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Distribution of Local Void Fraction of Air-Water Two-Phase Flow in a Vertical Channel*

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The present report describes an experimental study on the distributions of local void fraction along a cross section of a vertical channel in fully developed air-water two-phase flow at atmospheric pressure, using a probe method.

In bubble flow, a peculiar distribution which was fundamentally different from a power law distribution accepted in previous studies was observed. It had a peak of void fraction near the wall. The position of the maximum void fraction was at $(0.5 \sim 0.7)d$ from the wall where d was a bubble major axis.

The authors proposed an empirical formula which was in good agreement with the present and another experimental results and applicable not only to bubble flow but also to slug flow.

Comparisons between this formula and previous formulas were made in the experimental results.

1. Introduction

Many studies on the adiabatic two-phase flow in a tube have been reported. Most of them deal with the flow patterns, pressure drops and slip ratios in two-phase flow. As for the void fraction, there are a few studies on the distribution of its local value in a channel though many deal with the average void fraction. Analytical studies on the distribution of local void fraction of two-phase flow in a tube have been reported by Bankoff⁽¹⁾, Aoki et al.⁽³⁾, and Zuber et al.⁽⁵⁾, but an experimental one, by Petrick⁽²⁾ using a narrow gamma ray beam. Neal et al.⁽⁴⁾ tried to measure the distribution of void fraction in mercury-nitrogen two-phase flow by a probe method. Akagawa^{(7) (10) (11)} reported in his study on the air-water piston flow that the time averaged rate of gas phase of small bubbles suspended in liquid piston had a large value near the pipe wall. Yamagata et al.⁽⁸⁾ measured the distribution of bubble density of two-phase flow blowing air bubbles from pipe wall simulating an evaporative tube.

However, we have little knowledge on the distribution of local void fraction to date in spite of its fundamental importance for the analysis of the

phenomena in two-phase flow. The fact seems to be due to a lack of suitable and accurate measuring method.

In the present paper, an attempt was made to reveal the characteristics of the distribution of local void fraction in air-water two-phase vertical flow experimentally by using the probe method which had been previously developed by some of the authors. As the result, a peculiar distribution in bubble flow was observed and an empirical formula applicable to a wide range from bubble flow to slug flow was proposed.

Nomenclature

D : inner diameter of pipe mm

d : bubble major axis mm

f_g : local void fraction %

$\bar{f}_g$: average void fraction at a cross section %
 $f_g^* = f_g / f_{gc}$

Q_g : volumetric flow rate of gas cm³/sec

Q_l : volumetric flow rate of liquid cm³/sec

β : ratio of flow rates = $Q_g / (Q_g + Q_l)$

r : radial coordinate mm

R : inner radius of pipe mm

$r^* = r/R$

V_l : superficial liquid velocity = $(Q_l / \pi R^2) \times 10^{-2}$ cm/sec

y : distance from pipe wall = $(R - r)$ mm

$y^* = y/R = 1 - (r/R)$

subscripts

w : wall, c : center of pipe

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2. Experimental apparatus and procedure

Figure 1 shows an outline of the experimental apparatus. Water is circulated by the pump ①. The flow rate is adjusted by the valve ② and measured by the orifice flow meter ③. Air-water two-phase flow mixed in the mixing chamber ④ flows upwards to the separator ⑩ through the measuring sections of void fraction ⑦ and ⑨. Water separated from air flows back to the water tank ⑪ in which the free surface is exposed to the atmosphere. Air from the compressor and surge tank ⑫ is blown into the mixing chamber through the regulating valve ⑬ and the flow meter ⑭. A bubbler is set in the mixing chamber with a baffle plate so as to make the flow as uniform as possible. The bubbler is a hollow flat cylinder with many 0.3 mm-diam. holes drilled at a regular 3 mm pitch in the upper plate. The values of the hole diameter and pitch of its hexagonal arrangement were determined in advance by experimental survey to make the bubble diameter as uniform as possible for simplicity and reproducibility of the experimental conditions. Pipes of test section 6 and 1 m in vertical length from the mixing chamber were used and their diameters were changed in 19, 30 and 50 mm for the length of 6 m, and 30 and 60 mm for 1 m. Transparent pipes of polymethylmethacrylate were used for the measuring section to

Fig. 1 Experimental apparatus

observe the flow patterns and to take photographs.

