 - p_m b_1 / \chi)}{(a_2 - p_m b_2 / \chi)} \dots \dots \dots (9)$$

$A_{\delta=90^\circ}$ implies the nondimensional amplitude of the vibration of the journal when $\delta=90^\circ$. Its value is nearly equal to that of $A_{\max}$ when the bearing clearance is not so small. Applying $A_{\delta=90^\circ}=0.5$ to Eq. (9), we have

$$\epsilon_a = 0.5 \times \frac{(a_2 - p_m b_2 / \chi)}{(a_1 - p_m b_1 / \chi)} \dots \dots \dots (10)$$

Fig. 11

Fig. 12

In order to ascertain whether Eq. (10) is appropriate for calculation of the allowable eccentricity of center of gravity, ϵ_a calculated from Eq. (10) is illustrated in Fig. 13 in comparison with that calculated from Eq. (7) when $L/2R=1.0$ and $p_m=1.0$. It will be found from Fig. 13 that when C_r is larger than 0.03 mm, Eq. (10) may be used in place of Eq. (7). Then, as ϵ_a is a function of three variables of λ/p_m , χ/p_m and $L/2R$, it is shown in Fig. 14 as a function of the bearing parameters λ/p_m and χ/p_m when $L/2R=1.0$.

If a rigid rotor and an externally pressurized gas journal bearing are given, ϵ_a of each journal of the rotor can be known from this chart by the following procedures; Γ is calculated first from Eqs. (3) and (4); p_m is next obtained from Γ and supply pressure P_s by using Fig. 5. Angular resonant speed Ω_r is known approximately by measuring the natural frequency of the journal under the operating condition referenced as in Fig. 2. Nondimensional design parameters λ/p_m and χ/p_m are then calculated from these values by means of the equations:

$$\left. \begin{aligned} \frac{\chi}{p_m} &= \frac{\Omega_r^2 C_r M}{2\pi R^2 P_a p_m} \\ \frac{\lambda}{p_m} &= 6\mu \left(\frac{R}{C_r} \right)^2 \frac{\Omega_r}{P_a p_m} \end{aligned} \right\} \dots\dots\dots (11)$$

Fig. 13 Comparison between Eqs. (7) and (10)

where M is an equivalent mass of the journal.

Accordingly, if ϵ_a is illustrated once in the chart as a function of λ/p_m and χ/p_m for the given value of $L/2R$ by means of Eq. (10), we can easily know ϵ_a for various values of C_r and P_s from this chart.

Further, these analytical results can be applied to such a problem as estimation of the allowable alternating load whose frequency is synchronous to the rotor speed. For the purpose of safe operation even at the resonance, the amplitude of the alternating load must be smaller than F_a which is given by $F_a = \epsilon_a \times M C_r \Omega_r^2$.

6. Conclusion

Resonant characteristics of the journal supported in externally pressurized gas journal bearing and the allowable unbalance mass of the rotor for safe acceleration through resonant speed are investigated theoretically and experimentally. The theoretical results are found in good agreement with the experimental ones when the radial clearance is larger than 0.03 mm. Based on this analysis, a design chart for ready estimation of the allowable unbalance mass of rotor is presented.

References

- (1) J. W. Lund: *Trans. ASME, Ser. D*, Vol. 89, No. 1 (1967), p. 154.
- (2) K. Ono and A. Tamura: *Bulletin of JSME*, Vol. 11, No. 46 (1968), p. 706.
- (3) T. Tatara: *Trans. Japan Soc. Mech. Engrs.*, Vol. 34, No. 259 (1968), p. 560.
- (4) N. F. Rieger: *Text of RPI-MTI Gas Bearing Design Course*, (1967).

Fig. 14 Design chart for ϵ_a