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Thermodynamic Performance Analysis of Gas Turbine

Power Plants with Intercooler*

(1st Report, Theory of Intercooling and Performance of Intercooling Type Gas Turbine)

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One of the authors, Tanaka, has previously analyzed the thermodynamic performance of simple, regeneration, reheat and regeneration-reheat types of gas turbine power plants. In this previous report an approximate calculation method was presented.

Here the authors, using the same approximate calculation method, analyze the change in the thermodynamic performance when intercoolers are used in each type of gas turbine.

First, the theoretical simple intercooling cycle is analyzed. Then the specific output and thermal efficiency of the simple type gas turbine with intercooler is calculated approximately.

The specific output of the intercooling type can be significantly increased over that of the simple type; it is inferior to the rate of increase of the specific output of the reheat type.

The maximum thermal efficiency of the intercooling type is nearly equal to that of the simple type.

Other interrelations among the performance and the intercooler efficiency, pressure ratio, turbine inlet temperature and pressure losses are also investigated.

1. Introduction

The authors have previously analyzed the thermodynamic performance of simple, regeneration, reheat and regeneration-reheat types of gas turbine power plants. This report presented a method for approximately calculating the specific outputs, the thermal efficiencies and the optimum pressure ratios for the compressors⁽¹⁾.

Here the authors, using the same approximate calculation method, analyze the change in the thermodynamic performance when intercoolers are used in each type of gas turbine.

The theoretical simple intercooling cycle is analyzed both in the case of neglecting the pressure loss in the intercooler, and of considering it.

Furthermore, the specific output and thermal efficiency of the simple gas turbine with intercooler is approximately calculated. The specific output and thermal efficiency for this type is compared with those of the simple and reheat gas turbine.

These performance characteristics were also calculated using a more precise model, and the result will be given later in the 3rd report.

2. Theoretical simple cycle of intercooling and others

2.1 Theoretical simple cycle of intercooling

The reheat type increases the specific output of the cycle by increasing the expansion work. It is possible, however, to increase the specific output by decreasing the compression work required. The latter can be accomplished by dividing the complete compression process into two or more steps, and cooling the compressed air to a certain temperature after each compression by the intercooler.

Figure 1 (a) illustrates the arrangement of the components of the simple plant with one stage of intercooling, and Figs. 1 (b), (c) show the p - v diagram and the i - s diagram respectively.

Under the assumption that the pressure loss of the intercooler can be neglected, the case of one stage of intercooling will first be discussed.

The temperature effectiveness of the intercooler (intercooler efficiency), denoted by η_{ic} , is defined by

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$$\eta_{ic} = \frac{T_{B1} - T_{A2}}{T_{B1} - T_A} \quad (1)$$

where

T_A : air temperature at the first stage compressor inlet ($^{\circ}\text{K}$)

T_{B1} : air temperature at the first stage compressor outlet ($^{\circ}\text{K}$)

T_{A2} : air temperature at the second stage compressor inlet ($^{\circ}\text{K}$)

T_{B2} : air temperature at the second stage compressor outlet ($^{\circ}\text{K}$)

Thus the intercooler efficiency is expressed by the ratio: the temperature rise in the first stage compressor $T_{B1} - T_A$ to the cooling effect in the intercooler $T_{B1} - T_{A2}$.

To minimize the compression work the first stage compressor pressure ratio r_1 must be equal to the second stage compressor pressure ratio r_2 . Referring to Fig. 1 (b), (c)

(a) Component arrangement

(b) p - v diagram

(c) i - s diagram

Fig. 1 Simple cycle with intercooling (neglecting the pressure loss in the intercooler)

$$\frac{P_{B1}}{P_A} = \frac{P_B}{P_{A2}} = \left(\frac{P_B}{P_A} \right)^{\frac{1}{2}} = r^{\frac{1}{2}} \quad (2)$$

Since the pressure loss of the intercooler is neglected, P_{B1} equals P_{A2} , and the whole pressure ratio for the compressor is denoted by r .

From Eq. (1) the air temperature of the second stage compressor inlet T_{A2} is given by

$$\frac{T_{A2}}{T_A} = \frac{T_{B1}}{T_A} (1 - \eta_{ic}) + \eta_{ic} \quad (3)$$

and the air temperature at the first stage compressor outlet T_{B1} is given by

$$\frac{T_{B1}}{T_A} = 1 + \frac{1}{\eta_c} \left(r^{\frac{1}{2}} \left(\frac{\kappa-1}{\kappa} \right) - 1 \right) \quad (4)$$

For convenience, assume that the first and second stage compressor adiabatic efficiency η_{c1} and η_{c2} are equal to η_c , thus $\eta_{c1} = \eta_{c2} = \eta_c$

Substituting Eq. (4) into Eq. (3) gives the following expression:

$$\frac{T_{A2}}{T_A} = 1 + \frac{1 - \eta_{ic}}{\eta_c} \left(r^{\frac{1}{2}} \left(\frac{\kappa-1}{\kappa} \right) - 1 \right) \quad (5)$$

Introducing

$$\alpha = \frac{1 - \eta_{ic}}{\eta_c} \quad (6)$$

gives

$$\frac{T_{A2}}{T_A} = 1 + \alpha \left(r^{\frac{1}{2}} \left(\frac{\kappa-1}{\kappa} \right) - 1 \right) \quad (5')$$

Let $(\dot{W}_c / \dot{G}_a)_{ic}$ denote the compression work input required per unit air flow (kg/sec) in the first and second stage compressor. Then

$$\begin{aligned} \left(\frac{\dot{W}_c}{\dot{G}_a} \right)_{ic} &= \frac{1}{75} \frac{\kappa}{\kappa-1} R \frac{1}{\eta_c} [T_A \{ r^{(1/2)((\kappa-1)/\kappa)} - 1 \} \\ &\quad + T_{A2} \{ r^{(1/2)((\kappa-1)/\kappa)} - 1 \}] = \frac{1}{75} \frac{\kappa}{\kappa-1} R T_A \frac{1}{\eta_c} \\ &\quad \times [r^{(1/2)((\kappa-1)/\kappa)} - 1] [2 + \alpha \{ r^{(1/2)((\kappa-1)/\kappa)} - 1 \}] \end{aligned} \quad (7)$$

In the ideal case, of cooling the second stage compressor inlet T_{A2} to the initial inlet temperature T_A , since the efficiency of the intercooler $\eta_{ic} = 1.00$ ($\alpha = 0$), Eq. (7) becomes

$$\begin{aligned} \left(\frac{\dot{W}_c}{\dot{G}_a} \right)_{ic} &= \frac{1}{75} \frac{\kappa}{\kappa-1} R T_A \frac{1}{\eta_c} 2 [r^{(1/2)((\kappa-1)/\kappa)} - 1] \\ (\eta_{ic} = 1.00) &\quad \quad \quad (7') \end{aligned}$$

This is the case of minimum compression work input in the compressor with intercooler (intercooling type). On the other hand, the compression work of the simple type is given

$$\left(\frac{\dot{W}_c}{\dot{G}_a} \right)_0 = \frac{1}{75} \frac{\kappa}{\kappa-1} R T_A \frac{1}{\eta_c} [r^{\frac{\kappa-1}{\kappa}} - 1] \quad (8)$$

The compression for these two cases is discussed in Section 2.2. Referring to Fig. 1 (c), the air temperature of the second stage compressor outlet T_{B2} is given by

$$\begin{aligned} T_{B2} &= T_{A2} \left[1 + \frac{1}{\eta_c} \left(r^{\frac{1}{2}} \left(\frac{\kappa-1}{\kappa} \right) - 1 \right) \right] \\ &= T_A \left[1 + \{ r^{(1/2)((\kappa-1)/\kappa)} - 1 \} \left\{ \left(\frac{1}{\eta_c} + \alpha \right) \right. \right. \\ &\quad \left. \left. + \frac{\alpha}{\eta_c} \left(r^{(1/2)((\kappa-1)/\kappa)} - 1 \right) \right\} \right] \end{aligned} \quad (9)$$

On the other hand, the air temperature of the compressor outlet of the simple type T_B is given by

$$T_B = T_A \left\{ 1 + \frac{1}{\eta_c} \left(r^{\frac{\kappa-1}{\kappa}} - 1 \right) \right\} \dots \dots \dots (10)$$

It is apparent from Fig. 1 (c) that the temperature T_B is higher than T_{B2} .

Next we consider the pressure loss in the intercooler for which Fig. 2, illustrates the process on the i - s plane. Referring to Fig. 2 the pressure loss in the intercooler is given by

$$\frac{P_{B1} - P_{A2}}{P_{B1}} = \frac{\Delta P_{ic}}{P_{B1}}$$

Also in this case, to minimize the compression work, the first and second stage pressure ratio is made equal to r_c . Thus

$$\frac{P_{B1}}{P_A} = \frac{P_{B2}}{P_{A2}} = r_c \dots \dots \dots (11)$$

From Eq. (10) and Eq. (11) we obtain

$$r_c^2 = \frac{P_{B1}}{P_A} \frac{P_{B2}}{P_{A2}} = \frac{P_B}{P_A} \frac{P_{B1}}{P_{A2}} = \frac{r}{1 - \Delta P_{ic}/P_{B1}} \dots \dots (12)$$

Thus we obtain

$$r_c = r^{\frac{1}{2}} \sqrt{\frac{1}{1 - \Delta P_{ic}/P_{B1}}} \dots \dots \dots (12)'$$

$$r_c^{\frac{\kappa-1}{\kappa}} = ar^{\frac{1}{2} \left(\frac{\kappa-1}{\kappa} \right)}, \quad a = \left[\frac{1}{1 - \Delta P_{ic}/P_{B1}} \right]^{\frac{1}{2} \left(\frac{\kappa-1}{\kappa} \right)} \dots (13)$$

Hence, the compression work of this case is given by substituting Eq. (13) into Eq. (7)

$$\left(\frac{HP_c}{G_a} \right)_{ic} = \frac{1}{75} \frac{\kappa}{\kappa-1} RT_A \frac{1}{\eta_c} [ar^{(1/2) \left(\frac{\kappa-1}{\kappa} \right)} - 1] \times [2 + \alpha \{ ar^{(1/2) \left(\frac{\kappa-1}{\kappa} \right)} - 1 \}] \dots \dots \dots (14)$$

Let the pressure loss $\Delta P_{ic}/P_{B1} = 0.05$ and 0.10 . Then the corresponding values of a become $a = 1.007$ and 1.015 , so that the value of Eq. (14) becomes larger than for the case when the pressure loss ($a = 1.00$) is neglected.

In this case the temperature of the second stage compressor inlet T_{A2} is given by

$$T_{A2} = T_A [1 + \alpha \{ ar^{\frac{1}{2} \left(\frac{\kappa-1}{\kappa} \right)} - 1 \}] \dots \dots \dots (15)$$

Then the delivery temperature T_{B2} is given as

Fig. 2 i - s diagram considering the pressure loss in the intercooler

$$T_{B2} = T_{A2} \left[1 + \frac{1}{\eta_c} \{ ar^{(1/2) \left(\frac{\kappa-1}{\kappa} \right)} - 1 \} \right] = T_A \left[1 + \{ ar^{(1/2) \left(\frac{\kappa-1}{\kappa} \right)} - 1 \} \left\{ \left(\frac{1}{\eta_c} + \alpha \right) + \frac{\alpha}{\eta_c} (ar^{(1/2) \left(\frac{\kappa-1}{\kappa} \right)} - 1) \right\} \right] \dots \dots \dots (16)$$

It is apparent from Eq. (15) and Eq. (16) that T_{A2} and T_{B2} are higher than for the case when the pressure loss in the intercooler ($a = 1.00$) was neglected.