The principle of the probe method⁽¹²⁾ for measuring void fraction is based on utilizing the difference of electrical conductivity between the liquid and gas phases. The detector consists of an insulated needle ⑳ with exposed tip electrode projecting into the flow and a plate electrode which has area large enough to always contact with liquid phase in any bubble flow and slug flow. The change of the resistance depending on the tip electrode being dipped in the liquid or the gas phase is detected by a series resonance circuit. The fraction of time during which the gas phase exists at the tip of the needle is measured by an audio frequency signal oscillator ⑮, a discriminator ⑰, and an electronic counter ⑯. This is consistent with the void fraction averaged with time at the point.

The measurement was carried out at the cross sections ⑦ and ⑨ which were 0.65 m and 5.23 m high above the mixing chamber respectively for the pipe 6 m length, and at 0.8 m high for the pipe 1 m length.

As the diameter of the tip electrode was 0.1 mm, the error in this experimental conditions was estimated as within about 2 %⁽¹²⁾.

The average major axis of bubbles was found by its spectrum which was measured from the moment photographs of the flow.

The range of the experimental conditions was as follows: $60 \text{ cm}^3/\text{sec} \leq Q_L \leq 1800 \text{ cm}^3/\text{sec}$ ($8.5 \text{ cm}^3/\text{sec} \leq V_L \leq 575 \text{ cm}^3/\text{sec}$), $31 \text{ cm}^3/\text{sec} \leq Q_g \leq 380 \text{ cm}^3/\text{sec}$.

3. Results

Figure 2 shows an example of the radial

Fig. 2 Change of distribution of void fraction due to height

distributions of local void fraction measured at the low and high positions of the pipe 6 m length. The two curves with lower void fraction show the distributions at two heights in the same bubble flow and the other two with higher void fractions show those in the same slug flow. The differences in the distributions at high and low measuring positions for both flows are considered due to the flows at the low position being not fully developed yet under the effect of entrance region by disturbance of the bubbler. Therefore, the following major measurements were carried out at the high measuring position where the two-phase flow was considered to be fully developed⁽⁶⁾ as it was far enough from the mixing chamber.

Figure 3 shows typical radial distributions of void fraction in bubble flows. In the present experiments, considerably uniform bubbles in size were obtained as shown on the spectrums of bubble diameter in Fig. 4. The peculiar peaks of the void fraction appear near the wall in bubble flow with uniform bubble diameter as shown in Fig. 3. This was common characteristics to all bubble flows in the present experiments. However, the position of the peak is not definite even for the same pipe diameter, for instance in $D=30$ mm where one has the maximum value at $r^*=0.75$, and another, at $r^*=0.87$. As this was considered to be concerned with the bubble diameter, it was measured by moment photography of the flow. Figure 4 shows an example of the spectrums of the bubble major axis from which the average values d of the major axis were determined. Consequently, it was recognized

Fig. 3 Distributions of void fraction in bubble flow

that the peaks were located at the position $(0.5 \sim 0.7)d$ apart from the wall.

According to the analysis of high speed photographs, it is seen that the bubble velocities in outer region are less than those in central region, and bubbles are pushed outwards by velocity gradient and the yawing amplitude of the bubbles near the wall is limited to within a small distance adjacent to the wall. Consequently, the bubble density increases near the wall in bubble flow and the local void fraction tends to have a maximum value at about $(0.5 \sim 0.7)d$ apart from the wall because of no lateral movement of the bubbles contacting the wall.

Figure 5 shows radial distributions of void fraction in slug flows under a changed flow rate of gas for a constant flow rate of liquid. These distributions

Fig. 4 Spectrum of major axis of bubbles

Fig. 5 Distribution of void fraction in slug flow

have the maximum value at the center of the pipe and change monotonously along the radius.

