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Pipe Flows of Dilute Aqueous Polymer Solution*

(Part 2, Correlation of the Frictional Characteristics)

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A correlation of the frictional characteristics of dilute polymer solutions is established from the measurements of the velocity profile. The difference of profile from Newtonian fluids is found in the buffer layer. Analysis is made with much attention paid to the difference of the buffer layer. It is assumed that the Toms effect occurs only in the region, in which the shear rate is larger than the critical shear rate $\dot{\gamma}^*$. This critical shear rate $\dot{\gamma}^*$ is dependent on the concentration and the heading tank level under certain condition. Comparisons of the analytical results with the experimental results are made for two different pipe-sizes using solutions of 3.5 to 35 wppm. The results show: 1) Such parameters as the concentration, pipe diameter or heading tank water level have influence on the Toms effect only in unsaturated regions of the Toms effect; and 2) the pipe size which will be used for the model test to predict the characteristics in a large pipe is limited by $\dot{\gamma}^*$.

1. Introduction

In order to understand the mechanism of friction drag reduction by the Toms effect, it is necessary to grasp the internal structure of a flow. In a very dilute solution that has a viscosity almost equal to that of the solvent, the essential mechanism of the Toms effect may be made clear from the mean velocity distribution. The relation between the pipe friction coefficient and the Reynolds number can also be found by the mean velocity distribution. Although there are many studies on the Toms effect using the mean velocity distribution, a systematic grasp of the Toms effect has not been made sufficiently for the following reasons. In a very dilute solution, the Toms effect is so small that there is no distinct difference from the solvent. On the other hand, in a thick solution the Toms effect is large, but, it becomes difficult on account of extra-stress to measure the velocity. As an example of measurement of the velocity distribution which covers a wide range of concentrations, there is Wells⁽¹⁾ experiment with guar gum solutions. Ernst⁽²⁾, Shaver et al.⁽³⁾ carried out experiments with CMC solutions and Goren et al.⁽⁴⁾ with polyethylene oxide solutions. Meyer⁽⁵⁾ and Ernst⁽⁶⁾ proposed the correlation equation of the pipe friction coefficient to the Reynolds number in which account was taken of the difference of the

Toms effect due to the difference of pipe diameters. Virk et al.⁽⁷⁾, using an "effective slip" model, pointed out that the velocity profile is very similar to that of Newtonian fluids except in the wall vicinity. These studies are based on the fact that the velocity profile is similar to that of Newtonian fluids. That is to say, that the Kármán constant of logarithmic velocity distribution is unchanged. These analyses are not applicable, however, when the velocity profile of the polymer solution is no more similar to that of the Newtonian fluid in a whole layer on account of the reduction of turbulent core which is caused by increased drag reduction. Since the production and the dissipation of turbulent energy are important factors for the Toms effect, it is unreasonable to grasp the phenomena only in terms of the turbulent core where the production and the dissipation of turbulent energy are considerably small. Without making clear the structure of the inner layer, the Toms effect cannot be well understood. Therefore, it is necessary to obtain the detailed knowledges of the inner layer.

In this paper, the buffer layer is grasped in details through a systematic velocity distribution measurement using a very dilute aqueous solution of polyethylene oxide (trade name, Alkox-C). On the basis of the above results, it is tried to derive the correlation equation of the pipe friction coefficient. According to this equation, various phenomena caused by the Toms effect can be well explained externally and the analytical results obtained by this

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equation seem to be in good agreement with the experimental results.

