539. 388. 22 : 621. 771

Theory of Rolling for the Range of Mixed Friction*

(Part 1, A New Method of Calculation for the Rolling without Tension)

By Toshitaka TAMANO** and Samon YANAGIMOTO***

Bland and Ford's solution is used for calculating the roll force and torque in cold rolling of metal strip, and Sims' solution is used in hot rolling.

The present paper deals with a theoretical solution which is applicable to rolling under any condition of friction, by making certain mathematical approximations to the fundamental equations derived by Orowan.

This solution may be effective for explaining the rolling process in the range of mixed friction which would take place in warm rolling and hot rolling.

Further the solution should be used for the cold rolling in the range of frictional coefficients larger than 0.1.

1. Introduction

A number of researchers^{(2)~(8), (10)~(13)} have developed their rolling theory based on Karman's⁽¹⁾ or Orowan's⁽⁹⁾ equation. The solution derived by Orowan is taken into account the shearing stress caused by friction; thus it is applicable to rolling under any state of friction. This Orowan's solution has been confirmed to show an extremely good coincidence with measured values, while it is so hard to be applied to analysis of rolling process because of no mathematical approximation that it is not ready for immediate practical use. Bland and Ford's solution⁽¹⁰⁾ and Sims' solution⁽¹¹⁾ frequently used for calculating the roll force and torque are only applicable to the rolling process under the conditions of slipping friction and sticking friction, respectively.

The analysis of the rolling in the range of mixed friction has been undertaken by Troost et al.⁽¹²⁾ and Kalay et al.⁽¹³⁾. However Troost et al. assume the rolling as the compression between parallel plates in their analysis as in the case of Stone⁽⁷⁾, and having a disregard of the shearing stress caused by frictional stress. Kalay et al. give an approximate correction to the rolling pressure in the region of slipping friction only at the planes of exit and entry in Bland and Ford's solution. Thus these solutions

are not always satisfactory for all of actual rolling conditions.

This paper deals with a unified solution applicable to strip rolling under any condition of friction including Bland and Ford's solution and Sims' solution, by applying some mathematical approximations to Orowan's fundamental equation, and figures out the roll force function and the roll torque function which are required in practical work of rolling.

2. Notation

The following symbols are used.

R : working roll radius, mm

R' : radius of curvature of the elastically deformed roll, mm

h : thickness of the strip, mm

k : yield stress of the material in plane compression, kg/mm²

$\bar{k}$: mean yield stress over the arc of contact, kg/mm²

s : normal roll pressure, kg/mm²

τ : frictional stress acting on the roll surface, kg/mm²

f : horizontal compressive force per unit width acting on the section of the strip, kg/mm

t : compressive stress acting in the radial direction of the polar coordinates (ρ, θ) shown in Fig. 1, kg/mm²

$\tau_{\rho\theta}$: shearing stress in the polar coordinates shown in Fig. 1, kg/mm²

P : specific roll force per unit width, kg/mm

* Received 14th January, 1970.

** Research Engineer, Central Research Laboratories, Nippon Steel Corporation, Sagamihara.

*** Senior Research Engineer, Central Research Laboratories, Nippon Steel Corporation.

$$a = 2\tau/k \dots\dots\dots(10)$$

$$w(\phi, a) = \frac{1}{\sin \phi} \int_0^\phi \sqrt{1 - a^2 \left(\frac{\theta}{\phi}\right)^2} \cos \theta d\theta \dots\dots(11)$$

The frictional stress τ is derived from the assumptions (iv) and (v):

$$(a) \quad \tau = \mu s \text{ (in the region of slipping friction)} \dots\dots\dots(12)$$

$$(b) \quad \tau = \frac{1}{2}k \text{ (in the region of sticking friction)} \dots\dots\dots(13)$$

Orowan used Eqs. (5), (8) and (13) for deriving a theoretical solution of strip rolling.

In this report, an attempt is made to work out a theoretical solution applicable to rolling under any condition of friction without front and back tensions by applying the mathematical approximations shown in Eqs. (1) ~ (3) to the equations derived by Orowan.