2.2 Compression work input for simple and for intercooling types

The work input for compression of the intercooling type, neglecting the pressure loss in the intercooler, is given by Eq. (7) and that of the simple type is given by Eq. (8) and so the ratio between the compression works of these two types is given by

$$\left(\frac{HP_c}{G_a} \right)_{ic} / \left(\frac{HP_c}{G_a} \right)_0 = [r^{(1/2) \left(\frac{\kappa-1}{\kappa} \right)} - 1] \times [2 + \alpha \{ r^{(1/2) \left(\frac{\kappa-1}{\kappa} \right)} - 1 \}] / [r^{(\kappa-1)/\kappa} - 1] \dots (17)$$

Figure 3 presents the compression work ratio versus pressure ratio calculated from Eq. (17) for six values of intercooler efficiency η_{ic} , whence one assumes that the compressor adiabatic efficiency for the compression processes is equal to $\eta_c = 0.85$.

From Fig. 3 one sees that a reduction in work can be attained by adding one intercooler. At constant intercooler efficiency η_{ic} , larger benefits are obtained at higher pressure ratios, and at constant pressure ratios the work of compression ratio decreases with increasing intercooler efficiency. From Fig. 3 one can also see that it is easy to decrease the compressor work ratio to about 90 percent at higher pressure ratios.

Next, considering the pressure loss in the intercooler, the ratio of the compressor work is given from Eq. (14) and Eq. (8)

$$\left(\frac{HP_c}{G_a} \right)_{ic} / \left(\frac{HP_c}{G_a} \right)_0 = \frac{[ar^{(1/2) \left(\frac{\kappa-1}{\kappa} \right)} - 1] [2 + \alpha \{ ar^{(1/2) \left(\frac{\kappa-1}{\kappa} \right)} - 1 \}]}{[r^{(\kappa-1)/\kappa} - 1]} \dots \dots \dots (18)$$

Fig. 3 Compression work ratio of the compressor with and without intercooler (neglecting the pressure loss in the intercooler)

Figure 4 presents the results of Eq. (18) calculated for 5% and 10% of intercooler pressure loss and three values of intercooler efficiency η_{ic} . For the purpose of comparison Fig. 3 is also drawn in solid curves.

Figure 4 also shows the range in which the compression with intercooling requires more work than the normal compression process. For 5% and 10% pressure losses, these pressure ratios are about 2 and 4 respectively.

It is apparent that, in such a range of pressure ratios, more work is required in operating the compressor with intercooler. Thus the effect of the pressure loss appears strongly at a lower pressure ratio and this effect decreases with an increasing pressure ratio r . Therefore, the intercooler must be used at a higher pressure ratio and a small pressure loss is desirable.

The numerical values of the relations among the pressure loss in the intercooler $\Delta P_{ic}/P_{B1}$, intercooler efficiency η_{ic} , and compressor pressure ratio r , at 0.90 of compressor work ratio, are summarized in Table 1.

What is evident from Table 1 is that to obtain the effect of intercooling, it is necessary to decrease the pressure loss of the intercooler. Better results

Fig. 4 Complete pressure ratio of the compressor r
Compression work ratio of the compressor with and without intercooler (considering the pressure loss in the intercooler)

Table 1 The pressure loss $\Delta P_{ic}/P_{B1}$ and the intercooler efficiency η_{ic} corresponding to the compression work ratio $(\frac{HP_c}{G_a})_{ic} / (\frac{HP_c}{G_a})_o = 0.90$ and the variations of the whole pressure ratio of the compressor r .

Intercooler efficiency	Pressure loss in the intercooler $\Delta P_{ic}/P_{B1}\%$	Whole pressure ratio of the compressor
1.00	0	4.0
	5	6.0
	10	8.2
0.80	0	6.3
	5	10.5
	10	14.0
0.60	0	14.0
	5	24.3

are obtained, however, by increasing the intercooler efficiency η_{ic} .

2.3 Turbine output for reheat and for simple types

At first we consider the turbine output ratio of the reheat to the basic type. Figure 5 shows the i - s diagram of the one-stage reheat type when neglecting the pressure loss in the combustor.

From Fig. 5, it is assumed that the turbine inlet temperature of each stage is the same i.e., $T_{C1} = T_{C2}$, and the turbine expansion ratio of each stage is the same: $r_{t1} = r_{t2} = r^{\frac{1}{2}}$.

Thus the turbine outlet temperatures become equal i.e., $T_{D1} = T_{D2}$ and the turbine specific output can be written

$$(AL_t)_{rh} = (i_c - i_{D1}) + (i_{C1} - i_{D2}) = 2(i_c - i_{D1}) \quad (\text{kcal/kg})$$

The first-stage turbine outlet temperature can be expressed as

$$T_{D1} = T_c \left[1 - \eta_t \left\{ 1 - \left(\frac{1}{r} \right)^{\frac{1}{2} \left(\frac{\gamma-1}{\gamma} \right)} \right\} \right] \quad (19)$$

On the other hand, referring to Fig. 5, the turbine output of the simple type is

$$(AL_t)_o = i_c - i_D \quad (\text{kcal/kg})$$

The turbine outlet temperature is

$$T_D = T_c \left[1 - \eta_t \left\{ 1 - \left(\frac{1}{r} \right)^{\frac{\gamma-1}{\gamma}} \right\} \right] \quad (20)$$

Thus the turbine output ratio of these two types

Fig. 5 i - s diagram of the one-stage reheat type

Fig. 6 Ratio of turbine output of one-stage reheat type to that of simple type

can be expressed in the following approximate equation.

$$\left(\frac{P_t}{G_a}\right)_{rh} / \left(\frac{P_t}{G_a}\right)_0 = \frac{2(i_c - i_{D1})}{i_c - i_D} = \frac{2(T_c - T_{D1})}{T_c - T_D} \\ = 2 \left[1 - \left(\frac{1}{r}\right)^{\frac{1}{2}(\frac{\epsilon-1}{\epsilon})} \right] / \left[1 - \left(\frac{1}{r}\right)^{\frac{\epsilon-1}{\epsilon}} \right] \quad \dots\dots\dots (21)$$

where the turbine adiabatic efficiencies η_t of both types are equal.

It is apparent from Eq. (21) that the turbine output ratio is independent of the turbine inlet temperature T_c and turbine adiabatic efficiency η_t . Figure 6 shows the relation between the value of the turbine output ratio and the pressure ratio r . The turbine output ratio increases with increasing pressure ratio r .

Next, Fig. 7 shows the i - s diagram of the one-stage reheat type, considering the pressure loss in the combustors.

Referring to Fig. 7, the pressure losses of the first stage and second stage are expressed as follows.

$$\left. \begin{aligned} \frac{P_B - P_C}{P_B} &= \frac{\Delta P_C}{P_B} \\ \frac{P_{D1} - P_{C1}}{P_{D1}} &= \frac{\Delta P_{C1}}{P_{D1}} \end{aligned} \right\} \quad \dots\dots\dots (22)$$

Assuming that these pressure losses and the turbine expansion ratios of each stage are equal, then the expansion ratios can be written

$$\frac{P_C}{P_{D1}} = \frac{P_{C1}}{P_A} = r_t \quad \dots\dots\dots (23)$$

Thus we can write

$$\frac{P_B}{P_{D1}} = r^{\frac{1}{2}} = \frac{P_C}{P_{D1}} \frac{P_B}{P_C} = \frac{r_t}{1 - \Delta P_C / P_B}$$

Finally we can get

$$\left. \begin{aligned} r_t &= (1 - \Delta P_C / P_B) r^{\frac{1}{2}}, \quad \left(\frac{1}{r_t}\right)^{\frac{\epsilon-1}{\epsilon}} = b \left(\frac{1}{r}\right)^{\frac{1}{2}(\frac{\epsilon-1}{\epsilon})} \\ b &= \left[\frac{1}{1 - \Delta P_C / P_B} \right]^{\frac{\epsilon-1}{\epsilon}} \end{aligned} \right\} \quad (24)$$

Fig. 7 i - s diagram of one-stage reheat type (considering the pressure losses in the combustors)

Therefore, considering the pressure loss in the combustor, the first stage turbine outlet temperature can be written as

$$T_{D1} = T_c \left[1 - \eta_t \left\{ 1 - b \left(\frac{1}{r} \right)^{\frac{1}{2}(\frac{\epsilon-1}{\epsilon})} \right\} \right] \quad \dots\dots\dots (25)$$

Applying the same consideration, the turbine outlet temperature of the simple type is

$$T_D = T_c \left[1 - \eta_t \left\{ 1 - b \left(\frac{1}{r} \right)^{\frac{\epsilon-1}{\epsilon}} \right\} \right] \quad \dots\dots\dots (26)$$

Referring to Eq. (21), the turbine output ratio of these two types can be written as follows:

$$\left(\frac{P_t}{G_a}\right)_{rh} / \left(\frac{P_t}{G_a}\right)_0 = 2 \left[1 - b \left(\frac{1}{r} \right)^{\frac{1}{2}(\frac{\epsilon-1}{\epsilon})} \right] / \left[1 - b \left(\frac{1}{r} \right)^{\frac{\epsilon-1}{\epsilon}} \right] \quad \dots\dots\dots (27)$$

In this way, Eq. (27) is also independent of the turbine inlet temperature T_c and turbine adiabatic efficiency η_t .

Assuming the pressure losses in the combustors of both types to be 3% respectively, the pressure loss coefficient b of Eq. (27) becomes $b=1.009$. The broken curves of Fig. 6 show that the result of Eq. (27) corresponds to this case.

It is clear from Fig. 7 that the pressure loss in the combustor occurs once in the simple type and twice in the reheat type, so that the total turbine expansion ratio of the reheat type is smaller than that of the simple type. Therefore, to get the same turbine output ratio, the pressure ratio r must be greater than that of the simple type neglecting pressure loss.

2.4 Supplied heat for intercooling and for simple types

The heat supplied $(Q/G_a)_{ic}$ in the combustor of intercooling type is greater than that of the simple type $(Q/G_a)_0$, because the intercooler outlet temperature T_{B2} is smaller than that of the compressor outlet temperature T_B of the simple type for the same turbine inlet temperature T_c . [see Fig. 1(c)]

Neglecting the pressure loss in the intercooler, the ratio of heat supplied for the intercooling type to that for the simple type is given by

$$\left(\frac{Q}{G_a}\right)_{ic} / \left(\frac{Q}{G_a}\right)_0 = \frac{i_{Tc} - i_{T_{B2}}}{i_{Tc} - i_{T_B}} = \frac{T_c - T_{B2}}{T_c - T_B}$$

Rewriting the above equation, using Eq. (9) and

Fig. 8 Supplied heat ratio with and without intercooler (neglecting the pressure loss in the intercooler)

Eq. (10), we obtain

$$\frac{\left(\frac{Q}{G_a}\right)_{ic}}{\left(\frac{Q}{G_a}\right)_0} = \frac{\left(\frac{T_c}{T_A} - 1\right) - \left[r^{\frac{1}{2}\left(\frac{\epsilon-1}{\epsilon}\right)} - 1\right] \left[\left(\frac{1}{\eta_c} + \alpha\right) + \frac{\alpha}{\eta_c} \left(r^{\frac{1}{2}\left(\frac{\epsilon-1}{\epsilon}\right)} - 1\right)\right]}{\left(\frac{T_c}{T_A} - 1\right) - \frac{1}{\eta_c} \left[r^{\frac{\epsilon-1}{\epsilon}} - 1\right]} \quad (28)$$

Figure 8 shows the calculated result of Eq. (28), that is, the relation between $\left(\frac{Q}{G_a}\right)_{ic}/\left(\frac{Q}{G_a}\right)_0$ and r . The first stage compressor inlet temperature is set at $T_A = 288^\circ\text{K}$ and calculated for two values of turbine inlet temperature T_c and three values of intercooler efficiency η_{ic} .

