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Heat Conduction of Semiconductors*

(1st Report, Researches of the Fundamental Equations and the Boundary Conditions)

By KOZO KATAYAMA** and HIROMICHI UMEMIYA***

Thermal phenomena at the P-N junction of semiconductors are investigated, by experiments with Bithmus-Tellide, to verify the characteristic phenomena of semiconductors through which electric current and heat flow simultaneously.

The limits of application of Onsager's reciprocal-relation to this phenomenon, the fundamental equations and the boundary conditions of the heat conduction problems of semiconductors are revealed. The effect of the above verified thermal phenomenon (Peltier effect) upon the temperature rise of P-N junctions of semiconductor devices is discussed and the characteristic value of the degree of stability of devices is obtained.

1. Introduction

The problems of heat transfer in electronic devices utilizing semiconductors, such as transistors or diodes, have arisen increasingly. Fundamental phenomena of heat conduction occurring in semiconductors and at the P-N junctions of semiconductor elements when electric current is flowing through them, are important for the analysis of the above-mentioned problems.

In the previous report⁽¹⁾, the authors derived the equation of heat conduction of semiconductors and the boundary conditions at the junctions of semiconductor elements from the phenomenological equations in the thermodynamics of irreversible processes for the case of simultaneous flow of heat and electric current in semiconductors, and by solving these equations a measuring method of thermal properties of semiconductors by the utilization of Peltier effect was shown.

The Peltier effect, which is a characteristic phenomenon appearing at the junction of Ohmic contact, is a cross phenomenon arising from the interference of conduction of heat and electricity, and the Kelvin's relation $\pi = \alpha\theta$ can be obtained by applying the Onsager's reciprocal relation for the linear phenomenological relations, which are first-order approximations of the phenomena.

Therefore, the equation of heat conduction and the characteristic boundary condition of semiconductors (Peltier effect), derived in the analysis of the authors' previous report⁽¹⁾, are assumed to be valid in the domain of validity of linear phenomenological relations. But few experimental researches have been made concerning those domains of linear relations of semiconductors.

In this paper, as a first step to the investigation of the thermal phenomena at the P-N junction of semiconductors through which electric current flows, the rate of heat production at the Ohmic contact junction of P and N-type Bithmus tellide is measured, and the range of validity of the above mentioned equation of heat conduction and the boundary conditions of semiconductors are investigated.

The apparatus for this investigation is shown schematically in Fig. 1. P-type and N-type semiconductor plates of Bi_2Te_3 , and a plate of the standard specimen, pure Ni, are soldered and sandwiched

Fig. 1 Specimen

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** Professor, Faculty of Engineering, Tokyo Institute of Technology.

*** Lecturer, Faculty of Engineering, Yamagata University, Yonezawa.

between the copper blocks, in which subheaters are attached. For the measurements of the Seebeck coefficient and thermal conductivity by the comparison method, the specimens are heated by the subheaters.

When direct current flows in the specimen, heat generation or heat absorption occurs at the P-N junctions by Peltier effect.

At the steady state of heating process by Peltier effect, the maximum temperature of the junction θ_{0h} , and the minimum steady state temperature θ_{0c} in the cooling process, are measured respectively. By measuring θ_{0h} , θ_{0c} , Seebeck coefficient and thermal conductivity, sum of the heat production due to the Peltier effect can be calculated. By varying the ambient temperature of the specimen step by step, the ranges of validity of Kelvin's relation $\pi = \alpha\theta$, of the equation of heat conduction and of the boundary condition of semiconductors are investigated.

Electronic devices, such as transistors or diodes, are thermally destroyed and have their characteristic functions altered when the temperature of the P-N junctions rises over the restricted temperature. Such a temperature rise by Peltier effect at the junction of semiconductor elements is calculated from the solution of linear equation of heat conduction with the above verified boundary condition, and the characteristic current I_p , which is a measure for the thermal durability of a semiconductor element for the temperature rise by Peltier effect, is proposed.