Let us start with deriving fundamental equations for the roll pressure in the region of slipping friction. Substituting Eq. (12) into the equilibrium Eq. (5) and applying the approximation Eq. (1) to it gives

$$\frac{df}{d\phi} = 2R's(\phi \pm \mu) \dots\dots\dots(14)$$

Applying Eq. (1) to Eqs. (8) and (11) gives:
 $w(\phi, a) = w(0, a)$

$$= \int_0^1 \sqrt{1 - a^2 \gamma^2} d\gamma$$

$$= \frac{1}{2} \left[\frac{1}{a} \sin^{-1} a + \sqrt{1 - a^2} \right] = w(a) \dots\dots\dots(15)$$

$$f = hk \left[\frac{s}{k} - w(a) \right] \dots\dots\dots(16)$$

Substituting Eq. (16) into Eq. (14) gives

$$hk \frac{d}{d\phi} \left[\frac{s}{k} - w(a) \right] + \left[\frac{s}{k} - w(a) \right] \frac{d}{d\phi} (hk) = 2R's(\phi \pm \mu)$$

And applying the approximation Eq. (2) to it, the differential equation is written as

$$\frac{d}{d\phi} \left(\frac{s}{k} \right) - \frac{d}{d\phi} w(a) = 2 \frac{R'}{h} \frac{s}{k} (\phi \pm \mu) \dots\dots\dots(17)$$

By substituting the equation

$$a = 2\mu \frac{s}{k} \dots\dots\dots(18)$$

derived from Eqs. (10) and (12), and the equation representing the strip thickness

$$h = h_0 + R'\phi^2 \dots\dots\dots(19)$$

obtained by applying the approximations (1) to Eq. (9) and also Eq. (15) into Eq. (17) and integrating it, the fundamental equations representing the roll pressure in the region of slipping friction is obtained in the form

$$s/k = a/2\mu, \quad \tau/k = a/2 \dots\dots\dots(20)$$

$$\ln a + \mu z(a) = \ln (1 + \alpha^2 \phi^2) \pm 2\alpha \mu \tan^{-1} \alpha \phi + D \dots\dots\dots(21)$$

where

$$z(a) = \left(1 - \frac{1}{2a^2} \right) \sin^{-1} a + \frac{1}{2a} \sqrt{1 - a^2} \dots\dots\dots(22)$$

$$\alpha = \sqrt{\frac{R'}{h_0}} \dots\dots\dots(23)$$

In Eq. (21) D is an integration constant determined by the boundary conditions as described later. The 2nd term on the left side of Eq. (21) is introduced by taking into account the inhomogeneity of stress distribution caused by shearing stress. If this term is neglected, the equation will coincide with the fundamental equation in Bland and Ford's theoretical solution. Equations (20) and (21) hold only in the region of slipping friction or in the region of $0 < a < 1$ $\dots\dots\dots(24)$

There is a sticking friction between the roll surface and material in the region where Eq. (24) is not satisfied, making it necessary to analyze the roll pressure from Eqs. (5) and (13). The equation of equilibrium is reduced from Eqs. (5) and (13) and the equation of approximation (1).

$$\frac{df}{d\phi} = 2R' \left(s\phi \pm \frac{k}{2} \right) \dots\dots\dots(25)$$

On the other hand, from Eqs. (10), (13) and (15) the following equations are derived

$$a = 1, \quad w(1) = \frac{\pi}{4} \dots\dots\dots(26)$$

The horizontal force f is calculated from the above equations and Eq. (16).

$$f = hk \left[\frac{s}{k} - \frac{\pi}{4} \right] \dots\dots\dots(27)$$

Substituting Eq. (27) into Eq. (25) gives

$$k \frac{d}{d\phi} \left[h \left(\frac{s}{k} - \frac{\pi}{4} \right) \right] + h \left(\frac{s}{k} - \frac{\pi}{4} \right) \frac{dk}{d\phi} = 2R' \left(s\phi \pm \frac{k}{2} \right)$$