2. Measurement of velocity distribution

In flow of the polymer solutions an apparent dynamic pressure p_d' obtained by a Pitot tube method can be expressed as follows with the aid of Eqs. (22) and (23)

$$p_d'(r) = P_t(r, x) - P_w(x) = p_d(r) - \{P_{11}(r, x) - P_{22}(a, x)\} \quad (1)$$

Usually, the extra-stress term, $\{P_{11}(r, x) - P_{22}(a, x)\}$ is positive, thus p_d' is smaller than the true dynamic pressure p_d . Therefore, the possibility that the flow velocity can be obtained by the Pitot tube method depends upon the ratio of the extra-stress term to the dynamic pressure. With an increase of shear rate, the extra-stress term increases, so that the extra-stress term increases in the wall vicinity. That is, the error due to the use of p_d' is large in the wall vicinity. On the other hand, the true dynamic pressure p_d obtained by the Pitot tube is larger than the dynamic pressure at the point of measurement, since the dynamic pressure measured by the Pitot tube in shear flow is one on the higher velocity side by e . Young et al.⁽⁶⁾ gave the displacement e for a cylindrical Pitot tube by the following equation,

$$\left. \begin{aligned} e/D' &= 0.131 + 0.082d/D' \\ (D'/p_d)(dp_d/dy) &= 0.1 \sim 1.2 \end{aligned} \right\} \quad (2)$$

where D' is the outside diameter of the Pitot tube, d the inside diameter of the Pitot tube and y the distance from the wall. From Eq. (1), in the wall vicinity, p_d' is smaller than p_d due to the extra-stress and from Eq. (2) p_d is larger than that of the point of measurement. Then, it is thought that both effects cancel each other and the error becomes small. In the present paper, the velocity distribution calculated from p_d' is presented only when the mean velocity $\bar{u}$ calculated from p_d' agrees with the velocity $\bar{u}$ determined by the flow rate measurement of the weight method within four per cent error. Table 1 shows the error of the average velocity $\bar{u}$ calculated by p_d' relative to $\bar{u}$ and the error of the average velocity $\bar{u}'$ calculated by p_d' under considera-

tion of correcting the measured point by Eq. (2) relative to $\bar{u}$. Since quantities corrected by Eq. (2) are always negative, the corrections to 15 wppm solutions are not made. It is inferred that the extra-stress term is fairly large in 15 wppm solution. Therefore, a correction by Eq. (2) is made to the extent of 7 wppm solution.

3. Experimental apparatus and method

Experimental apparatus is the same as described in the previous paper⁽¹²⁾. The measurement of velocity distribution is made only in a large pipe of 24.82 mm I.D. A Pitot tube is a stainless steel tube with 1 mm O.D. and 0.7 mm I.D. The measurement of velocity distribution is made at a position of 180 diameters downstream from the pipe inlet using a traversing mechanism on which the position can be read to the extent of 0.025 mm. At the same time, a dial gauge is used. The contact of the Pitot tube with the wall is confirmed electrically and visibly. On account of the facility being of the intermittent flow type, the measurement cannot be made continuously. Therefore, the traversing measurement is made only on a half diameter. From preliminary experiment with water shown in Fig. 3, the distribution is assured to be symmetrical.

4. Experimental results

Figures 1 through 3 show the universal velocity distributions which are non-dimensionalized by the friction velocity u_* and the kinematic viscosity ν , where $u_* = \sqrt{\tau_w/\rho}$, τ_w is the wall shear stress and ρ is the density. Both ν and ρ are the properties for water estimated at solution temperature. Thus, $u^+ = u/u_*$, $y^+ = yu_*/\nu$, where u is the local mean velocity at a distance y from the wall.

As for water, measurement in the viscous sub-layer and the buffer layer is not made in the present experiment. In the turbulent core of water, the velocity distributions agree well with the logarithmic velocity distribution.

$$\left. \begin{aligned} u^+ &= A \ln y^+ + B, \quad y^+ \geq 30 \\ A &= 1/\kappa = 2.5, \quad B = 5.5 \end{aligned} \right\} \quad (3)$$