4. Formulation of empirical formula

Some previous investigators adopted the following relation of Eq. (1) on the radial distribution of the void fraction in vertical two-phase flow and determined a value for the exponent m .

$$g_o^* = y^{*1/m} \dots \dots \dots (1)$$

However, it is found that the power function of Eq. (1) is hardly applicable to the distribution in bubble flow though it may be adopted for the slug flow. Further, it is found that even for the distribution in slug flow a value of m can not cover satisfactorily the whole radius. Aoki et al.⁽⁹⁾ tried to give different values of m for y^* divided into three regions respectively. A good agreement with the experiments was obtained by putting m as a function of y^* , like $m = ay^*$ where a is a constant.

Considering the facts mentioned above, an expression like Eq. (2) was proposed as the empirical formula for radial distribution of void fraction including those from bubble flow which has a peak near the wall to slug flow,

$$f_o^* = y^{*1/(ay^*)} + ny^*e^{-ly^*} \dots \dots \dots (2)$$

where a , n , and l are constants to be determined by experiments.

4.1 Determination of the constant a

Figure 6 shows an example of the relation of the exponent m which is derived from Eq. (1) versus y^* . The inclination of the straight line gives the value of a . As shown in the figure, the values of m in the region $y^* > 0.8$ which is near the center of the pipe, deviate from the straight line. However, f_o approaches unity independently of the exponent m in the region where y^* is close to unity in Eq. (1). Since the values of m

Fig. 6 Example to determine the constant a

deviating from the straight line near $y^* = 1$ have negligible influence on the results but the values in the region $y^* < 0.8$, are most influential, a simple expression like $m = ay^*$ was adopted for the treatment of the experimental results.

The relation between the ratio of the flow rates β and the constant a found as described above, is shown in Fig. 7 in which the constant a is regarded to be independent of the pipe diameter D and is expressed as follows:

$$a = 200(\beta)^{1.5} \dots \dots \dots (3)$$

4.2 Determination of l

Equation (2) is composed of the first term varying monotonously and the second term expressing the peak of the void fraction in bubble flow. It was found that the constant l was expressed by Eq. (4), with consideration of Eq. (2) with the difference between the measured values of f_o^* and the first term using Eq. (3), the measured maximum values and their positions, and the average major axis of the bubbles.

$$l = 1.3D/d \dots \dots \dots (4)$$

4.3 Determination of n

The constant n is to determine the peak value of the void fraction in bubble flow though l gives its position. The relation between n and β is shown in Fig. 8 where n is expressed by a group of straight lines with same inclination and with D as parameter. They are formulated as follows:

$$D = 19 \text{ mm} \dots n = 0.08(\beta)^{-1.8} \dots \dots \dots (5)$$

$$D = 30 \text{ mm} \dots n = 0.45(\beta)^{-1.8} \dots \dots \dots (6)$$

$$D = 50 \text{ mm} \dots n = 2.30(\beta)^{-1.8} \dots \dots \dots (7)$$

$$D = 64 \text{ mm} \dots n = 4.00(\beta)^{-1.8} \dots \dots \dots (8)$$

As Fig. 9 is obtained for the relation between

Fig. 7 Relation between a and β

the constants A and D when the four equations are expressed in the form $n=A(\beta)^{-1.8}$, the value of A is formulated by the following relation.

$$A=0.01(D/10)^{3.3} \quad \text{.....(9)}$$

Therefore, n can be expressed by Eq. (10)

$$n=0.01(D/10)^{3.3}(\beta)^{-1.8} \quad \text{.....(10)}$$

The applicable range of Eq. (10) is

$$\beta > 0.006(D-15), \quad D > 15 \quad \text{.....(11)}$$

4.4 Determination of void fraction at pipe center f_{gc}

The plot of the measurements of f_{gc} versus $\beta/(1-\beta)$ is shown in Fig. 10 where f_{gc} has little dependency on pipe diameter. So it is expressed