Rearranging it by use of the approximation equation (3) and Eq. (19) before it is integrated, the fundamental equation for the roll pressure in the region of sticking friction is obtained as

$$\frac{d}{d\phi} \left(\frac{s}{k} \right) = 2 \frac{R'}{h} \left(\frac{\pi}{4} \phi \pm \frac{1}{2} \right) \dots\dots\dots(28)$$

$$\left. \begin{aligned} \frac{s}{k} &= \frac{\pi}{4} \ln (1 + \alpha^2 \phi^2) \pm \alpha \tan^{-1} \alpha \phi + D', \\ \frac{\tau}{k} &= \frac{1}{2} \end{aligned} \right\} \dots\dots\dots(29)$$

where

$$\alpha = \sqrt{\frac{R'}{h_0}} \dots\dots\dots(23)$$

In Eq. (29) D' is an integration constant determined by the boundary condition to be mentioned later. Equation (29) is identical with the equation in Sims' theoretical solution.

4.2 Rolling in the range of slipping friction

The rolling which satisfies Eq. (24) over the whole arc of contact will be called the rolling in the range of slipping friction hereinafter. The boundary conditions at the exit and entry can be defined in the case of no front and back tensions as

$$\left. \begin{aligned} \phi &= 0, & f_0 &= 0 \\ \phi &= \phi_i, & f_i &= 0 \end{aligned} \right\} \dots\dots\dots (30)$$

where the contact angle ϕ_i between rolls and material at the entry can be derived as

$$\phi_i = \sqrt{\frac{h_i - h_0}{R'}} = \frac{1}{\alpha} \sqrt{\frac{r}{1-r}} \dots\dots\dots (31)$$

Accordingly, the roll pressures on the exit side ($0 \leq \phi \leq \phi_n$) and on the entry side ($\phi_n \leq \phi \leq \phi_i$) for the rolling in the range of slipping friction can be derived from Eqs. (15), (16), (20), (23), (30) and (31), the results of which are shown as follows:

$$\left. \begin{aligned} (a) \quad & 0 \leq \phi \leq \phi_n \\ & \frac{s}{k} = \frac{a}{2\mu}, \quad \frac{\tau}{k} = \frac{a}{2} \\ & \ln a + \mu z(a) = \ln(1 + \alpha^2 \phi^2) + 2\alpha\mu \tan^{-1} \alpha \phi + D_0 \\ (b) \quad & \phi_n \leq \phi \leq \phi_i \\ & \frac{s}{k} = \frac{a}{2\mu}, \quad \frac{\tau}{k} = \frac{a}{2} \\ & \ln a + \mu z(a) = \ln(1 + \alpha^2 \phi^2) - 2\alpha\mu \tan^{-1} \alpha \phi + D_i \end{aligned} \right\} \dots\dots\dots (32)$$

where

$$z(a) = \left(1 - \frac{1}{2a^2}\right) \sin^{-1} a + \frac{1}{2a} \sqrt{1-a^2} \dots\dots\dots (22)$$

$$\alpha = \sqrt{\frac{R'}{h_0}} \dots\dots\dots (23)$$

$$\left. \begin{aligned} D_0 &= \ln a_0 + \mu z(a_0) \\ D_i &= D_0 - \ln \frac{1}{1-r} + 2\alpha\mu \tan^{-1} \sqrt{\frac{r}{1-r}} \\ a_0^2 - \mu [\sin^{-1} a_0 + a_0 \sqrt{1-a_0^2}] &= 0 \end{aligned} \right\} \dots\dots\dots (33)$$

The neutral plane in the case of the rolling in the range of slipping friction can be derived from the condition that the roll pressures on the exit side and entry side are equal to each other in Eq. (32):

$$\phi_n = \frac{1}{\alpha} \tan \left[\frac{1}{2} \tan^{-1} \sqrt{\frac{r}{1-r}} - \frac{1}{4\alpha\mu} \ln \frac{1}{1-r} \right] \dots\dots\dots (34)$$

The above equation is identical with Bland and Ford's solution. The specific roll force P and roll torque G are derived by putting the roll pressure calculated from Eqs. (22), (23), (32) and (33) into the equations

$$P = R' \left[\int_0^{\phi_n} (s - \tau \phi) d\phi + \int_{\phi_n}^{\phi_i} (s + \tau \phi) d\phi \right] \dots\dots\dots (35)$$

$$G = 2RR' \int_0^{\phi_i} s \phi d\phi \dots\dots\dots (36)$$

4.3 Rolling in the range of mixed friction

The rolling which satisfies Eq. (24) at some part of the arc of contact but does not satisfy it at the other part is called the rolling in the range of mixed friction.