Table 1 Accuracies of velocity distribution measurement

Water			Alkox-C 7 wppm			Alkox-C 15 wppm	
R_e	$\frac{\bar{u}-\bar{u}'}{\bar{u}}, \%$	$\frac{\bar{u}'-\bar{u}}{\bar{u}}, \%$	R_e	$\frac{\bar{u}-\bar{u}'}{\bar{u}}, \%$	$\frac{\bar{u}'-\bar{u}}{\bar{u}}, \%$	R_e	$\frac{\bar{u}-\bar{u}'}{\bar{u}}, \%$
10 620	2.36	1.51	8 721	0.94	-1.79	8 152	-1.46
11 800	0.70	0.35	9 946	3.10	0.04	9 238	-3.60
12 300	1.35	-0.18	11 320	2.43	-1.51	11 760	-2.84
15 100	0.68	-0.30	15 360	0.37	-2.41	15 910	-0.26
18 260	4.58	1.66	22 580	2.00	-0.27	21 090	-0.56
20 150	0.29	-0.50	27 840	1.21	-1.21	24 800	-2.42
24 150	0.62	-0.09	31 150	-0.20	-0.87	30 000	-2.81
27 850	-1.72	-2.25	37 590	2.90	-0.64	38 470	-2.36

Fig. 1 Universal velocity distributions

Fig. 2 Universal velocity distributions

Fig. 3 Universal velocity distributions

where κ is the Kármán constant. In Fig. 1 through 3, three kinds of curves are drawn by solid lines. One of them expresses the distribution in the viscous sublayer which is given by

$$u^+ = y^+ \dots\dots\dots (4)$$

The other is given by Eq. (3). The last one is given by the following equation that is assumed to be valid for all of the solutions:

$$u^+ = 10 \ln y^+ - 13 \dots\dots\dots (5)$$

In these figures one more curve is presented by a dashed line which expresses the following relation

$$u^+ \sim (1/\kappa) \ln y^+ \dots\dots\dots (6)$$

As is apparent from the experimental results Eq. (5) expresses one of distributions in the buffer layer. When the polymer solution is sufficiently dilute and therefore the drag reduction is small, u^+ in the most portions of a pipe section can be represented by Eq. (6) as is shown for the case of 0.5 wppm solution in Fig. 1. In this case, however, it is not clear whether the buffer layer by Eq. (5) exists or not. The results for 7 wppm solution with a high inlet water level which is shown in Fig. 2 include the buffer layer of Eq. (5). With an increasing Reynolds number, the turbulent core decreases and the transition point from the buffer layer to the turbulent core becomes more distinct. The inlet water level effect which is described in the previous paper appears clear in the velocity distribution. When the inlet water level is high, the turbulent core is small. In the results for 15 wppm solution shown in Fig. 3, the velocity distribution obtained by experiment near the wall is larger than that given by Eq. (4), maybe due to no correction of the measured point by Eq. (2). When correcting the experimental values by Eq. (2), the velocity distributions agree approximately with those calculated by Eqs. (4) and (5). At a high inlet water level, the turbulent core of Eq. (6) almost disappears. Since 15 wppm solution is no longer a very dilute solution, the viscoelasticity may exist on a large scale and will affect the mechanism of turbulent mixing. On the other hand, a polymer molecule will be able to behave as an isolated particle only in a very dilute solution unless the velocity gradient is large. Thus it will be concluded that in the turbulent core the mixing mechanism is controlled by the solvent. Therefore, the turbulent intensities which are normalized by the friction velocity not the local mean velocity will be larger than that of water.

From the above consideration the velocity distributions of polymer solutions are summarized as follows:

viscous sublayer

$$u^+ = y^+, \quad y^+ \leq \delta_v^+ \quad (4)$$

buffer layer

$$u^+ = A_1 \ln y^+ - B_1, \quad \delta_v^+ \leq y^+ \leq \delta_i^+ \quad (7)$$

$$u^+ = A_1' \ln y^+ + B_1', \quad \delta_i^+ \leq y^+ \leq \delta_i'^+ \quad (8)$$

turbulent core

$$u^+ = A_2 \ln y^+ + B_2, \quad \delta_i'^+ \leq y^+ \quad (9)$$

where $\delta_v^+ = 10$, $A_1 = 10$, $B_1 = 13$ and $A_2 = 1/\kappa = 2.5$ which are determined by experiments. In a state of large drag reduction, the buffer layer of Eq. (8) is very small. With increase of concentration, the turbulent core becomes different from that for Newtonian fluids.