Fig. 8 Relation between n and β

Fig. 9 Relation between A and D in the form $n=A(\beta)^{-1.8}$

well as follows.

$$\log f_{gc} = \frac{1.5 \left(\log 100 \frac{\beta}{1-\beta} \right)^{2.7}}{1 + 0.75 \left(\log 100 \frac{\beta}{1-\beta} \right)^{2.7}} \quad \text{.....(12)}$$

Consequently, the following empirical formula is obtained for the distributions of the void fraction.

$$f_g^* = y^* / \{200(\beta)^{1.5} y^*\} + 0.01 \left(\frac{D}{10} \right)^{3.3} (\beta)^{-1.8} y^* e^{-1.3(D/d)y^*} \quad \text{.....(13)}$$

where $y^* = y/R = 1 - (r/R)$

y : distance from pipe wall $= (R-r)$ mm

β : ratio of flow rates $= Q_g / (Q_g + Q_l)$

D : inner diameter of pipe mm

$f_g^* = f_g / f_{gc}$

f_{gc} : void fraction at pipe center [Eq. (12)]

f_g : local void fraction %

applicable range of the second term

$$\beta > 0.006(D-15), \quad D > 15$$

5. Comparison of the empirical formula with the previous formulas

5.1 The previous formulas for the distributions of void fraction

5.1.1 Petrick's empirical formula⁽²⁾

Petrick measured the distribution of void fraction by means of attenuation of narrow beam gamma rays for two dimensional two-phase flow, using a rectangular duct and proposed the following empirical formula.

$$f_g^* = (y^*)^m \quad \text{.....(14)}$$

where m is a function of the superficial velocity V_t , ft/sec as follows.

$$m = 0.024(V_t)^{66.6} / \bar{f}_g \quad \text{.....(15)}$$

5.1.2 Bankoff's form⁽¹⁾

Bankoff proposed the following form for the distribution of void fraction in the theoretical analysis of the wall shear stress and pressure drop in two-phase flow,

Fig. 10 Relation between f_{gc} and β

$$f_g^* = (y^*)^{1/m} \dots \dots \dots (16)$$

where the exponent $1/m$ is found from Eq. (17) which is derived from the relations on the mass flow rates of liquid and gas phases, average void fraction $\bar{f}_g$ and f_{gc} .

$$\frac{\bar{f}_g}{f_{gc}} = \frac{2m^2}{(m+1)(2m+1)} \dots \dots \dots (17)$$

5.1.3 Zuber et al.'s form⁽⁵⁾

Zuber et al. proposed the distribution of void fraction in the analysis on the average volumetric

Fig. 11 Comparison of measurements with formulas in bubble flow

Fig. 12 Comparison of measurements with formulas in bubble flow

concentration in two-phase flow considering the local relative velocity, as follows:

$$\frac{f_g - f_{gw}}{f_{gc} - f_{gw}} = 1 - (r^*)^m \dots \dots \dots (18)$$

where f_{gw} denotes the void fraction at the wall and m is found from Eq. (19) as $f_{gw} = 0$ in the present case.

$$\frac{\bar{f}_g}{f_{gc}} = \frac{m}{m+2} \dots \dots \dots (19)$$

5.1.4 Aoki et al.'s formula⁽³⁾

Aoki et al. obtained Eq. (20) in which the number of bubbles N in the unit volume was found theoretically considering the behavior of a spherical bubble with diameter a .

$$f_g = (4/3)\pi a^3 N \dots \dots \dots (20)$$

N is given by theoretical results for each of two regions divided along pipe radius.