The solid curves and the broken curves correspond to $T_c = 1000^\circ\text{K}$ and 1200°K respectively. From Fig. 8 it is easy to see that the larger the intercooler efficiency (namely the lower the combustor inlet temperature T_{B2}), the larger the supplied heat ratio, and that increasing the pressure ratio r tends to increase the cooling effect, so that the supplied heat ratio is also increased. Finally, at the same pressure ratio r , increasing the turbine inlet temperature T_c tends to decrease the supplied heat ratio.

The numerical values of the rate of decrease of the compression work ratio and its corresponding supplied heat ratio from Fig. 3 and Fig. 8 are summarized in Table 2.

It is clear from Table 2 that the greater the decrease rate of compressor work ratio, the greater is the corresponding values of compressor pressure ratio r and the heat supplied. This interrelation becomes more pronounced with the decrease of intercooler efficiency η_{ic} .

Next, considering the pressure loss in the intercooler, the following equation of heat supplied is obtained

$$\frac{\left(\frac{Q}{G_a}\right)_{ic}}{\left(\frac{Q}{G_a}\right)_0} = \frac{\left(\frac{T_c}{T_A} - 1\right) - \left[ar^{\frac{1}{2}\left(\frac{\epsilon-1}{\epsilon}\right)} - 1\right] \left[\left(\frac{1}{\eta_c} + \alpha\right) + \frac{\alpha}{\eta_c} \left(ar^{\frac{1}{2}\left(\frac{\epsilon-1}{\epsilon}\right)} - 1\right)\right]}{\left(\frac{T_c}{T_A} - 1\right) - \frac{1}{\eta_c} \left(r^{\frac{\epsilon-1}{\epsilon}} - 1\right)} \quad (29)$$

The only difference between Eq. (28) and Eq. (29) is in the numerator. Setting the pressure loss in the intercooler at 5%, the calculated results are shown by solid curves in Figs. 9 (a) (b) and the results of neglecting the pressure loss by broken curves.

From Fig. 9, it is clear that the supplied heat ratios are somewhat smaller than those in Fig. 8 where the pressure loss was neglected. This is because the second stage compressor outlet temperature T_{B2} is higher when considering the pressure loss ($a=1.007$) than when neglecting it ($a=1.00$), while

the simple type is unchanged.

The numerical values of the decrease rate of the compression work ratio and its corresponding supplied heat ratio from Fig. 4 and Figs. 9 (a) (b), are summarized in Table 3. Comparing the value of Table 2 when neglecting the pressure loss in the intercooler with that of Table 3, at the same intercooler efficiency η_{ic} and the same decrease of compressor work ratio, the corresponding pressure ratio r is larger than when neglecting the pressure loss. Therefore, the supplied heat ratios of Table 3 are greater than those of Table 2.

Table 2 Compression work ratio of intercooling type to simple type $\left(\frac{HP_c}{G_a}\right)_{ic}/\left(\frac{HP_c}{G_a}\right)_0$ and the corresponding supplied heat ratio $\left(\frac{Q}{G_a}\right)_{ic}/\left(\frac{Q}{G_a}\right)_0$

Turbine inlet temperature T_c	Inter-cooler efficiency η_{ic}	Compressor work ratio $\left(\frac{HP_c}{G_a}\right)_{ic}/\left(\frac{HP_c}{G_a}\right)_0$	Pressure ratio r	Supplied heat ratio $\left(\frac{Q}{G_a}\right)_{ic}/\left(\frac{Q}{G_a}\right)_0$
1000°K	1.00	0.90	4.00	1.16
		0.85	8.30	1.36
		0.80	16.90	1.86
	0.80	0.90	6.25	1.22
		0.85	16.50	1.65
		0.80	14.50	1.41
1200°K	1.00	0.90	4.00	1.12
		0.85	8.30	1.26
		0.80	16.90	1.52
	0.80	0.90	6.25	1.14
		0.85	16.50	1.39
		0.80	14.50	1.26

Fig. 9 Supplied heat ratio with and without intercooler

2.5 Supplied heat for reheat and for simple types

Referring to Fig. 5, the supplied heat ratio of the one-stage reheat type to the simple type, neglecting the pressure losses in the combustors, can be written as

$$\left(\frac{Q}{G_a}\right)_{rh} / \left(\frac{Q}{G_a}\right)_0 = \frac{(i_c - i_B) + (i_c - i_{D1})}{i_c - i_B} = 1 + \frac{T_C - T_{D1}}{T_C - T_B}$$

Substituting the first stage turbine outlet temperature T_{D1} from Eq. (19) and the compressor outlet temperature T_B of the simple type from Eq. (10) in to this equation, one gets

$$\left(\frac{Q}{G_a}\right)_{rh} / \left(\frac{Q}{G_a}\right)_0 = 1 + \frac{\eta_t \left[1 - \left(\frac{1}{r} \right)^{\frac{1}{2} \left(\frac{\epsilon-1}{\epsilon} \right)} \right]}{1 - \frac{T_A}{T_C} \left[1 + \frac{1}{\eta_c} \left(r^{\frac{\epsilon-1}{\epsilon}} - 1 \right) \right]} \dots\dots (30)$$

The solid curves of Fig. 10 show the results calculated by Eq. (30), the compressor inlet temperature is set at $T_A = 288^\circ\text{K}$, the turbine adiabatic efficiency at $\eta_t = 0.85$, and compressor adiabatic efficiency at $\eta_c = 0.85$. The absolute value of the

Table 3 Compression work ratio of intercooling type to simple type $\left(\frac{HP_c}{G_a}\right)_{ic} / \left(\frac{HP_c}{G_a}\right)_0$ and its corresponding supplied heat ratio $\left(\frac{Q}{G_a}\right)_{ic} / \left(\frac{Q}{G_a}\right)_0$ considering the 5% of pressure loss in the intercooler

Turbine inlet temperature T_c	Inter-cooler efficiency η_{ic}	Compressor work ratio $\left(\frac{HP_c}{G_a}\right)_{ic} / \left(\frac{HP_c}{G_a}\right)_0$	Pressure ratio r	Supplied heat ratio $\left(\frac{Q}{G_a}\right)_{ic} / \left(\frac{Q}{G_a}\right)_0$
1000°K	1.00	0.90	6.04	1.26
		0.85	11.05	1.50
		0.80	21.50	2.20
	0.80	0.90	9.82	1.34
		0.85	22.70	2.02
	0.60	0.90	19.50	1.58
1200°K	1.00	0.90	6.04	1.18
		0.85	11.05	1.34
		0.80	21.50	1.65
	0.80	0.90	9.82	1.23
		0.85	22.70	1.53
	0.60	0.90	19.50	1.32

Fig. 10 Supplied heat ratio of reheat type to simple type

supplied heat increases with increasing turbine inlet temperature T_c . On the contrary, the supplied heat ratio is decreased.

Comparing Fig. 10 with Fig. 8 (supplied heat ratio of intercooling type), the heat supplied ratio of the reheat type is seen to be greater than that of the intercooling type, at the same turbine inlet temperature T_c and at the same pressure ratio r .

The numerical values of the turbine output ratio and its corresponding supplied heat ratio for the reheat and the simple type from Fig. 10 and Fig. 6 are summarized in Table 4.

As is evident from Table 4, the rate of increase of the turbine output increases with increasing pressure ratio r , and the supplied heat ratio increases remarkably. Further, the absolute value of the heat supplied increases with increasing turbine inlet temperature T_c , but the supplied heat ratio decreases at the same rate as the turbine output increases.

Next, considering the pressure losses in the combustors, the supplied heat ratio of the one-stage reheat type and the simple type can be written as

$$\left(\frac{Q}{G_a}\right)_{rh} / \left(\frac{Q}{G_a}\right)_0 = 1 + \frac{\eta_t \left[1 - b \left(\frac{1}{r} \right)^{\frac{1}{2} \left(\frac{\epsilon-1}{\epsilon} \right)} \right]}{1 - \frac{T_A}{T_C} \left[1 + \frac{1}{\eta_c} \left(r^{\frac{\epsilon-1}{\epsilon}} - 1 \right) \right]} \dots\dots (31)$$

The difference between Eq. (30) and Eq. (31) exists only in the numerator. That is, the pressure

Table 4 Turbine output ratio of one-stage reheat type to simple type $\left(\frac{HP_t}{G_a}\right)_{rh} / \left(\frac{HP_t}{G_a}\right)_0$ and its corresponding supplied heat ratio $\left(\frac{Q}{G_a}\right)_{rh} / \left(\frac{Q}{G_a}\right)_0$

Turbine inlet temperature T_c	Ratio of turbine output $\left(\frac{HP_t}{G_a}\right)_{rh} / \left(\frac{HP_t}{G_a}\right)_0$	Pressure ratio r	Supplied heat ratio $\left(\frac{Q}{G_a}\right)_{rh} / \left(\frac{Q}{G_a}\right)_0$
1000°K	1.10	4.10	1.28
	1.15	8.25	1.52
	1.20	17.00	1.97
1200°K	1.10	4.10	1.25
	1.15	8.25	1.42
	1.20	17.00	1.70

Table 5 Turbine output ratio of one-stage reheat type to simple type $\left(\frac{HP_t}{G_a}\right)_{rh} / \left(\frac{HP_t}{G_a}\right)_0$ and its corresponding supplied heat ratio $\left(\frac{Q}{G_a}\right)_{rh} / \left(\frac{Q}{G_a}\right)_0$ considering the 3% of pressure loss in the combustor

Turbine inlet temperature T_c	Ratio of turbine output $\left(\frac{HP_t}{G_a}\right)_{rh} / \left(\frac{HP_t}{G_a}\right)_0$	Pressure ratio r	Supplied heat ratio $\left(\frac{Q}{G_a}\right)_{rh} / \left(\frac{Q}{G_a}\right)_0$
1000°K	1.10	5.50	1.36
	1.15	10.40	1.61
	1.20	20.30	2.15
1200°K	1.10	5.50	1.31
	1.15	10.40	1.48
	1.20	20.30	1.78

loss in the combustor of the simple type has no effect upon the supplied heat ratio.

The broken curves of Fig. 10 show the calculated result of Eq. (31), assuming as before a 3% pressure loss in the combustor.

It is clear from Eq. (31) that the supplied heat ratio in this case is somewhat smaller than that when neglecting the pressure loss ($b=1.00$). This occurs because the pressure loss in the first-stage combustor of the reheat type makes the first-stage turbine outlet temperature T_{D1} higher than when the pressure loss is neglected.