5. Derivation of correlation equation of pipe friction coefficient

In practice, what is to be known will be the relation between the pipe friction coefficient and the Reynolds number and a method of utilization of experimental results obtained for a certain pipe diameter. That is, a correlation equation of the pipe friction coefficient is demanded. The correlation equation by Meyer et al.⁽⁵⁾ is based on Eq. (9). They regarded the difference ΔB between B_2 and $B = 5.5$ as a function of the friction velocity and two constants peculiar to the solutions, where B is the equivalent value to B_2 for the Newtonian fluid and $\Delta B = B_2 - B$.

Now derive the equation. In this paper, the kind of polymer added is not considered. The velocity distributions in a pipe are approximated by Eqs. (4), (7) and (9). The correlation equation is derived by integrating Eqs. (4), (7) and (9) with respect to cross section and making the result equal to the flow rate. That is,

Fig. 4 Universal velocity distribution for polymer solution

$$Q = \pi a^2 \bar{u} = \int_{\delta_v}^a u(y) 2\pi r dy + \int_{\delta_i}^{\delta_i'} u(y) 2\pi r dy + \int_{\delta_i'}^a u(y) 2\pi r dy \quad (10)$$

Where A is the pipe radius, $\bar{u}$ the mean flow velocity, and $a = y + r$. In Eq. (10), δ_v^+ , A_1 and B_2 are determined by experiments. But δ_i^+ and B_2 have yet to be determined. Since the velocities by Eqs. (7) and (9) should agree at $y = \delta_i$, B_2 is determined by the following equation.

$$B_2 = (A_1 - A_2) \ln \delta_i^+ - B_1 \quad (11)$$

Still δ_i remains unknown. In order to find the value of δ_i it is necessary to find out the condition under which the velocity distribution changes from Eqs. (7) to Eq. (9) at $y = \delta_i$. Now, since the point of δ_i is far away from the viscous sublayer, the effect of viscosity on the phenomena at this point may be considered to be negligible. The absolute velocity at this point is not determined only by momentum and density at the same point, but also is dependent on the viscous sublayer. Therefore, the kinematical quantity which describes the phenomena at δ_i is not the velocity itself, but is the velocity difference or its gradient which can be considered with the influences of viscous sublayer excluded. On the other hand, when considering the turbulent suppression by polymer molecules, the dynamical quantity considered in the first place is shear rate. In a pipe flow, it is equivalent to the velocity gradient. Therefore, the condition at $y = \delta_i$ is given by the velocity gradient. Further, the following assumption is made for the velocity gradient at this point: In the fully developed flow, the drag reduction by polymer molecules occurs only where the shear rate is over the critical shear rate, γ^* , peculiar to the solving state of polymer molecules.

This assumption may be an extension of Virk's onset point hypothesis⁽¹⁾. The difference between Virk's hypothesis and the present one is in the fact that γ^* is used instead of the wall shear stress $\tau_w^* = \mu \gamma_w^*$ at onset point, and that the Toms effect occurs in the region where $\gamma \geq \gamma^*$. In this way, the condition at $y = \delta_i$ may be given by the following equation.

$$\left. \frac{du}{dy} \right|_{y=\delta_i} = \gamma^* \quad (12)$$

There is a problem about which of Eqs. (7) and (9) should be adopted to calculate the velocity gradient. The region of the velocity distribution given by Eq. (7) is taken to be larger than the real region. So for the purpose to suppress γ^* by a large value, the velocity gradient is assumed to be determined by Eq. (7)

$$\gamma^* = \left. \frac{du}{dy} \right|_{y=\delta_i} = \frac{A_1 u_*}{\delta_i} = \frac{A_1 u_*^2}{\nu \delta_i^+} \quad (13)$$