In such a case of rolling, the roll pressure in the region of slipping friction can be obtained in the same way as in the case of the rolling in the slipping friction. The friction transition planes ϕ_f and ϕ_b are calculated by putting

$$\frac{s}{k} = \frac{1}{2\mu} \dots\dots\dots (37)$$

into the equation for the roll pressure in the region of slipping friction. The integral constant D' in Eq. (29) for the roll pressure in the region of sticking friction can be derived from the condition that the roll pressure is continuous on the transition planes of friction.

The roll pressures in respectively the region of slipping friction on the exit side ($0 \leq \phi < \phi_f$), the region of sticking friction on the exit side ($\phi_f \leq \phi \leq \phi_n$), the region of sticking friction on the entry side ($\phi_n \leq \phi \leq \phi_b$) and the region of slipping friction on the entry side ($\phi_b < \phi \leq \phi_i$) are defined by the equations

$$\left. \begin{aligned} & 0 \leq \phi < \phi_f \\ & \frac{s}{k} = \frac{a}{2\mu}, \quad \frac{\tau}{k} = \frac{a}{2} \\ & \ln a + \mu z(a) = \ln(1 + \alpha^2 \phi^2) + 2\alpha\mu \tan^{-1} \alpha \phi + D_0 \\ & \phi_f \leq \phi \leq \phi_n \\ & \frac{s}{k} = \frac{\pi}{4} \ln(1 + \alpha^2 \phi^2) + \alpha \tan^{-1} \alpha \phi + D_f', \quad \frac{\tau}{k} = \frac{1}{2} \\ & \phi_n \leq \phi \leq \phi_b \\ & \frac{s}{k} = \frac{\pi}{4} \ln(1 + \alpha^2 \phi^2) - \alpha \tan^{-1} \alpha \phi + D_b', \quad \frac{\tau}{k} = \frac{1}{2} \\ & \phi_b < \phi \leq \phi_i \\ & \frac{s}{k} = \frac{a}{2\mu}, \quad \frac{\tau}{k} = \frac{a}{2} \\ & \ln a + \mu z(a) = \ln(1 + \alpha^2 \phi^2) - 2\alpha\mu \tan^{-1} \alpha \phi + D_i \end{aligned} \right\} \dots\dots\dots (38)$$

where

$$z(a) = \left(1 - \frac{1}{2a^2}\right) \sin^{-1} a + \frac{1}{2a} \sqrt{1-a^2} \dots\dots\dots (22)$$

$$\alpha = \sqrt{\frac{R'}{h_0}} \dots\dots\dots (23)$$

$$\left. \begin{aligned} D_0 &= \ln a_0 + \mu z(a_0) \\ D_i &= D_0 - \ln \frac{1}{1-r} + 2\alpha\mu \tan^{-1} \sqrt{\frac{r}{1-r}} \\ a_0^2 - \mu [\sin^{-1} a_0 + a_0 \sqrt{1-a_0^2}] &= 0 \end{aligned} \right\} \dots\dots\dots (33)$$

$$\left. \begin{aligned} & \ln(1 + \alpha^2 \phi_f^2) + 2\alpha\mu \tan^{-1} \alpha \phi_f = \frac{\pi}{4} \mu - D_0 \\ & \ln(1 + \alpha^2 \phi_b^2) - 2\alpha\mu \tan^{-1} \alpha \phi_b = \frac{\pi}{4} \mu - D_i \\ & \phi_n = \frac{1}{\alpha} \tan \left[\frac{1}{2} (\tan^{-1} \alpha \phi_f + \tan^{-1} \alpha \phi_b) \right. \\ & \quad \left. - \frac{\pi}{8\alpha} \ln \left(\frac{1 + \alpha^2 \phi_b^2}{1 + \alpha^2 \phi_f^2} \right) \right] \\ & D_f' = \frac{1}{2\mu} - \frac{\pi}{4} \ln(1 + \alpha^2 \phi_f^2) - \alpha \tan^{-1} \alpha \phi_f \\ & D_b' = \frac{1}{2\mu} - \frac{\pi}{4} \ln(1 + \alpha^2 \phi_b^2) + \alpha \tan^{-1} \alpha \phi_b \end{aligned} \right\} \dots\dots\dots (39)$$