Considering the 3% of pressure loss, the numerical values of the ratio of the turbine output of the reheat type to that of the simple type and the corresponding value of the supplied heat ratio from

$$\left(\frac{P_E}{G_a}\right)_{ic} = \left(\frac{P_E}{G_a}\right) - \left(\frac{P_E}{G_a}\right)_{ic} = \frac{1}{75} \frac{\kappa}{\kappa-1} RT_A \left[\eta_t \frac{T_c}{T_A} \left\{ 1 - \left(\frac{1}{r}\right)^{(\kappa-1)/\kappa} \right\} - \frac{1}{\eta_c} \{ r^{(1/2)((\kappa-1)/\kappa)} - 1 \} \{ 2 + \alpha [(1/2)((\kappa-1)/\kappa) - 1] \} \right] \dots (32)$$

when the intercooler efficiency $\eta_{ic}=1.00$, α becomes zero. Thus

$$\left(\frac{P_E}{G_a}\right)_{ic} = \frac{1}{75} \frac{\kappa}{\kappa-1} RT_A \left[\eta_t \frac{T_c}{T_A} \left\{ 1 - \left(\frac{1}{r}\right)^{(\kappa-1)/\kappa} \right\} - \frac{2}{\eta_c} \{ r^{(1/2)((\kappa-1)/\kappa)} - 1 \} \right] \quad (\eta_{ic}=1.00) \dots (32)'$$

The first stage compressor inlet temperature is set at $T_A=288^\circ\text{K}$, the turbine adiabatic efficiency at $\eta_t=0.85$, and the first and second-stage compressor adiabatic efficiencies at $\eta_{c1}=\eta_{c2}=\eta_c=0.85$ and calculated for two values of turbine inlet temperature T_c and three values of intercooler efficiency η_{ic} .

Figures 11 (a) and (b) show the result of Eq. (32), and for the purpose of comparison the result of the simple type and one-stage reheat type are shown by broken curves. For these two types also the pressure loss in the combustor is neglected.

It is clear from Figs. 11 (a) and (b) that the specific output of the intercooling type occupies an intermediate position between the simple and the one-stage reheat types. At the same pressure ratio r , the intercooling type and the simple type have the same turbine output P_E/G_a , but the intercooling type (see Section

Fig. 10 and Fig. 6 are summarized in Table 5.

Comparing the supplied heat ratio values of Table 5 with those of Table 4 and considering the pressure losses in the combustors, one sees that it increases with increasing pressure ratio r at the same turbine output ratio.

3. Performance of the intercooling type

3.1 Analysis by the approximate calculation of specific output

3.1.1 Specific output (neglecting the pressure loss)

Neglecting the pressure losses in the intercooler and combustor, the specific output of the intercooling type is expressed in the approximate calculation formula.

(a)

(b)

Fig. 11 Specific output of the intercooling type (neglecting the pressure losses in the intercooler and combustor)

2.2) decreases its compression work $(\dot{W}_c/\dot{G}_a)$ by intercooling, and so the work saved in the compression appears as a quite large additional specific output $\dot{W}_E/\dot{G}_a$.

The greater the intercooler efficiency η_{ic} , the more effective is the cooling. So the specific output increases. Figure 11 also shows that the greater the intercooler efficiency η_{ic} , the higher is the optimum pressure ratio r , giving the maximum specific output.

Next, comparing the simple type with the reheat type, for increasing the specific output, it is clear that to attain a positive increase of turbine output by reheating is more effective than to decrease the compression work by intercooling.

In this approximate calculation, it is of interest that the optimum pressure ratio to the maximum specific output of the reheat type r_{cmL} is equal to that of the intercooling type at the intercooler efficiency $\eta_{ic}=1.00$, for which the details will be described later.

Tables 6 (a) and (b) show the numerical values of the maximum specific output and optimum pressure ratio from Fig. 11 (a) and (b) and they also tabulate the compressor work ratio, the supplied heat ratio and the turbine output ratio corresponding to this compression ratio as read from Fig. 3, Fig. 6, Fig. 7, Fig. 10.

It is clear from Table 6 that the maximum specific output of the intercooling type is 27% to 12%

greater than that of the simple type, and this value increases with increasing intercooler efficiency η_{ic} . This rate of increase is nearly equal at two different turbine inlet temperatures T_c of 1000°K and 1200°K.

But the optimum pressure ratio to the maximum specific output $(r_{cmL})_{ic}$ is greater than that for the simple type. The increase is between 25% and 71% for the simple type when $T_c=1000^\circ\text{K}$, and between 25% and 96% when $T_c=1200^\circ\text{K}$.

The compression work ratio corresponding to the maximum specific output decreases by 18% to 7%. On the other hand, the specific output increases by 12% to 27%. Therefore, it is clear that the decrease of compressor work is much greater than the increase of specific output. In this case, the supplied heat ratio is greater than that of the simple type by 16% to 40%.

Next, comparing the simple type with the reheat type (Table 6), the maximum specific output in this case is greater than that of the simple type by 41% to 42%. And the pressure ratio for this case is equal to that of the intercooling type when the intercooler efficiency $\eta_{ic}=1.00$.

Furthermore, the turbine output ratio in this case is greater than that of the simple type by 1.15 to 1.18 times, and supplied heat ratio is greater than that by 1.53 to 1.58 times.

The pressure ratio corresponding to the maximum specific output $(r_{cmL})_{ic}$ is readily determined by

Table 6
(a) Maximum specific output of the intercooling type and simple type and their corresponding pressure ratios
(Neglecting the pressure losses in the intercooler and combustor)

Turbine inlet temperature T_c	Intercooler efficiency η_{ic}	Maximum specific output		Pressure ratio		Compressor work ratio corresponding to this pressure ratio $\frac{(\dot{W}_c/\dot{G}_a)_{ic}}{(\dot{W}_c/\dot{G}_a)_0}$	Supplied heat ratio corresponding to this pressure ratio $\frac{(\dot{Q}/\dot{G}_a)_{ic}}{(\dot{Q}/\dot{G}_a)_0}$
		$\frac{(\dot{W}_E)}{(\dot{G}_a)_{ic}}$	$\frac{(\dot{W}_E/\dot{G}_a)_{ic}}{(\dot{W}_E/\dot{G}_a)_0}$	$(r_{cmL})_{ic}$	$\frac{(r_{cmL})_{ic}}{(r_{cmL})_0}$		
1000°K	1.00	200	1.27	8.56	1.71	0.85	1.38
	0.80	187	1.18	7.20	1.44	0.89	1.25
	0.60	177	1.12	6.29	1.25	0.93	1.16
	Simple type	158	1.00	5.00	1.00	1.00	1.00
1200°K	1.00	314	1.26	13.09	1.96	0.82	1.40
	0.80	295	1.18	10.32	1.50	0.87	1.24
	0.60	279	1.12	9.15	1.33	0.90	1.16
	Simple type	250	1.00	6.88	1.00	1.00	1.00

(b) Maximum specific output of the reheat type and simple type and their corresponding pressure ratios
(Neglecting the pressure loss in the combustor)

Turbine inlet temperature T_c	Type	Maximum specific output		Pressure ratio		Turbine output ratio corresponding to this pressure ratio $\frac{(\dot{W}_t/\dot{G}_a)_{rh}}{(\dot{W}_t/\dot{G}_a)_0}$	Supplied heat ratio corresponding to this pressure ratio $\frac{(\dot{Q}/\dot{G}_a)_{rh}}{(\dot{Q}/\dot{G}_a)_0}$
		$\frac{(\dot{W}_E)}{(\dot{G}_a)_{rh}}$	$\frac{(\dot{W}_E/\dot{G}_a)_{rh}}{(\dot{W}_E/\dot{G}_a)_0}$	$(r_{cmL})_{rh}$	$\frac{(r_{cmL})_{rh}}{(r_{cmL})_0}$		
1000°K	Reheat type	222	1.41	8.56	1.71	1.15	1.53
	Simple type	158	1.00	5.00	1.00	1.00	1.00
1200°K	Reheat type	355	1.42	13.09	1.96	1.18	1.58
	Simple type	250	1.00	6.88	1.00	1.00	1.00

differentiating Eq. (32) with respect to r and setting the result equal to zero.

Thus we get the following biquadratic equation with respect to $\left(\frac{1}{r}\right)^{\frac{\kappa-1}{2\kappa}}$

$$\eta_t \frac{T_c}{T_A} \left(\frac{1}{r}\right)^{2[(\kappa-1)/\kappa]} - \frac{1}{\eta_c} (1-\alpha) \left(\frac{1}{r}\right)^{(1/2)[(\kappa-1)/\kappa]} - \frac{\alpha}{\eta_c} = 0 \quad (33)$$

When the intercooler efficiency $\eta_{ic}=1.00$, α becomes zero. Substituting $\alpha=0$ for Eq. (33), we get

$$(r_{cmL})_{ic} = \left[\eta_c \eta_t \frac{T_c}{T_A} \right]^{\frac{2}{3} \left(\frac{\kappa-1}{\kappa} \right)} \quad (\eta_{ic}=1.00) \quad (34)$$

Here, we get the interesting result that Eq. (34) is equal to that for the one-stage reheat type.

The general solution of the biquadratic Eq. (34) is

$$\left\{ \begin{aligned} \left[\frac{1}{(r_{cmL})_{ic}} \right]^{(1/2)[(\kappa-1)/\kappa]} &= \gamma - i\delta, \quad \gamma = u + v, \quad u = [-q + \sqrt{q^2 + p^3}]^{1/3}, \quad v = [-q - \sqrt{q^2 + p^3}]^{1/3} \\ p &= \frac{1}{12} \frac{T_A}{\eta_c \eta_t T_c} \alpha, \quad q = -\frac{1}{128} \left(\frac{T_A}{\eta_c \eta_t T_c} \right)^2 (1-\alpha)^2, \quad \delta = \left[\gamma^2 + \frac{1}{48} \left\{ -\frac{T_A}{\eta_c \eta_t T_c} (1-\alpha) \right\} \right]^{1/2} \end{aligned} \right\} \quad (35)$$

From this general solution we can get $(r_{cmL})_{ic}$ when the intercooler efficiency $\eta_{ic}=0.60$ and 0.80 . But the numerical calculation of Eq. (33) is more simple than that using Eq. (35), because we can easily get the value of the specific output approximately from Figs. 11 (a) and (b).

3.1.2 Specific output (considering the pressure loss)

From Eq. (14) the compressor work ratio, considering the pressure loss in the intercooler, is given by

$$\left(\frac{P_c}{G_a} \right)_{ic} = \frac{1}{75} \frac{\kappa}{\kappa-1} RT_A \frac{1}{\eta_c} [ar^{(1/2)[(\kappa-1)/\kappa]} - 1] [2 + \alpha \{ ar^{(1/2)[(\kappa-1)/\kappa]} - 1 \}] \quad (14)$$

where $a = \left[\frac{1}{1 - \Delta P_{ic}/P_{B1}} \right]^{\frac{1}{2} \left(\frac{\kappa-1}{\kappa} \right)} \quad (13)$

Here ΔP_{ic} denotes the pressure loss in the intercooler.