Nomenclatures

$D = \frac{IQ_p}{\lambda l}$: gradient of the temperature at the junction $A^{-1}cm^2deg$

$f(\theta)$: temperature function of the Peltier coefficient $^{\circ}K$

i : electric current density $A cm^{-2}$

l : length of specimen cm

q : energy flux $W cm^{-2}$

Q_p : Peltier heat flux $W cm^{-2}$

V : electric potential V

α : Seebeck coefficient $\mu V deg^{-1}$

α_{ij} : relative Seebeck coefficient between materials i and j

θ : temperature $^{\circ}C$

Θ : absolute temperature $^{\circ}K$

θ_1, θ_2 : ambient temperature $^{\circ}C, ^{\circ}K$

θ_0, θ_0 : temperature of the P-N junction $^{\circ}C, ^{\circ}K$

θ_{0h}, θ_{0h} : steady state temperature at the P-N junction in the heating process $^{\circ}C, ^{\circ}K$

θ_{0c}, θ_{0c} : steady state temperature at the P-N junction in the cooling process $^{\circ}C, ^{\circ}K$

$\Delta\theta_{max} = \theta_{0h} - \theta_{0c}$: maximum temperature difference at the P-N junction

$\bar{\theta}_0 = (\theta_{0h} + \theta_{0c})/2$
 $\bar{\theta}_0 = (\theta_{0h} + \theta_{0c})/2$: mean temperatures $^{\circ}C, ^{\circ}K$

π : coefficient of Peltier effect μV

λ : thermal conductivity $W cm^{-1} deg^{-1}$

$\kappa = \lambda_N / \lambda_P$

$\bar{\lambda} = (\lambda_N + \lambda_P)/2$

σ : electric conductivity

Suffixes

N: suffix of N-type semiconductor

P: suffix of P-type semiconductor

i and q change its symbols ($\pm$) according to the direction of flow in the one dimensional flow.

2. Theory of heat conduction of semiconductors

We investigate the temperature distribution in the specimen in Fig. 1, where direct current flows. The following assumptions are employed in the analysis to simplify the calculation:

(i) Heat flux and electric current are assumed to be one-dimensional flows, and heat losses from the surface of the specimen and thermocouples are assumed to be negligibly small.

(ii) Electric contact at the junction is assumed to be Ohmic.

(iii) The temperature at the junctions of copper blocks or standard specimen is kept constant.

(iv) The specimens are homogeneous.

(v) Variation of thermal and electrical properties of the specimens with temperature is assumed to be negligible in the range of measurement.

2.1 The irreversible thermodynamics and the cross phenomena at the junction of semiconductors

As shown in the previous report, when heat and current flow simultaneously, the energy flux q and the electric current flux i are expressed by the following phenomenological equations from the irreversible thermodynamics⁽²⁾.

$$i = -\sigma(\text{grad } V + \alpha \text{ grad } \theta) \dots\dots\dots (1)$$

$$q = (V + \alpha\theta)i - \lambda \text{ grad } \theta \dots\dots\dots (2)$$

From the energy balance $q_N = q_P$ at the P-N junction,

$$\lambda_N \frac{d\theta_N}{dx} - \lambda_P \frac{d\theta_P}{dx} = \alpha_{NP} \theta_0 i \dots\dots\dots (3)$$

From the definition of the Peltier coefficient $Q_p = \pi i$ and Eq. (3), the Kelvin's relation $\pi_{NP} = \alpha_{NP} \theta$ is obtained. Equations (1)~(3) are derived from the assumption of linear relations and are first-order approximation of thermoelectric phenomena.

In this paper, the cross phenomenon at the junction is considered to be expressed by $f(\theta)$ instead of θ in Eq. (3).

$$\lambda_N \frac{d\theta_N}{dx} - \lambda_P \frac{d\theta_P}{dx} = \alpha_{NP} f(\theta_0) i \dots\dots\dots (4)$$

In this case, the phenomenological equations

corresponding to Eqs. (1) and (2) are

$$i = -\sigma \left(\text{grad} V + \alpha \frac{df(\theta)}{d\theta} \text{grad} \theta \right) \dots\dots\dots (5)$$

$$q = [V + \alpha f(\theta)]i - \lambda \text{grad} \theta \dots\dots\dots (6)$$

2.2 The steady state equation of heat conduction in semiconductors

In the case of one dimensional flow, the steady state equation of heat conduction is obtained from the law of conservation of energy and electric current.