The roll force P and torque G can be derived by substituting the roll pressures calculated from the above-mentioned equations into Eqs. (35) and (36).

4.4 Rolling in the range of sticking friction

The rolling in the case where there is sticking friction over the whole arc of contact is called the rolling in the range of sticking friction. The roll pressure in this case can be calculated by the following equations derived from Eqs. (29) and (31), which are identical with Sims' solution:

$$\left. \begin{aligned} 0 \leq \phi \leq \phi_n \\ \frac{s}{k} = \frac{\pi}{4} + \frac{\pi}{4} \ln(1 + \alpha^2 \phi^2) + \alpha \tan^{-1} \alpha \phi \\ \frac{\tau}{k} = \frac{1}{2} \\ \phi_n \leq \phi \leq \phi_t \\ \frac{s}{k} = \frac{\pi}{4} + \frac{\pi}{4} \ln(1 + \alpha^2 \phi^2) - \alpha \tan^{-1} \alpha \phi \\ - \frac{\pi}{4} \ln \frac{1}{1-r} + \alpha \tan^{-1} \sqrt{\frac{r}{1-r}} \\ \frac{\tau}{k} = \frac{1}{2} \end{aligned} \right\} \dots (40)$$

where

$$\alpha = \sqrt{\frac{R'}{h_0}} \dots (23)$$

$$\phi_n = \frac{1}{\alpha} \tan \left[\frac{1}{2} \tan^{-1} \sqrt{\frac{r}{1-r}} - \frac{\pi}{8\alpha} \ln \frac{1}{1-r} \right] \dots (41)$$

The roll force P and torque G can also be obtained by substituting the roll pressures derived from Eqs. (40) and (41) into Eqs. (35) and (36).

5. Calculation results and considerations

The results of the above-mentioned analysis from the basis of the results of calculation of the roll pressure distribution over the arc of contact, the roll force and torque under various rolling conditions described hereinafter.

5.1 Relation between rolling conditions and the state of friction

The analysis in this paper was made on the premise that the rolling conditions are in one to one correspondence to the state of friction. The state of friction may take the form of slipping friction, mixed friction or sticking friction depending on geometrical and physical rolling factors (such as radius of roll, thickness of the strip before and after rolling, rolling temperature, yield stress of materials in plane compression and the surface and lubricating conditions of roll and material). As seen from the equations of the solution, three variables α , μ and r suffice as the rolling conditions in this report. Needless to say, it is important to investigate the relation between the three variables and the (physical) rolling factors, but in this report an examination will be only made as to the relation between

the three variables and the state of friction.

The neutral plane gets closer to the plane of exit as the coefficient of friction decreases. The condition at which the neutral plane coincides with the exit is called the limit of rollability. The condition expressing the limit is derived from Eq. (34).

$$2\alpha\mu \tan^{-1} \sqrt{\frac{r}{1-r}} = \ln \frac{1}{1-r} \dots (42)$$

The limit is shown by the broken lines in Fig. 2. Rolling is impracticable under the rolling condition with smaller coefficient of friction below these lines.