Therefore, δ_i is expressed by unknown quantity u_* and constants γ^* and A_1 . Upon integrating Eq. (10) using the above equation, the following equation results.

$$\begin{aligned} & \left(\frac{A_1 - A_2}{4} \right) \left(\frac{A_1 \nu}{\gamma^* a^2} \right)^2 R_a^4 - (A_1 - A_2) \left(\frac{A_1 \nu}{\gamma^* a^2} \right) R_a^3 \\ & + \left\{ \left(\frac{A_1 - A_2}{2} \right) \ln \left(\frac{A_1 \nu}{\gamma^* a^2} \right) - \frac{3}{4} A_2 - \frac{B_1}{2} \right\} R_a^2 \\ & + \left(A_1 - \frac{A_2}{2} \right) R_a^2 \ln R_a + \left(\frac{\delta_v^{+2}}{2} - \frac{R_e}{4} \right) R_a \\ & - \delta_v^{+2} \left(\frac{\delta_v^+}{3} - \frac{A_1}{2} \ln \delta_v^+ + \frac{A_1}{4} + \frac{B_1}{2} \right) = 0 \quad \dots (14) \end{aligned}$$

where $R_a = u_* a / \nu = R_e \sqrt{\lambda/32}$, $R_e = 2\bar{u}a/\nu$. Since A_1 , B_1 , δ_v^+ and γ^* are determined by experiment, in Eq. (14) the Reynolds number R_e is a parameter and R_a is unknown. Therefore, by giving R_e , λ is determined. Equation (14) is non-dimensional, so it can be applied to an arbitrary pipe diameter. Now consider about Eq. (14).

i) The applicable range of Eq. (14)

Consider the upper limit of the applicable range of Eq. (14); δ_i cannot be larger than the pipe radius a . The limit is given by Eq. (13), that is,

$$\delta_i = A_1 u_* / \gamma^* < a \quad \dots (15)$$

The contribution of the core region to flow rate is small. Reynolds number which is determined by the following equation.

$$A_1 u_* / \gamma^* = a, \quad R_a = u_* a / \nu = a^2 \gamma^* / A_1 \nu \quad \dots (16)$$

is the maximum Reynolds number where Eq. (14) can be applied. It may be concluded that the Toms effect saturates at this Reynolds number. In the flow with a larger than this Reynolds number, the velocity distribution in a pipe may be expressed by Eqs. (4) and (7), so Eq. (14) is reduced to the following form. Since the sixth term of the left hand side of Eq. (14) is smaller than the other terms, it can be neglected.

$$\begin{aligned} & - \left(\frac{3}{4} A_1 + \frac{B_1}{2} \right) R_a + \left(\frac{\delta_v^{+2}}{2} - \frac{R_e}{4} \right) \\ & + \left(\frac{A_1}{2} \right) R_a \ln R_a = 0 \quad \dots (17) \end{aligned}$$

Further, when $\delta_v^{+2}/2$ in the second term is negligible in relation to R_e , Eq. (17) becomes as follows from $R_a = R_e \sqrt{\lambda/32}$,

$$\begin{aligned} & \frac{1}{\sqrt{\lambda}} = \frac{A_1}{\sqrt{8}} \ln R_e \sqrt{\lambda} \\ & - \left(\frac{1}{\sqrt{8}} \right) \left\{ \left(\frac{5}{2} \ln 2 + \frac{3}{2} \right) A_1 + B_1 \right\} \quad \dots (18) \end{aligned}$$

Equation (17) and (18) do not contain γ^* , thus they are independent of the concentration and the inlet water level of heading tank. Further, these equations do not contain the pipe diameter, so they are independent of the pipe diameter. For the saturated flow described by Eqs. (17) or (18), λ can be determined only by the Reynolds number as

Newtonian fluids. Accordingly, Eq. (17) gives the maximum drag reduction which can be attained by Alkox-C solutions. When the pipe diameter and Reynolds number are given, the most effective solution using Alkox-C is one that has γ^* which satisfies Eqs. (16) and (17) at the same time.