When the conservation laws at steady state $\text{div } q = 0$, $\text{div } i = 0$ are substituted into Eqs. (5), (6),

$$\lambda \frac{d^2 \theta}{dx^2} + \frac{d\lambda}{d\theta} \left(\frac{d\theta}{dx} \right)^2 - \frac{d\alpha}{d\theta} f(\theta) i \frac{d\theta}{dx} + \frac{i^2}{\sigma} = 0 \dots\dots\dots (7)$$

Equation (7) is a one-dimensional steady state equation of heat conduction. When the electric current and the temperature distribution are small enough, the 2nd and 3rd terms of the left hand side are small and Eq. (7) reduces to Eq. (8).

$$\lambda \frac{d^2 \theta}{dx^2} + \frac{i^2}{\sigma} = 0 \dots\dots\dots (8)$$

2.3 Problems of heat conduction of semiconductors

We calculate the steady state temperature at the P-N junction when direct current flows through the specimen in Fig. 1.

From Eq. (8), the equations of heat conduction of semiconductors in steady state are

$$\lambda_N \frac{d^2 \theta_N}{dx^2} + \frac{i^2}{\sigma_N} = 0 \quad (x \leq 0) \dots\dots\dots (9)$$

$$\lambda_P \frac{d^2 \theta_P}{dx^2} + \frac{i^2}{\sigma_P} = 0 \quad (x \geq 0) \dots\dots\dots (10)$$

Boundary conditions are

at P-N junction:

$$\lambda_N \frac{d\theta_N}{dx} - \lambda_P \frac{d\theta_P}{dx} = \alpha_{NP} f(\theta) i \quad (x=0) \dots\dots\dots (11)$$

$$\theta_N = \theta_P = \theta_c \quad (x=0) \dots\dots\dots (12)$$

at both ends of semiconductors [from assumption (iii)]:

$$\theta_N = \theta_{-l} \quad (x = -l) \dots\dots\dots (13)$$

$$\theta_P = \theta_l \quad (x = l) \dots\dots\dots (14)$$

Solving Eqs. (9) and (10) with the boundary conditions (11)~(14), we obtain the temperature at the P-N junction.

$$\begin{aligned} & \theta_0(\lambda_N + \lambda_P) - f(\theta_0) \lambda \alpha_{NP} \\ & = \left\{ \left(\frac{i^2}{\sigma} \right)_N + \left(\frac{i^2}{\sigma} \right)_P \right\} l^2 + \lambda_N \theta_{-l} + \lambda_P \theta_l \dots\dots\dots (15) \end{aligned}$$

When we write the temperature at the junction in heating process as θ_{0h} , in cooling process as θ_{0c} respectively,

$$\begin{aligned} & \theta_{0h}(\lambda_N + \lambda_P) - \lambda \alpha_{NP} f(\theta_{0h}) i \\ & = \left\{ \left(\frac{i^2}{\sigma} \right)_N + \left(\frac{i^2}{\sigma} \right)_P \right\} l^2 + \lambda_N \theta_{-l} + \lambda_P \theta_l \dots\dots\dots (16) \end{aligned}$$

$$\begin{aligned} & \theta_{0c}(\lambda_N + \lambda_P) + \lambda \alpha_{NP} f(\theta_{0c}) i \\ & = \left\{ \left(\frac{i^2}{\sigma} \right)_N + \left(\frac{i^2}{\sigma} \right)_P \right\} l^2 + \lambda_N \theta_{-l} + \lambda_P \theta_l \dots\dots\dots (17) \end{aligned}$$

From Eqs. (16) and (17), the maximum temperature difference $\Delta \theta_{\max}$ is

$$\begin{aligned} \Delta \theta_{\max} &= \theta_{0h} - \theta_{0c} = \lambda \alpha_{NP} i \frac{f(\theta_{0h}) + f(\theta_{0c})}{\lambda_N + \lambda_P} \\ &= \frac{\lambda \alpha_{NP} i f(\bar{\theta}_0)}{\bar{\lambda}} = D i \dots\dots\dots (18) \end{aligned}$$

where

$$\bar{\lambda} = \frac{\lambda_N + \lambda_P}{2}, \quad \bar{f}(\bar{\theta}_0) = \frac{f(\theta_{0h}) + f(\theta_{0c})}{2}$$

When electrical current flux i is small, $\bar{\theta}_0$ is nearly equal to the ambient temperature θ_1 , and we can put

$$\bar{f}(\bar{\theta}_0) \doteq f(\bar{\theta}_0) = f(\theta_1)$$

Eq. (18) is reduced as follows.

$$\bar{\lambda} = \frac{\lambda \alpha_{NP} f(\theta_1)}{D} \dots\dots\dots (19)$$

From

$$\kappa = \lambda_P / \lambda_N$$

$$\lambda_N = \frac{2}{1 + \kappa} \frac{\lambda \alpha_{NP} f(\theta_1)}{D} \dots\dots\dots (20)$$

$$\lambda_P = \frac{2\kappa}{1 + \kappa} \frac{\lambda \alpha_{NP} f(\theta_1)}{D} \dots\dots\dots (21)$$

or

$$f(\theta_1) = D \bar{\lambda} / \lambda \alpha_{NP} \dots\dots\dots (22)$$

In the previous report, we regard the Peltier heat as known, $f(\theta) = \theta$, and we utilize the Peltier effect for measurement of the heat conductivity and thermal diffusivity of semiconductors.