The turbine output, considering the pressure loss in the combustor, is given by

$$\frac{P_t}{G_a} = \frac{1}{75} \frac{\kappa}{\kappa-1} RT_c \eta_t \left[1 - b \left(\frac{1}{r} \right)^{\frac{\kappa-1}{\kappa}} \right] \quad (36)$$

where $b = \left[\frac{1}{1 - \Delta P_c/P_B} \right]^{\frac{\kappa-1}{\kappa}} \quad (37)$

and ΔP_c denotes the pressure loss in the combustor.

Therefore, the specific output, considering the pressure loss, is given by

$$\left(\frac{P_E}{G_a} \right)_{ic} = \frac{1}{75} \frac{\kappa}{\kappa-1} RT_A \left[\eta_t \frac{T_c}{T_A} \left\{ 1 - b \left(\frac{1}{r} \right)^{(\kappa-1)/\kappa} \right\} - \frac{1}{\eta_c} \{ ar^{(1/2)[(\kappa-1)/\kappa]} - 1 \} \{ 2 + \alpha [ar^{(1/2)[(\kappa-1)/\kappa]} - 1] \} \right] \quad (38)$$

Fig. 12 Specific output of the intercooling type (considering the pressure losses in the intercooler and combustor)

when the intercooler efficiency $\eta_{ic}=1.00$,

$$\left(\frac{HP_E}{G_a}\right)_{ic} = \frac{1}{75} \frac{\kappa}{\kappa-1} RT_A \left[\eta_c \frac{T_c}{T_A} \left\{ 1 - b \left(\frac{1}{r} \right)^{(\kappa-1)/\kappa} \right\} - \frac{2}{\eta_c} \{ a r^{(1/2)(\kappa-1)/\kappa} - 1 \} \right] \dots (38)'$$

Setting the pressure loss in the intercooler $\Delta P_{ic}/P_{B1}=0.05$ and the pressure loss in the combustor $\Delta P_c/P_B=0.03$, with other assumptions being the same as before, Figs. 12 (a) and (b) show the results calculated from Eqs. (38) (38)'. For purposes of comparison, the specific output of the simple type and reheat type, considering the 3% pressure loss in the combustor, are also shown by the broken curves. In this case the pressure coefficients of Eq. (38) become $a=1.007$ and $b=1.008$.

The general shapes of these curves are similar to those of Figs. 11 (a), (b) neglecting the pressure losses in the intercooler and combustor. But in general, the absolute value decreases because of pressure loss. The curves for specific output for the intercooling type occupy an intermediate position between the simple and one-stage reheat types. But the optimum pressure ratio corresponding to the maximum specific output of the reheat type $(r_{cmL})_{rh}$ is somewhat larger than that of the intercooling type $(r_{cmL})_{ic}$ at the intercooler efficiency of $\eta_{ic}=1.00$. This will be explained in detail later.

The numerical values of the maximum specific output and its corresponding pressure ratio from Figs. 12 (a), (b) are summarized in Tables 7 (a), (b).

Also the numerical values of the compressor work ratio, the supplied heat ratio and the turbine output ratio corresponding to this pressure ratio, read from Fig. 4, Fig. 6, Fig. 9 (a) (b), Fig. 10, are tabulated.

As is evident from Tables 7 (a), (b), the maximum specific output of the intercooling type is greater than that of the simple type by 7% to 23%. This rate of increase is not so large as that when the pressure loss is neglected (12% to 27%).

When compared to the simple type at the same pressure ratio, the compressor work ratio

$$\left(\frac{HP_c}{G_a}\right)_{ic} / \left(\frac{HP_c}{G_a}\right)_0$$

is greater when considering the pressure loss than when neglecting it.

This difference is evident from comparison of Table 7 (a) and Table 6 (b). Comparing the simple type and reheat type, one sees that the rate of increase of the specific output rises from 36% to 39%, because the reheat type suffers two different pressure losses in the combustors. But when neglecting the pressure losses in the combustors, an increase of 41% to 42% is obtained as shown in Table 6.

The turbine output ratio for this case increases by a factor of 1.136 to 1.169. But this increase is smaller than when neglecting the pressure loss, namely 1.153 to 1.182. The maximum specific output of the intercooling type is greater than that of the simple type, as was mentioned above.

Table 7
(a) Maximum specific output of the intercooling type and simple type and their corresponding pressure ratios
(Considering the pressure losses in the intercooler and combustor)

Turbine inlet temperature T_c	Intercooler efficiency η_{ic}	Maximum specific output		Pressure ratio		Compressor work ratio corresponding to this pressure ratio $\left(\frac{HP_c}{G_a}\right)_{ic} / \left(\frac{HP_c}{G_a}\right)_0$	Supplied heat ratio corresponding to this pressure ratio $\left(\frac{Q}{G_a}\right)_{ic} / \left(\frac{Q}{G_a}\right)_0$
		$\left(\frac{HP_E}{G_a}\right)_{ic}$	$\frac{(HP_E/G_a)_{ic}}{(HP_E/G_a)_0}$	$(r_{cmL})_{ic}$	$\frac{(r_{cmL})_{ic}}{(r_{cmL})_0}$		
1000°K	1.00	185	1.22	8.57	1.69	0.87	1.37
	0.80	172	1.13	7.19	1.41	0.92	1.26
	0.60	161	1.06	6.25	1.23	0.96	1.14
	Simple type	151	1.00	5.08	1.00	1.00	1.00
1200°K	1.00	298	1.23	13.13	1.88	0.84	1.53
	0.80	278	1.14	10.86	1.55	0.89	1.24
	0.60	261	1.07	8.95	1.28	0.95	1.14
	Simple type	243	1.00	6.99	1.00	1.00	1.00

(b) Maximum specific output of the reheat type and simple type
(Considering the pressure loss in the combustor)

Turbine inlet temperature T_c	Type	Maximum specific output		Pressure ratio		Turbine output ratio corresponding to this pressure ratio $\left(\frac{HP_t}{G_a}\right)_{rh} / \left(\frac{HP_t}{G_a}\right)_0$	Supplied heat ratio corresponding to this pressure ratio $\left(\frac{Q}{G_a}\right)_{rh} / \left(\frac{Q}{G_a}\right)_0$
		$\left(\frac{HP_E}{G_a}\right)_{rh}$	$\frac{(HP_E/G_a)_{rh}}{(HP_E/G_a)_0}$	$(r_{cmL})_{ic}$	$\frac{(r_{cmL})_{ic}}{(r_{cmL})_0}$		
1000°K	Reheat type	206	1.36	8.73	1.72	1.14	1.53
	Simple type	151	1.00	5.08	1.00	1.00	1.00
1200°K	Reheat type	337	1.39	13.36	1.91	1.17	1.58
	Simple type	243	1.00	6.99	1.00	1.00	1.00

The optimum pressure ratio corresponding to the maximum specific output $(r_{cmL})_{ic}$ is given from Eq. (32)

$$\eta_t \frac{T_c}{T_A} \left(\frac{1}{r} \right)^{2[(\kappa-1)/\kappa]} - \frac{1}{\eta_c} \frac{a}{b} (1-\alpha) \left(\frac{1}{r} \right)^{(1/2)[(\kappa-1)/\kappa]} - \frac{1}{\eta_c} \frac{a^2}{b} \alpha = 0 \quad (39)$$

Here, we get the biquadratic equation; when intercooler efficiency $\eta_{ic}=1.00$, α become zero. When substituting $\alpha=0$ into Eq. (39), we have

$$(r_{cmL})_{ic} = \left[\eta_c \eta_t \frac{T_c}{T_A} \frac{b}{a} \right]^{\frac{2}{3} \left(\frac{\kappa-1}{\kappa} \right)} \quad (\eta_{ic}=1.00) \quad (40)$$

Setting $\eta_{ic}=1.00$, Eq. (40) corresponds completely with that for the reheat type when considering the pressure losses in the combustors.

Assuming an intercooler efficiency of $\eta_{ic}=1.00$, the optimum pressure ratio corresponding to the maximum specific output $(r_{cmL})_{ic}$ of the intercooling type is smaller than that of the reheat type $(r_{cmL})_{rh}$, because the pressure coefficient a is greater than 1.

The general solution of the biquadratic Eq. (39) is

$$\left[\frac{1}{(r_{cmL})_{ic}} \right]^{(1/2)[(\kappa-1)/\kappa]} = \gamma - i\delta, \quad \gamma = u + v, \quad u = [-q + \sqrt{q^2 + p^3}]^{1/3}, \quad v = [-q - \sqrt{q^2 + p^3}]^{1/3} \quad (41)$$

$$p = \frac{1}{12} \frac{T_A}{\eta_c \eta_t T_c} \frac{a^2}{b} \alpha, \quad q = -\frac{1}{128} \left(\frac{T_A}{\eta_c \eta_t T_c} \right)^2 \left(\frac{a}{b} \right)^2 (1-\alpha)^2, \quad \delta = \left[\gamma^2 + \frac{1}{48} \left\{ -\frac{T_A}{\eta_c \eta_t T_c} \frac{a}{b} (1-\alpha) \right\} \right]^{1/2}$$

From this general solution we can get the value corresponding to the case of $\eta_{ic}=0.80$ and 0.60 , but as described before, the numerical calculation of Eq. (39) is more simple.

3.2 Analysis by the approximate calculation of thermal efficiency

3.2.1 Thermal efficiency (neglecting the pressure loss)

First we describe the thermal efficiency neglecting the pressure losses in the intercooler and combustor. As described in Section 2.4, the heat supplied in the combustor of the intercooling type is larger than that supplied for the simple type, because of the cooling effect in the intercooler.

From Eq. (28) the heat supplied per unit rate of air is

$$\left(\frac{Q}{G_a} \right)_{ic} = i_{TC} - i_{TB2} = \frac{\kappa}{\kappa-1} ART_A \left[\left(\frac{T_c}{T_A} - 1 \right) - \{ r^{(1/2)[(\kappa-1)/\kappa]} - 1 \} \left\{ \left(\frac{1}{\eta_c} + \alpha \right) + \frac{\alpha}{\eta_c} [r^{(1/2)[(\kappa-1)/\kappa]} - 1] \right\} \right] \quad (42)$$

The specific output in this case is given from Eq. (32), and so after equalizing the dimension of Eq. (32) and Eq. (42), we get the thermal efficiency.

$$(\eta)_{ic} = \eta_B \frac{\eta_t \frac{T_c}{T_A} \left[1 - \left(\frac{1}{r} \right)^{(\kappa-1)/\kappa} \right] - \frac{1}{\eta_c} [r^{(1/2)[(\kappa-1)/\kappa]} - 1] [2 + \alpha \{ r^{(1/2)[(\kappa-1)/\kappa]} - 1 \}]}{\left(\frac{T_c}{T_A} - 1 \right) - [r^{(1/2)[(\kappa-1)/\kappa]} - 1] \left[\left(\frac{1}{\eta_c} + \alpha \right) - \frac{\alpha}{\eta_c} \{ r^{(1/2)[(\kappa-1)/\kappa]} - 1 \} \right]} \quad (43)$$

Fig. 13 Thermal efficiency of the intercooling type (neglecting the pressure losses in the intercooler and combustor)

where η_B denotes the combustion efficiency in the combustor.