The next step is to consider the lower limit of the applicable range of Eq. (14). When the Toms effect does not occur in a very dilute solution, the pipe friction of the solution can not be larger than that of the solvent. Therefore Eq. (14) can be applied to the extent of the Reynolds number at which the turbulent pipe friction by Eq. (14) agrees with that for Newtonian fluids.

By Blasius' equation,

$$\lambda = 0.316 4 R_e^{-0.25} \quad \dots (19)$$

the lower limit Reynolds number satisfies Eqs. (14) and (19) at the same time.

ii) Determination of γ^*

γ^* is dependent on the concentration and the water level of heading tank under certain solutions. Then, it is necessary to find out γ^* for each solution. The velocity gradient which is equal to γ^* can be regarded equal to that at the onset point.

That is, it can be determined by the pipe friction of the onset point and the Reynolds number. The Toms effect does not always occur from the turbulent flow given by Eq. (19). However, Eq. (14) is available up to the intersecting point of Eq. (14) with Eq. (19). Thus, it makes no great difference to use the Reynolds number at which the extrapolated curve of λ - R_e given by experiments intersects with the curve of Eq. (19). Therefore, γ^* is determined by the following equation.

$$\gamma^* \cong \gamma_w = \frac{du}{dy} \Big|_{y=0} = \frac{\lambda R_e^{*2} \nu}{8 D^2} = \frac{(0.316 4)}{8 D^2} \nu R_e^{*1.75} \quad \dots (20)$$

It is true that γ^* should be determined from the velocity distributions as the intersecting point of Eq. (7) with Eq. (9) based on the definition of Eq. (12), but it is difficult in practice as shown later.

6. Comparison with experimental results

Equations (14) and (17) are compared with the experimental values. Since it may be assumed that A_1 , B_1 and δ_v^+ are independent of the concentration, both the effects of the concentration and the inlet water level are expressed only in terms of γ^* . According to the experimental results and Eq. (16), when the pipe diameter is small, the Toms effect saturates earlier, so it is very rare for γ^* to be found out clearly. This fact makes difficult the application of the data obtained in a small pipe to the case of a large one.

All values of γ^* are found out by Eq. (20). In Table 2, values of γ^* are shown. Since Eq. (20) is not concerned with Eq. (14), it cannot be said that the curve of Eq. (14) passes the point (λ, R_e) given by Eq. (20). Figure 5 shows the relation between Reynolds number and γ^* obtained from the velocity distributions for 7 wppm solution with the high inlet water level. When the Reynolds number is comparatively small, γ^* is nearly constant and almost equal to the value by Eq. (20). As is seen above, γ^* can be determined by the velocity distribution only under very restricted conditions.

Table 2 Critical shear rate γ^* by Eq. (20) with D 24.82 ϕ

Concentration wppm	Inlet water level mm	R_e	γ^* sec $^{-1}$
3.5	135	11 000	764
	735	4 000	129
7	135	6 000	266
	735	2 200	45.4
15	135	3 000	78.8
	735	1 630	27.0
35	—	1 350	19.4

Fig. 5 Critical shear rate for 7 wppm

Fig. 6 Comparison of the predicted correlation of frictional characteristics with experimental results for 3.5 wppm Alkox-C solution in water

Comparisons with experimental results are made for 3.5 wppm, 7 wppm, 15 wppm and 35 wppm solutions, which are shown in Fig. 6 through Fig. 9. For 3.5 wppm solution in Fig. 6 Eq. (14) is in good