In the region, where the relation $f(\theta) = \theta$ is valid, from Eqs. (20) and (21), thermal conductivity of semiconductors can be obtained.

On the contrary, when $\bar{\lambda}$ is known, the temperature function of the Peltier effect $f(\theta)$ is obtained from Eq. (22) and the region of validity of the linear phenomenological relation can be determined.

3. Measuring apparatus and measuring method

The Peltier heat and its temperature dependency are measured on the basis of the analysis in the previous chapter, using the values of the thermal conductivity of semiconductors measured by the steady comparison method.

3.1 Measuring apparatus

Measuring apparatus is shown in Fig. 1 and Fig. 2 schematically. P-type and N-type Bi_2Te_3 are chosen as the semiconductor specimen and pure nickel-rod is soldered as a standard specimen on the other side of N-type Bi_2Te_3 as shown in Fig. 1. A thin copper plate was soldered at the P-N junction for the temperature measurement and all the specimens were sandwiched between two copper blocks

which were used for heat reservoirs. The junctions were soldered with low-temperature solder to make Ohmic contacts.

The contact resistance of the junctions will change these values with the change in the direction of the direct electric current when the contact is not Ohmic. But, in this apparatus, such rectifying phenomena were not observed in the examination of the state of contact. So, the assumption (ii) of the previous chapter is satisfied.

Thermocouples ④ were 0.1 mm ϕ C-C. The specimen was set in a vacuum vessel to minimize the heat loss. The pressure in the vacuum vessel was about 10^{-4} torr., and the heat losses from the thermocouples and the surface of the specimen were so small at low temperature that we considered the previous assumption(ii) as being satisfied. For the measurement of low temperature range, the vacuum vessel was set in a liquid Nitrogen bath.

The metal vessel was heated by a main electric heater ②. The ambient temperature of the specimen was controlled by changing the output of the heater. The subheater ⑤ to make a temperature gradient in the specimen was attached in the upper copper block.

3.2 Measuring methods

When direct current flows in the specimen, heat absorption or heat generation occurs by the Peltier effect. From the analysis of chapter 2, the temperature function $f(\theta)$ of the Peltier effect is determined by Eq. (22) as follows.

$$f(\theta) = \frac{D\bar{\lambda}}{l\alpha_{NP}}$$

where D , named the gradient of the temperature difference, is obtained from Eq. (18) by measuring the maximum temperature θ_{oh} and minimum temperature θ_{oc} at the P-N junction. The temperature function $f(\theta)$ is determined from the length of the specimen l , Seebeck coefficient α_{NP} , mean thermal conductivity $\bar{\lambda}$, and the above mentioned D . In the range of linear phenomenological equations (1) and

(2), $f(\theta)$ will coincide with θ .

The outline of the measuring method of the above mentioned experiments to determine $f(\theta)$ is similar to that of the authors' previous report.

3.2.1 Measuring method of the Seebeck coefficient α

When the steady state temperature gradient is attained in the specimen by the subheater ⑤, corresponding electric potential s distributed in the specimen. The electric potential difference ΔV , and temperature difference $\Delta\theta$ between two boundaries of the semiconductor specimen are

$$\Delta V_N = \alpha_{Nm} \Delta\theta_N \quad (23)$$

$$\Delta V_P = \alpha_{Pm} \Delta\theta_P \quad (24)$$

$$\alpha_{NP} = \alpha_{Nm} - \alpha_{Pm} = \frac{\Delta V_N}{\Delta\theta_N} - \frac{\Delta V_P}{\Delta\theta_P} \quad (25)$$

The Seebeck coefficient α_{NP} can be determined by measuring the electrical potential difference ΔV , and temperature difference $\Delta\theta$ between two boundaries of the specimen from Eq. (25).