When the intercooler efficiency $\eta_{ic}=1.00$, α becomes zero, thus

$$(\eta)_{ic} = \eta_B \frac{\eta_{ic} \frac{T_c}{T_A} \left[1 - \left(\frac{1}{r} \right)^{(\kappa-1)/\kappa} \right] - \frac{2}{\eta_c} \left[r^{(1/2)(\kappa-1)/\kappa} - 1 \right]}{\left(\frac{T_c}{T_A} - 1 \right) - \frac{1}{\eta_c} \left[r^{(1/2)(\kappa-1)/\kappa} - 1 \right]} \quad (\eta_{ic}=1.00) \quad (43)'$$

Figures 13 (a), (b) show the results calculated from Eq. (43) setting the same condition as the specific output and assuming the combustion efficiency $\eta_B=0.95$. For purposes of comparison, the result of the simple type and the reheat type are also drawn with the broken curves.

As is evident from Figs. 13 (a) (b), the thermal efficiency of the simple type $(\eta)_0$ is larger than that of the intercooling type $(\eta)_{ic}$, before the pressure ratio of the former reaches its maximum value $(\eta_{max})_0$. In the region of the pressure ratio beyond the maximum, the thermal efficiency of the simple type decreases rapidly, but the intercooling type maintains a greater value than the simple type. Also when comparing the reheat type and the simple type, the thermal efficiency of the simple type $(\eta)_0$ is greater than that of the reheat type $(\eta)_{rh}$ before the former reaches its maximum value, and in this region of pressure ratio, the value of the reheat type is somewhat smaller than that of the intercooling type.

In the neighborhood of the maximum value

$(\eta_{max})_{rh}$, the thermal efficiency of the reheat type is the greatest, and its optimum pressure ratio $(r_{cmr})_{rh}$ is also the highest.

The numerical values of the maximum thermal efficiency and its optimum pressure ratio from Figs. 13 (a), (b) are summarized in Tables 8(a), (b). Furthermore, the numerical value of the compressor work ratio, the supplied heat ratio, and the turbine output ratio at this pressure ratio are tabulated.

As is evident from Tables 8 (a), (b) there are no large differences between the maximum thermal efficiency of the intercooling type $(\eta_{max})_{ic}$ and that of the simple type $(\eta_{max})_0$ and the value of the former increases with increasing intercooler efficiency η_{ic} .

Because of the effect of intercooling, the compressor work ratio decreases by a factor of 0.79 to 0.92, and increases the specific output. On the other hand, the heat supplied in the combustor increases by a factor of 1.59 to 1.24. Therefore, these two effects cancel each other and the maximum thermal efficiency remains nearly equal to that of the simple type.

Also in comparing the reheat and the simple types, the specific output increases as the turbine output ratio increases by a factor of 1.178 to 1.215, but in the same manner the heat supplied increases by 1.73 to 1.82. Therefore, these two effects cancel each other and the maximum thermal efficiencies of these two types becomes nearly equal.

Table 8
(a) Maximum thermal efficiency of the intercooling type and simple type and their corresponding pressure ratios
(Neglecting the pressure losses in the intercooler and combustor)

Turbine inlet temperature T_c	Intercooler efficiency η_{ic}	Maximum thermal efficiency		Pressure ratio		Supplied heat ratio corresponding to this pressure ratio $\left(\frac{Q}{G_a} \right)_{ic} / \left(\frac{Q}{G_a} \right)_0$	Compressor work ratio corresponding to this pressure ratio $\left(\frac{HP_c}{G_a} \right)_{ic} / \left(\frac{HP_c}{G_a} \right)_0$
		$(\eta_{max})_{ic}$	$\frac{(\eta_{max})_{ic}}{(\eta_{max})_0}$	$(r_{cmr})_{ic}$	$\frac{(r_{cmr})_{ic}}{(r_{cmr})_0}$		
1000°K	1.00	0.240	1.04	11.64	1.43	1.54	0.83
	0.80	0.234	1.02	10.54	1.30	1.38	0.87
	0.60	0.230	1.00	9.46	1.16	1.24	0.92
	Simple type	0.230	1.00	8.13	1.00	1.00	1.00
1200°K	1.00	0.292	1.03	19.28	1.50	1.59	0.79
	0.80	0.287	1.01	16.50	1.28	1.40	0.85
	0.60	0.282	0.99	14.76	1.15	1.26	0.95
	Simple type	0.285	1.00	12.84	1.00	1.00	1.00

(b) Maximum thermal efficiency of the reheat type and simple type
(Neglecting the pressure loss in the combustor)

Turbine inlet temperature T_c	Type	Maximum thermal efficiency		Pressure ratio		Supplied heat ratio corresponding to this pressure ratio $\left(\frac{Q}{G_a} \right)_{rh} / \left(\frac{Q}{G_a} \right)_0$	Turbine output ratio corresponding to this pressure ratio $\left(\frac{HP_t}{G_a} \right)_{rh} / \left(\frac{HP_t}{G_a} \right)_0$
		$(\eta_{max})_{rh}$	$\frac{(\eta_{max})_{rh}}{(\eta_{max})_0}$	$(r_{cmr})_{rh}$	$\frac{(r_{cmr})_{rh}}{(r_{cmr})_0}$		
1000°K	Reheat type	0.244	1.06	12.40	1.53	1.73	1.18
	Simple type	0.230	1.00	8.13	1.00	1.00	1.00
1200°K	Reheat type	0.296	1.04	21.00	1.64	1.82	1.22
	Simple type	0.285	1.00	12.84	1.00	1.00	1.00

The pressure ratio at the maximum thermal efficiency $(r_{cm\eta})_{ic}$ of the intercooling type is determined by differentiating Eq. (43) with respect to r and setting the result equal to zero. Thus we get the following fifth-degree equation with respect to $\left(\frac{1}{r}\right)^{\frac{1}{2}(\frac{\kappa-1}{\kappa})}$

$$\eta_i \frac{T_c}{T_A} \left[\frac{T_c}{T_A} + (1-\alpha) \left(\frac{1}{\eta_c} - 1 \right) \right] \left(\frac{1}{r} \right)^{5/2[(\kappa-1)/\kappa]} - \frac{3}{2} \eta_i \frac{T_c}{T_A} \left[\frac{1-2\alpha}{\eta_c} + \alpha \right] \left(\frac{1}{r} \right)^{2[(\kappa-1)/\kappa]} - 2 \eta_i \frac{T_c}{T_A} \frac{\alpha}{\eta_c} \left(\frac{1}{r} \right)^{(3/2)[(\kappa-1)/\kappa]} - \left[\frac{T_c}{T_A} \left\{ \frac{1-\alpha}{\eta_c} - \frac{1}{2} \eta_i \left(\frac{1-2\alpha}{\eta_c} + \alpha \right) \right\} - \frac{1}{\eta_c} \left\{ (1-\alpha) \frac{1}{2} \alpha^2 \right\} + \frac{1}{2} \left(\frac{1}{\eta_c} \right)^2 \alpha \right] \left(\frac{1}{r} \right)^{(\kappa-1)/\kappa} - \frac{\alpha}{\eta_c} \left[\frac{T_c}{T_A} (1-\eta_i) - \frac{1}{\eta_c} - (1-\alpha) \right] \left(\frac{1}{r} \right)^{(1/2)[(\kappa-1)/\kappa]} - \frac{1}{2} \frac{\alpha}{\eta_c} \left[\frac{1}{\eta_c} - \alpha \right] = 0 \quad (44)$$

when $\eta_{ic}=1.00$, α become zero and Eq. (44) reduces to the following cubic equation.

$$\eta_i \frac{T_c}{T_A} \left[\frac{T_c}{T_A} + \frac{1}{\eta_c} - 1 \right] \left(\frac{1}{r} \right)^{3/2[(\kappa-1)/\kappa]} - \frac{3}{2} \eta_i \frac{T_c}{T_A} \frac{1}{\eta_c} \left(\frac{1}{r} \right)^{(\kappa-1)/\kappa} - \frac{1}{\eta_c} \left[\frac{T_c}{T_A} \left(1 - \frac{1}{2} \eta_i \right) - 1 \right] = 0 \quad (\eta_{ic}=1.00) \quad (44)'$$

The general solution of Eq. (44) must be solved by numerical calculation, but Eq. (44)' can be rewritten as follows.

$$\left[\frac{T_c}{T_A} \left(1 - \frac{1}{2} \eta_c - 1 \right) \right] r^{(3/2)[(\kappa-1)/\kappa]} + \frac{3}{2} \eta_i \frac{T_c}{T_A} r^{(1/2)[(\kappa-1)/\kappa]} - \eta_c \eta_i \left[\frac{T_c}{T_A} + \frac{1}{\eta_c} - 1 \right] = 0 \quad (45)$$

Thus we can get the general solution of Eq. (45)

$$\left. \begin{aligned} (r_{cm\eta})_{ic}^{(1/2)[(\kappa-1)/\kappa]} &= u + v, \quad u = [-q + \sqrt{q^2 + p^3}]^{1/3}, \quad v = [-q - \sqrt{q^2 + p^3}]^{1/3} \\ p &= \frac{1}{2} \frac{\eta_i \frac{T_c}{T_A}}{\frac{T_c}{T_A} \left(1 - \frac{1}{2} \eta_i \right) - 1}, \quad q = -\frac{1}{2} \frac{\eta_c \eta_i \frac{T_c}{T_A} \left[\frac{T_c}{T_A} + \frac{1}{\eta_c} - 1 \right]}{\frac{T_c}{T_A} \left(1 - \frac{1}{2} \eta_i \right) - 1} \end{aligned} \right\} \quad (46)$$

3.2.2 Thermal efficiency (considering the pressure loss)

Equation (42) for the heat supplied in the combustor, considering the pressure loss in the intercooler, becomes

$$\left(\frac{Q}{G_a} \right)_{ic} = \frac{\kappa}{\kappa-1} ART_A \left[\left(\frac{T_c}{T_A} - 1 \right) - \{ ar^{(1/2)[(\kappa-1)/\kappa]} - 1 \} \left\{ \left(\frac{1}{\eta_c} + \alpha \right) + \frac{\alpha}{\eta_c} [ar^{(1/2)[(\kappa-1)/\kappa]} - 1] \right\} \right] \quad (47)$$

Equalizing the dimension of Eq. (47) and Eq. (38) of the specific output, the equation for the thermal efficiency can be written

$$(\eta)_{ic} = \eta_B \frac{\eta_i \frac{T_c}{T_A} \left[1 - b \left(\frac{1}{r} \right)^{(\kappa-1)/\kappa} \right] - \frac{1}{\eta_c} [ar^{(1/2)[(\kappa-1)/\kappa]} - 1] [2 + \alpha \{ ar^{(1/2)[(\kappa-1)/\kappa]} - 1 \}]}{\left(\frac{T_c}{T_A} - 1 \right) - [ar^{(1/2)[(\kappa-1)/\kappa]} - 1] \left[\left(\frac{1}{\eta_c} + \alpha \right) + \frac{\alpha}{\eta_c} \{ ar^{(1/2)[(\kappa-1)/\kappa]} - 1 \} \right]} \quad (48)$$

Fig. 14 Thermal efficiency of the intercooling type (considering the pressure losses in the intercooler and combustor)

When the intercooler efficiency $\eta_{ic}=1.00$, α becomes zero. Thus

$$(\eta)_{ic} = \eta_B \frac{\eta_{ic} \frac{T_c}{T_A} \left[1 - b \left(\frac{1}{r} \right)^{\frac{\kappa-1}{\kappa}} \right] - \frac{2}{\eta_c} \left[ar^{\frac{1}{2}} \left(\frac{\kappa-1}{\kappa} \right) - 1 \right]}{\left(\frac{T_c}{T_A} - 1 \right) - \frac{1}{\eta_c} \left[ar^{\frac{1}{2}} \left(\frac{\kappa-1}{\kappa} \right) - 1 \right]} \quad \dots\dots\dots(48)'$$

Figures 14 (a), (b) show the results calculated from Eq. (48), under the same conditions as before, and assuming the pressure loss in the intercooler to be 5%, and that in the combustor to be 3%. For purposes of comparison, the results of the simple type and reheat type, considering the 3% of pressure loss in the combustor, are also drawn in broken curves.