Fig. 7 Comparison of the predicted correlation of frictional characteristics with experimental results for 7 wppm Alkox-C solution in water

Fig. 8 Comparison of the predicted correlation of frictional characteristics with experimental results for 15 wppm Alkox-C solution in water

Fig. 9 Comparison of the predicted correlation of frictional characteristics with experimental results for 35 wppm Alkox-C solution in water

agreement with the experimental values for the large pipe. In the case of a low inlet water level, however, the agreement is not so good. In Fig. 7, such a variation of λ is also presented as is calculated by Eq. (14) for the small pipe using γ^* obtained for the large pipe. In the case of a low water level, this result agrees well with the experimental one, but for a high water level this result is lower than the experimental one. In the small pipe, since the effect of a inlet water level is small, it is felt that the Toms effect is nearly in saturation. It may be considered that this discrepancy will be due to the fact that the approximated velocity distributions are not given with sufficient accuracy. When the Reynolds number is large, the variation of λ approaches Eq. (17). Thus it is necessary to take account of Eq. (8). For 7 wppm solution in Fig. 7, the agreement is good for a whole region. In comparison with 3.5 wppm solution, this case saturates earlier. The experimental values with the high inlet water level for the small pipe are a little smaller than the calculated values. It is also considered that the approximated velocity distribution is not given with sufficient accuracy. δ_v^+ and A_1 being larger, the agreement is better. Or this discrepancy may be caused by a tentative assumption of similarity of the inlet water level effect using the inlet water level H . For 15 wppm in Fig. 8, the same tendency is seen as the above. In this case, the discrepancy may be due to the region of the Reynolds number where the Toms effect is not saturated and the turbulent core of Eq. (9) is not already a Newtonian one. For 35 wppm solution in Fig. 9, the influence of the pipe diameter is saturated from the onset point of the Toms effect. The inlet water level effect does not occur, neither. It is felt that the influence of the pipe diameter will appear for a much larger pipe, but it is difficult to predict this from the result for the pipe of $D = 24.82\phi$. The reason why the experimental values agree well with Eq. (14) and Eq. (17) may be in the fact that the increase of δ_v^+ and decrease of A_1 cancel each other rather than the good approximation of velocity distribution. This fact is also inferred from the velocity distribution of 70 wppm solution which is obtained tentatively by Eq. (29) in Fig. 1. Furthermore, for 35 wppm solution, the results due to the neglect of viscosity increase may be included. The scatter of the experimental values with a low inlet water level in the large pipe is comparatively large. It is not considered due to this scatter that the experimental error is larger than that for the high inlet water level. This scatter may result from the following fact that, as seen from the analysis, the increase of inlet water level and the smallness

of pipe diameter make R_{e*} small and promote the saturation with the result that the scatter decreases apparently. That is to say, because the unsaturated region of the Toms effect for the small pipe with the high inlet water level is made narrower than that for the large pipe with the low inlet water level, the accuracy of experimental values may look different apparently.

7. Discussion

In Eqs. (14) and (17), the kind of polymer is not considered. It is important in what form the characteristics of polymer appears. According to this paper, as the Toms effect saturates, the influences of the concentration and others disappear. Therefore, it may be predicted that the characteristics appear most remarkably in the unsaturated region. Among the four constants of Eq. (14), δ_v^+ is determined by A_1 and B_1 . Thus A_1 , B_1 and γ^* prescribe the flow of polymer solutions. A_1 and B_1 are determined by polymer. As is seen from Eq. (20), γ^* will be determined by the external condition.

It is, usually, important whether it is possible or not to apply the results of the model experiment by a small pipe to the case of large pipe. And it becomes possible when the relation shown by Eq. (14) exists. Therefore, when there has already been the relation of Eq. (17) for a small pipe, it is difficult to apply the experimental values for the small pipe to that for the large one. For example, from data obtained for $D=9.0\phi$ with the low inlet water level in 7 wppm solution, the case of $D=24.82\phi$ can be predicted, but it is difficult to use the results of the high inlet water level. Accordingly, a pipe for which the model experiment is applicable is limited by γ^* .