3.2.2 Measuring method of κ and thermal conductivity λ

When steady state temperature distribution is attained in the specimen, under the assumption (i),

$$\lambda_N \left(\frac{d\theta_N}{dx} \right)_{x=0} = \lambda_P \left(\frac{d\theta_P}{dx} \right)_{x=0}$$

κ is obtained as follows

$$\kappa = \lambda_P / \lambda_N = \frac{\Delta\theta_N}{\Delta\theta_P} = \frac{\theta_0 - \theta_{-l}}{\theta_l - \theta_0} \quad (26)$$

where θ_{-l} , θ_0 and θ_l are the temperatures of the junctions at $x = -l$, 0 and l respectively.

Similarly the thermal conductivities of the semiconductor specimens are determined by that of the standard specimen.

$$\lambda_N = \frac{l}{L} \frac{\Delta\theta_m}{\Delta\theta_N} \lambda_m \quad (27)$$

$$\lambda_P = \frac{l}{L} \frac{\Delta\theta_m}{\Delta\theta_P} \lambda_m \quad (28)$$

$$\bar{\lambda} = \frac{1}{2}(\lambda_N + \lambda_P) = \frac{l\Delta\theta_m}{2L} \left(\frac{1}{\Delta\theta_N} + \frac{1}{\Delta\theta_P} \right) \lambda_m \quad (29)$$

where $\Delta\theta$: temperature difference between the junctions.

$$\Delta\theta_m = \theta_{-l} \sim \theta_{-l-L}$$

$$\Delta\theta_N = \theta_0 \sim \theta_{-l}$$

$$\Delta\theta_P = \theta_l \sim \theta_0$$

L : length of the standard specimen.

m : suffix of the standard specimen.

So, κ , $\bar{\lambda}$, λ_N , and λ_P can be obtained by measuring the temperature differences $\Delta\theta_N$, $\Delta\theta_P$, and $\Delta\theta_m$.

4. Experimental results

4.1 Experimental examination of the measuring conditions

In the previous analysis, we made five assumptions in Chapter 2. For the assumptions (iii) and

Fig. 2 Measuring apparatus

(V), we need to check the measuring conditions by experiments.

4.1.1 Experimental examination of the assumption (V)

The specimen was made long enough to obtain large temperature changes by the Peltier effect with smaller electric current, as shown in Fig. 3.

The temperature of the P-N junction was measured by changing the electric current flux. The results are shown in Fig. 4. The mean temperature $\bar{\theta}_0$ of the P-N junction is nearly equal to the ambient temperature θ_1 when the electric current flux is smaller than 5 A/cm^2 and the assumption for Eq. (22) is shown to be satisfied.

The maximum temperature difference $\Delta\theta_{\max}$,

Fig. 3 Specimen of the preliminary experiment

Fig. 4 Steady state temperature at junction by changing the electric current

Fig. 5 Maximum temperature difference $\Delta\theta$

shown in Fig. 5, was calculated from the data in Fig. 4. In Fig. 5, it is clear that $\Delta\theta_{\max}$ is proportional to the current flux i in the range of $i < 5 \text{ A/cm}^2$ which coincides with Eq. (18). As the electric current flux increases, $\Delta\theta_{\max}$ becomes large, and the experimental points of $\Delta\theta_{\max}$ deviate from the above linear relation, because the nonlinear terms in Eq. (7) can not be neglected in the range of large i . The differences between the experimental points and the straight line are negligibly small when the experimental values of $\Delta\theta_{\max}$ are smaller than 20 deg. So, the equations of heat conduction are approximated by Eq. (9) and (10) when $\Delta\theta_{\max} < 20 \text{ deg}$ and $i < 5 \text{ A/cm}^2$.

4.1.2 Experimental examination of the assumption (iii)

The actual specimens for the experiments were made so thin that the heat losses from the side surfaces of the specimens could be neglected. The dimensions of the specimen plates were thickness 3~4 mm and diameter 5~6 mm. Using the above thin specimen, the maximum temperature difference $\Delta\theta_{\max}$ and the temperatures at the edges of the specimens were measured in the range of small electric current flux $i < 5 \text{ A/cm}^2$. The results are shown in Fig. 6. The temperature differences at the edges were negligibly small compared with the maximum temperature difference $\Delta\theta_{\max}$. So, the assumption (iii) could be satisfied under these conditions.

4.2 The experimental results

The dimensions of the specimen and measuring conditions are chosen such as satisfy the following conditions given in the preceding paragraphs.