5.2 Comparison of empirical formula (13) with the previous formulas

Comparison of the measurements and the

Fig. 13 Comparison of measurements with formulas in bubble flow

Fig. 14 Comparison of measurements with formulas in bubble flow

empirical formula (13) with the formulas (14) to (20) by previous investigators is made as follows:

5.2.1 On the bubble flow

Figures 11 ~ 14 show them in bubble flow. The measured distributions of void fraction in bubble flow are considerably different from the predictions by previous formula because of the existence of the peak near the wall which has not been clear in the previous studies. The prediction by Zuber's form becomes negative and approaches $-\infty$ at the center and that by Bankoff's form, approaches asymptotically $+\infty$ at the wall because the exponents m in both forms are determined by the ratio of average void fraction $\bar{f}_g$ and the value at the center f_{gc} under the premise that $f_{gc} > \bar{f}_g$ but it does not always follow that $f_{gc} > \bar{f}_g$ on account of the existence of a peak near the wall.

On the other hand, the exponent m in the Petrick's formula takes so large value when the liquid superficial velocity V_L is large and the average

void fraction is small that it does not predict proper distribution as shown in Fig. 11 where m is 1.65×10^5 . In the example shown in Fig. 11, the maximum value at the peak of void fraction is more than three times the value at the center which is several percent void fraction as seen for $\beta = 0.065$ [$1/(1-\beta) = 0.0695$] in Fig. 10.

Figure 14 shows the distribution at transient region from bubble flow to slug flow, which preserves a vestige of the peak. Though the values of β in Fig. 11 and Fig. 14 are approximately equal, their flow patterns are as different as between a perfect bubble flow in Fig. 11 and a transient flow in Fig. 14 where the pipe diameter is smaller than in Fig. 11. The fact is considered to mean that the bubbles in a pipe with smaller diameter are easier to coalesce and consequently the flow transfers to slug flow earlier than ones in a pipe with larger diameter.

5.2.2 On the slug flow

Figures 15 and 16 show the comparison of the

Fig. 15 Comparison of measurements with formulas in slug flow

Fig. 16 Comparison of measurements with formulas in slug flow

Fig. 17 Comparison of Petrick's experiment with his formula and the formula (13)

Fig. 18 Comparison of Petrick's experiment with his formula and the formula (13)

measurements in slug flow with the empirical formula (13) and the other previous formulas. Aoki et al.'s formula is omitted here because it needs the bubble radius which is not definite in slug flow. In the empirical formula (13), no bubble major axis is necessary for the calculation in slug flow since the value of β is so large that the second term becomes negligibly small. The empirical formula (13) is in good agreement with the measurements as well as the bubble flow.

5.3 Comparison of the empirical formula (13) with Petrick's measurements

Figures 17 and 18 show the comparison of Petrick's measurements with his empirical formula and the formula (13) using the experimental conditions. The values by Petrick's formula vary with the liquid superficial velocity and average void fraction even if β is constant as shown in Fig. 17. They deviate from the measured values with closer approach to the pipe wall and f_{θ}^* falls steeply towards zero where r^* exceeds about 0.9. The present formula is also in good agreement.

According to the comparisons described above, it is recognized that the empirical formula (13) expresses well the distributions of void fraction in a vertical channel for a wide range of the conditions of two-phase flow.

6. Conclusions

The distributions of void fraction in vertical air-water two-phase flow for the patterns from bubble flow to slug flow were studied by a probe method varying the pipe diameter. The results are summarized as follows:

(1) The distribution of void fraction in fully developed bubble flow has a peak at $(0.5 \sim 0.7)d$ apart from the pipe wall where d denotes the average major axis of the bubbles.

(2) The distribution of void fraction in slug flow is regarded to follow a power law distribution.

(3) The empirical formula (13) on the distribution of void fraction which would be applicable for a wide range from bubble flow to slug flow was obtained by systematic measurements. The formula is in good agreement with the present experiments including bubble flow which has a peak near the wall and slug flow, and with Petrick's experiments.

(4) It is possible to predict the distribution of void fraction using the present empirical formula when the ratio of flow rates β and average major axis of bubbles for bubble flow and only the ratio of flow rates for slug flow are given through many previous formulas need average value of void fraction for prediction.

(5) The void fraction at the center of pipe is predicted from Eq. (12) independently of pipe diameter when the ratio of flow rates is given. Thus, the distribution of absolute values of void fraction can be predicted using formulas (12) and (13).

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