8. Conclusions

In this paper, the velocity distributions are measured and analyzed. The correlation equation of coefficient of pipe friction is derived. The following conclusions can be drawn from these results.

(1) In a very dilute solution, the Toms effect occurs in a field of the larger shear rate than the critical shear rate. And the contributions of the Toms effect to the velocity distributions appear as a modification of the internal layer.

(2) The influences of the pipe diameter and the inlet water level on the Toms effect are considered to be transient. These influences disappear at the saturation point of the Toms effect.

(3) The essential elucidation of this phenomenon is expected from the experimental study of the fluctuating quantities and the establishment of

rheology equation for a very dilute solution. Especially, the measurements of the turbulent quantities in the buffer layer seem to offer an interesting problem. Furthermore, the measurements in the large wave number region of energy spectra will make a very valuable investigation.

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Appendix

(Velocity measurement in polymer solution)

Consider a fully developed pipe flow. The stress at an arbitrary point O (r, φ, x) is given by the local Cartesian stress component τ_{ij} at this point⁽⁹⁾. That is

$$\begin{pmatrix} \tau_{11} & \tau_{12} & 0 \\ \tau_{21} & \tau_{22} & 0 \\ 0 & 0 & \tau_{33} \end{pmatrix} = \begin{pmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{pmatrix} + \begin{pmatrix} P_{11} & \tau_{12} & 0 \\ \tau_{12} & P_{22} & 0 \\ 0 & 0 & P_{33} \end{pmatrix} \quad (21)$$

where, $p = -1/3\tau_{ii}$ and $P_{ii} = 0$. The former, p is an isotropic pressure, the latter P_{ii} is an extra normal stress. The pressure P_w ^{(10) (11)} which can be measured through the wall static pressure holes is

$$P_w(x) = -\tau_{22}(a) = p(x) - P_{22}(a) \quad (22)$$

The pressure P_t which can be measured by a Pitot tube across the cross section with respect to the same x is

$$\begin{aligned} P_t(r, x) &= -\tau_{11}(r) + \frac{\rho}{2}u^2(r) \\ &= p(x) - P_{11}(r) + p_d(r) \quad (23) \end{aligned}$$

The pressure P_{s2} measured by a static pressure tube which has static pressure holes only in the direction of r is

$$P_{s2}(r, x) = -\tau_{22}(r) = p(x) - P_{22}(r) \quad (24)$$

Along the pipe axis, the velocity gradient is zero. Then, $P_{22}(0) = 0$ and $P_{s2}(0, x) = p(x)$. Accordingly $P_{22}(r)$ is given by the next equation:

$$P_{22}(r) = P_{s2}(0, x) - P_{s2}(r, x) \quad (25)$$

Similarly, $P_{33}(r)$ is found out by a static pressure tube which has static pressure holes in the direction of φ .

$$P_{33}(r) = P_{s3}(0, x) - P_{s3}(r, x) \quad (26)$$

where $P_{s3}(0, x) = P_{s2}(0, x) = p(x)$. Then $P_{11}(r)$ is given by the next equation

$$P_{11}(r) = -\{P_{22}(r) + P_{33}(r)\} \quad (27)$$

Therefore, the dynamic pressure p_d is found out by Eqs. (22), (23) and (27)

$$\begin{aligned} p_d(r) &= P_t(r, x) + P_{s2}(r, x) + P_{s3}(r, x) \\ &\quad - 3P_{s2}(0, x) \quad (28) \end{aligned}$$

Using Weissenberg's hypothesis, i.e., $P_{22} = P_{33}$, it becomes

$$p_d(r) = P_t(r, x) + 2P_{s2}(r, x) - 3P_{s2}(0, x) \quad (29)$$

which shows that the velocity can be determined by the conventional method.

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