The condition for the rolling in the range of slipping friction is that Eq. (24) holds over the whole arc of contact. Taking the distribution of roll pressure over the arc of contact into consideration, this condition is reduced to

$$0 < a_n < 1$$

This condition is also rewritten into equations and inequality with respect to the rolling factors (α , μ and r) by use of Eqs. (32) and (33).

$$\left. \begin{aligned} \ln(1 + \alpha^2 \phi_n^2) + 2\alpha\mu \tan^{-1} \alpha \phi_n - \frac{\pi}{4} \mu + D_0 < 0 \\ 4\alpha\mu \tan^{-1} \alpha \phi_n - 2\alpha\mu \tan^{-1} \sqrt{\frac{r}{1-r}} + \ln \frac{1}{1-r} = 0 \\ D_0 = \ln a_0 + \mu \left[\left(1 - \frac{1}{2a_0^2} \right) \sin^{-1} a_0 + \frac{1}{2a_0} \sqrt{1 - a_0^2} \right] \\ a_0^2 - \mu [\sin^{-1} a_0 + a_0 \sqrt{1 - a_0^2}] = 0 \end{aligned} \right\} \dots (43)$$

The relation between (α , μ and r) is calculated from Eq. (43) as represented by the space below the solid lines in Fig. 2. Accordingly, the rolling in the range of slipping friction takes place under the condition between the broken line and the solid line in Fig. 2. The limit of rolling in the range of

Fig. 2 Relation between the rolling conditions and the state of friction

mixed friction can be expressed from the condition that the range of slipping friction is at the exit and entry by

$$a_0^2 - \mu[\sin^{-1}a_0 + a_0\sqrt{1-a_0^2}] = 0$$

$$0 < a_0 < 1$$

The limiting condition for the rolling in the range of mixed friction is derived from the above equation and the inequality.

$$\mu < \frac{2}{\pi} \quad (44)$$

Hence, the rolling in the range of mixed friction takes place under the condition between the one-dot chain line and the solid line in Fig. 2.

The condition for the rolling in the range of sticking friction is defined by

$$\mu \geq \frac{2}{\pi} \quad (45)$$

as represented by the space above the one-dot chain line.

Since the solid lines in Fig. 2 show the limit of the rolling in the range of slipping friction, Bland and Ford's theoretical solution is only applicable to the rolling condition below the solid lines. Further, it must be noted to that even in the rolling in this range a large error may arise if the coefficient of friction is so large that the frictional stress cannot be neglected against the yield stress in shear.

5.2 Distribution of roll pressure

The solution presented in this report is compared with Orowan's exact solution in Fig. 3. It will be seen in this figure that there is little difference between the two in the distribution of roll pressure even when yield stress varies considerably in the roll gap.

The solution in this report is compared with Bland and Ford's solution and Sims' solution in Figs. 4 and 5. In Fig. 4, it is natural that there should be

Fig. 3 Comparison with Orowan's solution

a great difference between the present solution and Sims' solution because of a difference in the friction state, while it is interesting to see that the solution in this report differs not a little from Bland and Ford's solution. This shows that the solution in this report should be used for the rolling in the range of slipping friction too. Figure 5 shows the rolling in the range of mixed friction (about 45.5% of the arc of contact fall within the region of slipping friction and the small circles denote the friction transition planes). It will be seen in this figure that there is a considerable difference between the solution in this report and both of the Bland and Ford and Sims' solutions.

Figures 6 and 7 show how the maximum roll pressure, which is the most important value of roll pressure, will vary with the rolling conditions (α , μ and r). The maximum roll pressure increases

Fig. 4 Comparison with Bland & Ford's and Sims' solutions (I)

Fig. 5 Comparison with Bland & Ford's and Sims' solutions (II)

in the coefficient of friction for the rolling in the range of slipping friction, while it increases slowly for the rolling in the range of mixed friction.

5.3 Functions of roll force and roll torque

Although it is desirable to calculate the roll force and torque from Eqs. (35) and (36), they can be calculated conveniently if the roll force function F_P and the roll torque function F_G are obtained beforehand for various rolling conditions by introducing the mean two-dimensional yield stresses $\bar{k}_P$ and $\bar{k}_G$

$$\left. \begin{aligned} \bar{k}_P &= \frac{1}{\phi_i} \int_0^{\phi_i} k d\phi \\ \bar{k}_G &= \frac{2}{\phi_i^2} \int_0^{\phi_i} k \phi d\phi \end{aligned} \right\} \dots\dots\dots (46)$$

into

$$\left. \begin{aligned} P &= \sqrt{R'(h_i - h_0)} \bar{k}_P F_P(\alpha, \mu, r) \\ G &= R(h_i - h_0) \bar{k}_G F_G(\alpha, \mu, r) \end{aligned} \right\} \dots\dots\dots (47)$$