Specimen: P and N-type Bi_2Te_3

Dimensions: diameter 5~6 mm, length 3~5 mm for P- and N-type Bi_2Te_3

Flux of electric current: $i \leq 5 \text{ A/cm}^2$

Maximum temperature difference: $\Delta\theta_{\max} \leq 20 \text{ deg}$

4.2.1 The measured results of α , λ_N and λ_P

The Seebeck coefficient α_{NP} was obtained from Eq. (28) by measuring temperature and electric potential distributions. The experimental results,

Fig. 6 Maximum temperature differences at P-N junction $\Delta\theta_{\max}$ and at the edges of specimen

the temperature dependence of the Seebeck coefficient are shown in Fig. 7. The differences of α_{NP} between different specimen are rather large, but the tendencies of the temperature dependence of α_{NP} are similar and attain maximum values at about 370°K and are proportional to the absolute temperature below 330°K.

The thermal conductivities were measured by the comparison method using pure Nickel as the standard specimen. The results of $\bar{\lambda}$, by Eq. (32), are shown in Fig. 8.

The tendencies of temperature dependence of the mean thermal conductivity $\bar{\lambda}$ coincide with the results measured by Goldsmid⁽³⁾ and Walker⁽⁴⁾, which varied as $1/\theta$ in the temperature range 200~300°K.

4.2.2 The measuring results of the temperature function $f(\theta)$ of the Peltier coefficient

Temperature dependences of the gradient of temperature difference D were determined by Eq.

Fig. 7 Typical data of Seebeck coefficient (P-N, Bi_2Te_3)

Fig. 8 Data of thermal conductivity (P-N, Bi_2Te_3)

(18). The results are shown in Fig. 9. Using the experimental results of gradient of the temperature difference D , of Seebeck coefficient α_{NP} and of the mean thermal conductivity $\bar{\lambda}$, the temperature function $f(\theta)$ was calculated by Eq. (22), and the results are shown in Fig. 10. It is seen that $f(\theta)$ agrees with the straight line $f(\theta)=\theta$ in the temperature range 200~300°K.

From the above results, it is revealed that the Onsager's reciprocal relation is applicable to the cross effect of simultaneous flow of heat and electricity at the P-N junction of Bi_2Te_3 semiconductors in the ranges $i < 5 \text{ A/cm}^2$ and $\theta = 200 \sim 300^\circ\text{K}$.

Outside of the above temperature range, some different problems as follows occur.

In the region of lower temperature $\theta < 200^\circ\text{K}$

(i) The accuracy and reliability of temperature measurements by thermocouples decrease.

(ii) The Seebeck coefficient decreases and the temperature rise due to the Peltier effect is very small.

In the region of high temperature $\theta > 300^\circ\text{K}$

(i) The quality of the specimen changes.

Fig. 9 Gradient of temperature difference D (P-N, Bi_2Te_3)

Fig. 10 Results of $f(\theta)$ (P-N, Bi_2Te_3)

(ii) The contact state at the junction is changed as the solder melts away.

For further study of the thermoelectric effect and heat conduction in semiconductors, it is necessary to improve the experimental apparatus about the above mentioned points.

4.2.3 Results of measurements of thermal conductivity by utilizing the Peltier effect

To confirm the results of the above experiments, thermal conductivity was calculated from the measuring results of the gradient of temperature difference D and Seebeck coefficient α_{NP} by Eq. (19). The results are shown in Fig. 11.

It is seen that the calculated results from the data measured by utilization of the Peltier effect agree with the previous data of $\bar{\lambda}$ in 3.2.2, measured by comparison method. Therefore, the fundamental equation of heat conduction and the boundary condition derived in the authors' previous report⁽¹⁾ are appropriate in the range of temperatures 200~300°K and the electric current flux $i < 5 \text{ A/cm}^2$.

5. The effect of Peltier heat for the temperature rise of P-N junction of semiconductor devices

When the temperature at the P-N junctions of semiconductor devices rises over a critical temperature which is a characteristic value for a specified semiconductor, the functions of the devices are lost or the junction will be thermally broken. The life time of a device is determined by the maximum temperature rise of the P-N junction. But direct measurements of the temperatures at the junction are very difficult because of their small sizes. By the measurements of electric circuit, the temperature rise by Peltier heat cannot be separated from that of Joulean heating due to the contact resistances. In the process of electric rectification, the electric current flows in one direction, and only the heating process of Peltier effect occurs at the junction and the cooling process doesn't occur.