The general shapes of these curves are similar to those of Figs. 13 (a) (b), neglecting the pressure loss.

In general, however, the absolute value decreases on account of pressure loss. Furthermore, the pressure ratio $r_{cm\eta}$ at the maximum thermal efficiency of each type decreases slightly.

As is evident from Eq. (48), the heat supplied, considering the pressure loss in the intercooler, is slightly smaller than that of neglecting this pressure loss. But from Eq. (38), the drop in specific output, considering the pressure losses in the intercooler and combustor, is larger than that when neglecting it. Therefore, in general, the thermal efficiency is smaller when considering the pressure loss than when

neglecting it.

The thermal efficiency of the simple type η_0 is greater than that of the intercooling type η_{ic} and the reheat type η_{rh} before the pressure ratio of the simple type reaches its optimum value. Furthermore, this difference is greater than that when neglecting the pressure loss in Figs. 13 (a), (b).

The numerical values of the maximum thermal efficiency and its corresponding pressure ratio from Figs. 14 (a), (b) are summarized in Tables 9 (a), (b). Further, the numerical value of the compressor work ratio, supplied heat ratio and the turbine output ratio corresponding to this pressure ratio are tabulated.

As is evident from Tables 9 (a), (b), when the intercooler efficiency $\eta_{ic}=1.00$, the maximum thermal efficiency $(\eta_{max})_{ic}$ of the intercooling type is nearly equal to that of the simple type $(\eta_{max})_0$. When the intercooler efficiency $\eta_{ic}=0.80$ and 0.60 the maximum thermal efficiency is nearly equal to that of the simple type. As when neglecting the pressure loss, the maximum thermal efficiency of the reheat type $(\eta_{max})_{rh}$ is a little greater than any of the others.

The optimum pressure ratio to the maximum thermal efficiency of the intercooling type $(r_{cm\eta})_{ic}$ is determined by differentiating Eq. (48) with respect to r , thus we get the following fifth-degree equation with respect to $\left(\frac{1}{r} \right)^{\frac{1}{2}} \left(\frac{\kappa-1}{\kappa} \right)$

Table 9
(a) Maximum thermal efficiency of the intercooling type and simple type and their corresponding pressure ratios
(Considering the pressure losses in the intercooler and combustor)

Turbine inlet temperature T_c	Intercooler efficiency η_{ic}	Maximum thermal efficiency		Pressure ratio		Supplied heat ratio corresponding to this pressure ratio $\left(\frac{Q}{G_a} \right)_{ic} / \left(\frac{Q}{G_a} \right)_0$	Compressor work ratio corresponding to this pressure ratio $\left(\frac{HP_c}{G_a} \right)_{ic} / \left(\frac{HP_c}{G_a} \right)_0$
		$(\eta_{max})_{ic}$	$\frac{(\eta_{max})_{ic}}{(\eta_{max})_0}$	$(r_{cm\eta})_{ic}$	$\frac{(r_{cm\eta})_{ic}}{(r_{cm\eta})_0}$		
1000°K	1.00	0.223	1.01	11.47	1.41	1.54	0.85
	0.80	0.216	0.98	10.10	1.25	1.36	0.90
	0.60	0.210	0.95	8.97	1.11	1.22	0.94
	Simple type	0.221	1.00	8.11	1.00	1.00	1.00
1200°K	1.00	0.279	1.01	19.02	1.51	1.56	0.81
	0.80	0.272	0.98	16.04	1.27	1.36	0.87
	0.60	0.266	0.96	14.13	1.12	1.23	0.92
	Simple type	0.277	1.00	12.12	1.00	1.00	1.00

(b) Maximum thermal efficiency of the reheat type and simple type
(Considering the pressure loss in the combustor)

Turbine inlet temperature T_c	Type	Maximum thermal efficiency		Pressure ratio		Supplied heat ratio corresponding to this pressure ratio $\left(\frac{Q}{G_a} \right)_{rh} / \left(\frac{Q}{G_a} \right)_0$	Turbine output ratio corresponding to this pressure ratio $\left(\frac{HP_t}{G_a} \right)_{rh} / \left(\frac{HP_t}{G_a} \right)_0$
		$(\eta_{max})_{rh}$	$\frac{(\eta_{max})_{rh}}{(\eta_{max})_0}$	$(r_{cm\eta})_{rh}$	$\frac{(r_{cm\eta})_{rh}}{(r_{cm\eta})_0}$		
1000°K	Reheat type	0.229	1.04	12.29	1.52	1.71	1.16
	Simple type	0.221	1.00	8.11	1.00	1.00	1.00
1200°K	Reheat type	0.284	1.04	20.94	1.73	1.80	1.20
	Simple type	0.277	1.00	12.12	1.00	1.00	1.00

$$\begin{aligned} \eta_i \frac{T_c}{T_A} \left[\frac{T_c}{T_A} + (1-\alpha) \left(\frac{1}{\eta_c} - 1 \right) \right] b \left(\frac{1}{r} \right)^{(5/2)\{(k-1)/k\}} - \frac{3}{2} \eta_i \frac{T_c}{T_A} \left[\frac{1}{\eta_c} (1-2\alpha) + \alpha \right] ab \left(\frac{1}{r} \right)^{2\{(k-1)/k\}} \\ - 2 \eta_i \frac{T_c}{T_A} \frac{\alpha}{\eta_c} a^2 b \left(\frac{1}{r} \right)^{(3/2)\{(k-1)/k\}} - \left[\frac{T_c}{T_A} \left\{ \frac{1-\alpha}{\eta_c} - \frac{1}{2} \eta_i \left(\frac{1-2\alpha}{\eta_c} + \alpha \right) \right\} - \frac{1}{\eta_c} \left\{ (1-\alpha) + \frac{1}{2} \alpha^2 \right\} \right] \\ + \frac{1}{2} \left(\frac{1}{\eta_c} \right)^2 \alpha \left[\frac{1}{r} \right]^{(k-1)/k} - \frac{\alpha}{\eta_c} \left[\frac{T_c}{T_A} (1-\eta_i) - \frac{1}{\eta_c} - (1-\alpha) \right] a^2 \left(\frac{1}{r} \right)^{1/2\{(k-1)/k\}} - \frac{1}{2} \frac{\alpha}{\eta_c} \left[\frac{1}{\eta_c} - \alpha \right] a^3 = 0 \end{aligned} \quad (49)$$

When $\eta_{ic}=1.00$ α becomes zero and Eq. (49) reduces to the following cubic equation.

$$\eta_i \frac{T_c}{T_A} \left[\frac{T_c}{T_A} + \frac{1}{\eta_c} - 1 \right] b \left(\frac{1}{r} \right)^{(3/2)\{(k-1)/k\}} - \frac{3}{2} \eta_i \frac{T_c}{T_A} \frac{1}{\eta_c} ab \left(\frac{1}{r} \right)^{(k-1)/k} - \frac{1}{\eta_c} \left[\frac{T_c}{T_A} \left(1 - \frac{1}{2} \eta_i \right) - 1 \right] a = 0 \quad (49)'$$

The general solution of Eq. (49) must be obtained by numerical calculation, but when $\eta_{ic}=1.00$ Eq. (49)' can be rewritten as

$$\frac{T_c}{T_A} \left[\left(1 - \frac{1}{2} \eta_i \right) - 1 \right] r^{(3/2)\{(k-1)/k\}} + \frac{3}{2} \eta_i \frac{T_c}{T_A} b r^{1/2\{(k-1)/k\}} - \eta_i \frac{T_c}{T_A} \left[\frac{T_c}{T_A} + \frac{1}{\eta_c} - 1 \right] \frac{b}{a} = 0 \quad (50)$$

Thus we can get the general solution of Eq. (50)

$$(r_{cmr})_{ic}^{(1/2)\{(k-1)/k\}} = u + v, \quad u = [-q + \sqrt{q^2 + p^3}]^{1/3}, \quad v = [-q - \sqrt{q^2 + p^3}]^{1/3} \\ p = \frac{1}{2} \frac{\eta_i \frac{T_c}{T_A} b}{\frac{T_c}{T_A} \left(1 - \frac{1}{2} \eta_i \right) - 1}, \quad q = -\frac{1}{2} \frac{\eta_i \eta_c \frac{T_c}{T_A} \left[\frac{T_c}{T_A} + \frac{1}{\eta_c} - 1 \right] b}{\frac{T_c}{T_A} \left(1 - \frac{1}{2} \eta_i \right) - 1} \quad (51)$$

4. Conclusion

As the result of the approximate calculation, the following conclusions have been drawn.

1) The specific output of the intercooling type, as compared with the simple type, can be increased quite a bit, but the increase is inferior to the rate of increase of the reheat type. The optimum pressure ratio corresponding to the maximum specific output is greater than that of the simple type. Furthermore, the specific output increases with increasing intercooler efficiency η_{ic} , and at the same time, the optimum pressure ratio corresponding to the maximum specific output also moves to a higher value. When $\eta_{ic}=1.00$, this pressure ratio is nearly equal to that of the reheat type. Therefore, as in the case of the reheat type, to increase the specific output the intercooling type must be used in the region of fairly higher pressure ratios.

2) The maximum thermal efficiency of the

intercooling type is nearly equal to that of the simple type, and decreases more than the simple type with decreasing intercooler efficiency η_{ic} . The simple type, after reaching its maximum thermal efficiency, decreases rapidly in this region of the pressure ratio. But the intercooling type maintains a fairly high thermal efficiency in this region. This relationship also holds good when comparing the reheat and the simple types. The pressure ratio corresponding to the specific output is not identical with that of the thermal efficiency. In general, the latter has a value somewhat greater than the former. Therefore, when operating the intercooler at the pressure ratio corresponding to the maximum specific output, the thermal efficiency is lower than that of the simple type. The reheat type also shows the same tendency.

Reference

- (1) K. Tanaka: *Trans. Japan Soc. Mech. Engrs.*, Vol. 33, No. 253 (1967), p. 1450.

Discussion

K. MIWA: (1) In this paper, the meanings of the author's "approximate calculation method" and the "precise calculation method" are not clear. Explain the definitions of these terms.

R. KAWAHARA and T. IWAYA: (2) Generally the air for the cooling and the sealing of the turbine system is supplied by the air bled from the compressor. I think that particularly in the range of $T_c > 1000^\circ\text{K}$ the rate of air for cooling is not negligible. Therefore, I would like to know the author's opinion on how the specific output and thermal efficiency change with the change of the rate of bled air.