The roll force function F_P is derived by putting the roll pressure calculated from Eq. (32), (38) or (40) into

$$F_P = \frac{1}{\phi_i} \left[\int_0^{\phi_n} \left(\frac{s}{k} - \frac{\tau}{k} \phi \right) d\phi + \int_{\phi_n}^{\phi_i} \left(\frac{s}{k} + \frac{\tau}{k} \phi \right) d\phi \right] \dots\dots\dots (48)$$

The function of the roll torque F_G is derived by putting the roll pressure calculated from Eqs. (32), (38) or (40) into

$$F_G = \frac{2}{\phi_i^2} \int_0^{\phi_i} \frac{s}{k} \phi d\phi \dots\dots\dots (49)$$

The results of calculation of the roll force function and the roll torque function by use of Eqs. (48) and (49) are shown in Figs. 8 to 11. As seen in these figures, the roll force function and the roll torque function show the same tendency as the maximum roll pressure with regard to the coefficient of friction.

The results of calculation were obtained by use of an electronic computer IBM 1130 and the roll force function and the roll torque function were calculated by the trapezoidal integration by dividing

Fig. 6 Effect of the rolling conditions on maximum roll pressure (I)

Fig. 8 Effect of the rolling conditions on the force function (I)

Fig. 7 Effect of the rolling conditions on maximum roll pressure (II)

Fig. 9 Effect of the rolling conditions on the roll force function (II)

Fig. 10 Effect of the rolling conditions on the roll torque function (I)

the length of the arc of contact into 100 equal parts.

6. Conclusions

A theoretical solution applicable to strip rolling process under any state of friction has been derived from the fundamental equations of Orowan's exact solution by applying some mathematical approximations to them. The solution in this report is effective for explaining characteristics of rolling process in the range of mixed friction.

This solution in the range of sticking friction is identical with Sims' solution. In the range of slipping friction, the solution in this report should also be applied for the rolling in the range of frictional coefficient larger than 0.1 where Bland and Ford's solution is less accurate.

This solution will be extended to the case of strip rolling process with front and back tensions in a subsequent report. It is also planned to apply this solution to experimental study on the rolling in the range of larger coefficients of friction.

Fig. 11 Effect of the rolling on the roll torque function (II)

References

- (1) T. V. Karman: *Z. AMM*, Bd. 5, Ht. 2 (1952), S. 139.
- (2) S. Ekelund: *Steel*, Vol. 93 (1933), p. 27.
- (3) A. Nadai: *Jour. Appl. Mech.*, Vol. 6, No. 2 (1939), p. A-54.
- (4) A. Tselikov: *Metallurgia*, No. 6 (1939), p. 61.
- (5) A. Geleji: *Arch. Eisenhüttenwes.*, Jg. 34, Ht. 8 (1963-8), S. 565.
- (6) R. Hill: *Mathematical Theory of Plasticity*, (1950), p. 188, Oxford Univ. Press.
- (7) M. D. Stone: *Iron Steel Engr.*, Vol. 30 (1953-2), p. 61.
- (8) K. Tong and G. Sachs: *Jour. Mech. Phys. Solid*, Vol. 6 (1959), p. 35.
- (9) E. Orowan: *Proc. Inst. Mech. Engr.*, Vol. 150, No. 4 (1943), p. 140.
- (10) D. R. Bland and H. Ford: *Proc. Inst. Mech. Engr.*, Vol. 159, No. 39 (1948), p. 144.
- (11) R. B. Sims: *Proc. Inst. Mech. Engr.*, Vol. 168 (1954), p. 191.
- (12) A. Troost and K. Holling: *Arch. Eisenhüttenwes.*, Jg. 33, Ht. 7 (1962), S. 427.
- (13) A. K. E. H. A. E. Kalay and L. G. M. Sparling: *Jour. Iron Steel Inst.*, Vol. 206, No. 2 (1968-2), p. 152.