In the case of analysing the effect of Peltier

Fig. 11 Calculated results of thermal conductivity by Eq. (19) (P-N, Bi_2Te_3)

heat on the temperature rise at P-N junction, it is sufficient to use the linear fundamental equation and the linear boundary conditions.

The temperature at the P-N junction due to Peltier effect is by Eq. (16)

$$\theta_{oh} = \frac{\left(\frac{1}{\sigma_N} + \frac{1}{\sigma_P}\right) l^2 i^2 + \lambda_N \theta_{-l} + \lambda_P \theta_l}{2\bar{\lambda} - l\alpha_{NP}i}$$

In the case of neglecting the Peltier effect, the temperature θ_{oh}' at the P-N junction is

$$\theta_{oh}' = \frac{\left(\frac{1}{\sigma_N} + \frac{1}{\sigma_P}\right) l^2 i^2 + \lambda_N \theta_{-l} + \lambda_P \theta_l}{2\bar{\lambda}}$$

The ratio $P(i)$ of the temperature rise due to the Peltier effect to the θ_{oh}' is

$$P(i) = \frac{\theta_{oh} - \theta_{oh}'}{\theta_{oh}'} = \frac{2\bar{\lambda}}{2\bar{\lambda} - l\alpha_{NP}i} - 1 = \frac{I_P}{I_P - i} - 1$$

where

$$I_P = \frac{2\bar{\lambda}}{l\alpha_{NP}}$$

The smaller I_P of device, the greater the temperature rise due to Peltier effect, therefore, I_P is a characteristic current density which is a measure of the degree of durability of devices for Peltier heating. The characteristic current density I_P of the specimen used for measurement of $f(\theta)$ in this report was about 300 A/cm². The dependence of $P(i)$ on i/I_P is shown in Fig. 12. The work electric current flux i must be kept within the range smaller than the characteristic current density I_P in order to avoid the thermal damages of semiconductor devices.

6. Conclusions

In this report, thermal phenomena at the P-N junction of semiconductors were investigated to verify the characteristic cross phenomena of semiconductors through which electrical current and heat flux flow simultaneously.

Fig. 12 The effect of Peltier heating for temperature rise of P-N junction of semiconductor elements

Theoretical analyses on the basis of the irreversible thermodynamics and the range of validity of application of Onsager's reciprocal relation to the Peltier effect, the fundamental equations and the boundary conditions of heat conduction in semiconductor elements were made in the case of Bi_2Te_3 .

The following conclusions are obtained.

(i) When the Peltier heat Q_π is given as

$$Q_\pi = \alpha_{NP} f(\theta_0) i$$

the temperature function $f(\theta)$ of Peltier effect can be obtained by measuring the mean thermal conductivity $\bar{\lambda}$, Seebeck coefficient α_{NP} and the variation of temperature at the junction, from the solution of heat conduction of semiconductors when direct current flows in the specimen. And the range of validity of Onsager's reciprocal relation for the thermoelectric phenomena can be determined experimentally by measuring the temperature function $f(\theta)$.

(ii) The criteria for satisfying the assumptions which are made in the above analysis are made clear by experiments. From the results of the experiments with long specimens, P- and N-type Bi_2Te_3 , the following conditions are determined.

Dimensions of specimens (for P- and N-type Bi_2Te_3),

diameter : 5~6 mm

length : 3~5 mm

electric current density i :

$$i \leq 5 \text{ A/cm}^2$$

maximum temperature difference $\Delta\theta_{\max}$:

$$\Delta\theta_{\max} \leq 20 \text{ deg}$$

(iii) The experimental results of $f(\theta)$ coincide with θ in the range of temperatures $200^\circ\text{K} < \theta < 300^\circ\text{K}$ for Bi_2Te_3 .

The range of validity of the Onsager's reciprocal relation applicable for the Peltier effect, and that of the fundamental equations and the boundary conditions of the specimens for this research was determined.

(iv) The ratio of temperature rise of P-N junction $P(i)$ due to the above verified Peltier heat was investigated by the analysis of the linear equation of heat conduction and was shown to be

$$P(i) = \frac{I_P}{I_P - i} - 1$$

where

$$I_P = (2\bar{\lambda}) / (l\alpha_{NP})$$

and it was shown that the characteristic current density I_P was a measure of degree of thermal durability of semiconductor devices for the temperature rise by the Peltier effect.

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