E. ISHIBASHI: (3) In this paper, the calculation is performed for different combustor outlet temperatures. On that occasion the rate of fuel flow leads to an increase of the working fluid flow. Have you considered this point? Further, I think the particular properties of the working fluid should be considered in the calculation of specific output.

G. T. SATO and K. MIWA: (4) In the case of the intercooling type, the combustor inlet temperature T_{B2} falls and the supplied heat ratio increases. Thus at a lower pressure ratio ($r < 10$) the thermal efficiency of the simple type is larger than that of the

intercooling type. In the case of the reheat type, the supplied heat ratio increases and at the same time the turbine outlet temperature T_{D2} rises, so the amount of heat escaping in the exhaust gases increases. For this reason it is also advantageous to operate the simple type at a lower pressure ratio. Thus in order to remove the defects of each type, a regenerative heat exchanger should be used, for example, intercooling-regeneration, regeneration-reheating, and intercooling-regeneration-reheating. Therefore, in practice, it is always necessary to consider some combination with regeneration. What do you think about these points?

R. KAWAHARA: (5) In this paper, the intercooling type and the reheat type are discussed separately and these analyses are based on the assumption that the pressure ratio and expansion ratio for each type are equal. In general, most of reheat types are the 2-shaft intercooling-reheat type and in this case the pressure ratio of each compression stage can be made equal, but can not be made equal for the expansion ratio by means of the power balance of the compressor and turbine. Therefore, it would be profitable to add an analysis of the combination of the 2-shaft type. I would like to know the opinion of the authors on this point.

G. T. SATO and T. IWAYA: (6) To minimize the total compression work, the division of the compressor pressure ratio of each stage is not equal, when η_{ic} is not equal to 1. How does the specific output changes according to the division of pressure ratio? Also, the pressure ratio division to maximize the thermal efficiency is different from that of the specific output. I would like to know more about this difference and its effect upon the total performance.

G. T. SATO and E. ISHIBASHI: (7) The thermal effectiveness of the intercooler η_{ic} is defined by Eq. (1), the denominator of this equation is defined as a temperature rise in the first-stage compressor. But from the essential meaning of the temperature effectiveness of the intercooler, I think that the temperature T_A in the denominator should be considered to be the cooling-water temperature, which is based on the assumption that the cooling water temperature is equal to atmospheric temperature. What do you think about this?

R. KAWAHARA: (8) The temperature effectiveness of the intercooler η_{ic} is expressed by Eq. (1). In general, the temperature T_A in Eq. (1) is the temperature of the cooling medium (usually cold water) at the inlet of the intercooler. I think this temperature is usually lower than T_A from 5° to 10°C. Explain the effect of this temperature difference upon the analysis.

N. KAWAHARA and E. ISHIBASHI: (9) When atmospheric temperature and humidity are high, may one consider the effect of the humidity of air intake to be negligibly small?

G. T. SATO and N. KAWAHARA: (10) If the authors perform these calculations in such detail, I think it is better to treat the pressure loss as a function of η_{ic} , corresponding to the value of η_{ic} . From this point of view, I see the possibility of causing a misunderstanding, when the pressure loss and the intercooler efficiency η_{ic} are separately discussed, for example, "to obtain the effect of the intercooling it is necessary to decrease the pressure loss of the intercooler. Better results are obtained, however, by increasing the intercooler efficiency η_{ic} ." at the end of the left hand column on page 1213.

K. MIWA: (11) The thermal efficiency of the intercooling type (or reheat type) is lower than that of the simple type without regeneration and that with no pressure loss, so that the difference of the thermal efficiency between these two types depends on the loss of each component, and especially depends on the efficiency of the compressor and turbine. Therefore, there remains the question of how to reach any conclusion about the thermal efficiency from the result of a specified combination of η_c and η_i . If possible, explain how the calculated result would vary over certain ranges of these values.

G. T. SATO: (12) Among other factors, I think the increase of pressure ratio due to the temperature drop at the compressor inlet and the improvement of the compressor efficiency due to the drop in the pressure ratio of each stage should be still more carefully considered. What do you think about this?

K. MIWA: (13) In this paper, the calculation is performed for the turbine inlet temperature T_c of 1 000°K and 1 200°K, but to see better the effect of T_c and also to evaluate performance at the higher inlet temperatures recently made possible, I hope you will add, for example, the calculated result for $T_c = 1 400^\circ\text{K}$.

T. IWAYA: (14) From Figs. 12 and 14, it would seem reasonable, in practice for the designer to select some pressure ratio close to that corresponding to the maximum thermal efficiency as the optimum design point of the intercooling type, because specific output does not decrease very much in the range $\eta_{ic} > 0.80$. Isn't there any change in the behavior of this cycle under the effects of the difference in the particular properties of the working fluid and in the temperature in the compressor and the turbine?

T. IKUI: (15) Following the previous report, the troublesome calculation on the gas turbine has

been performed in this paper. The intercooling type, however, is sub-classified into single-shaft and double-shaft types. Furthermore, there are high pressure and low pressure types in the turbine and combustor. It is consequently very tedious to calculate these separately. Therefore, I would like to suggest to adopt either the general calculation chart (carpet diagram) covering various types of gas turbine or the method using a computer solution.

Authors' closure

(1) The approximate calculation method presented in this paper is based on the following assumptions:

1) The working fluid is air, and its specific heat and specific heat ratio are constant with temperature.

2) There is no difference in the mass flow of working fluid at any point in the cycle.

3) The adiabatic efficiency of the compressor η_c , the adiabatic efficiency of the turbine η_t and the combustion efficiency of the combustor η_B are considered respectively.

4) The pressure losses in the intercooler and the combustor are considered.

On the other hand, the precise calculation method is based on the following assumptions:

1) There is a change in the substance of the working fluid. From the compressor inlet to the combustor inlet there is air, and from the combustor outlet to the turbine outlet there is a combustion gas. Furthermore, the mass increase of the working fluid due to the fuel injection is considered.

2) The specific heat and the specific heat ratio of the working fluid change with temperature.

3) The adiabatic efficiency of the compressor η_c , the adiabatic efficiency of the turbine η_t and the combustion efficiency of the combustor η_B are considered respectively.

4) The pressure losses in the intercooler and the combustor are considered.

A comparison of these two methods will be presented in a 3rd report.

(2) The approximate calculation method presented in this paper, assumes no difference in the mass flow of working fluid at any point in the cycle, and the amount of the air bled is not considered. In general, however, it can also be assumed that the amount of air bled for cooling is about 2% of the total amount of air flow, and that it is equal to the weight of fuel supplied in the combustor. In the precise calculation, the authors intend to consider the effect of the bled air ratio.

(3) The approximate calculation analysis presented in this paper is based on the assumption that

the mass flow of working fluid is constant as it flows through the cycle. In the precise calculation, the authors consider the point you mention and it will be treated in a 3rd report.

(4) The authors agree and the study of a combination with regeneration has been already completed. In this first report, the case without regeneration was investigated in order to examine the special character of intercooling. From the 2nd report on, the cases with regeneration and with reheat, etc, are to be taken up.

(5) In this paper, the intercooling type and the reheat type are discussed separately, in order to stress the special character of intercooling. Since the compression and expansion ratios are not equal in an actual two-shaft gas turbine, the authors intend to report on these points later.

(6) The pressure at which the intercooling should best be done may be determined in the following ways: to get the maximum specific output, to get the maximum thermal efficiency, and to get the minimum compression work. Furthermore, it is necessary to consider the division of turbine output or the matching of each component. Therefore, we cannot determine this absolutely unconditionally. Provided that the total pressure ratio of the compressor is r , the pressure ratio and the adiabatic efficiency of the first-stage compressor are r_1 and η_{c1} respectively, and the pressure ratio and the adiabatic efficiency of the second-stage compressor are r_2 and η_{c2} respectively. Then the first-stage pressure ratio, r_1 corresponding to the minimum compression work, when $\eta_{ic} \neq 1.00$ is given by

$$r_1 = \left(\frac{\eta_{c1} - 1 + \eta_{ic}}{\eta_{c2} - 1 + \eta_{ic}} r^{(\kappa-1)/\kappa} \right)^{\kappa/2(\kappa-1)},$$

where $r = r_1 r_2$

and when $\eta_{c1} = \eta_{c2}$, then we have $r_1 = r^{1/2}$

Next, the first stage pressure ratio r_1 corresponding to the maximum thermal efficiency is determined from the following biquadratic equation with respect to $r_1^{(\kappa-1)/\kappa}$

$$\begin{aligned} & \left[\frac{T_c}{T_A} \left(\frac{1 - \eta_{c1} - \eta_{c2}}{\eta_{c1} \eta_{c2}} \right) + \left(1 - \frac{1}{\eta_{c2}} \right) \left\{ \frac{1}{\eta_{c1}} + \frac{1 - \eta_{ic}}{\eta_{c1}} \frac{T_c}{T_A} \right. \right. \\ & \quad \times \eta_t \left(1 - \frac{1}{r^{(\kappa-1)/\kappa}} \right) \left. \right\} + \frac{1 - \eta_{ic}}{\eta_{c1}^2 \eta_{c2}} r^{(\kappa-1)/\kappa} \left. \right] r_1^{2(\kappa-1)/\kappa} \\ & \quad - 2 r^{(\kappa-1)/\kappa} \frac{1 - \eta_{ic} - \eta_{c1}}{\eta_{c1}^2 \eta_{c2}} r_1^{(\kappa-1)/\kappa} + r^{(\kappa-1)/\kappa} \\ & \quad \times \frac{\eta_{c1} - 1 + \eta_{ic}}{\eta_{c1} \eta_{c2}} \left[\frac{T_c}{T_A} \left\{ 1 - \eta_t \left(1 - \frac{1}{r^{(\kappa-1)/\kappa}} \right) \right\} \right. \\ & \quad \left. \left. - 1 - \frac{\eta_{ic}}{\eta_{c1}} \right] = 0 \end{aligned}$$

Setting $\eta_{ic} = 1.00$ and $\eta_{c1} = \eta_{c2} = \eta_t = 1.00$, then the above equation is simplified as

$$\frac{T_c}{T_A} r_1^{2(\frac{\kappa-1}{\kappa})} - 2 r^{\frac{\kappa-1}{\kappa}} r_1^{\frac{\kappa-1}{\kappa}} - \frac{T_c}{T_A} + 2 r^{\frac{\kappa-1}{\kappa}} = 0$$

In general, there is a difference between the optimum dividing point of the pressure ratio corresponding to the specific output and the thermal efficiency. To raise the thermal efficiency, the first-stage pressure ratio r_1 is selected somewhat lower than that of the minimum compression work. Then the total compression work increases to some degree; at the same time, however, the amount of heat supplied decreases with a rising temperature of the air leaving the compressor. Here the decrease in the value of the heat supplied is greater than the increase in the value of the total compression work, and so the thermal efficiency is raised.

As a numerical example, consider the turbine inlet temperature $T_c = 900^\circ, 1000^\circ, 1100^\circ, 1200^\circ\text{K}$, total pressure ratio $r = 4$ and assuming $\eta_{ic} = 1.00$, $\eta_{c1} = \eta_{c2} = \eta_t = 1.00$.

The first-stage pressure ratio corresponding to the maximum specific output is $r_1 = \sqrt{4} = 2$, independently of the turbine inlet temperature T_c .

Next, the first-stage pressure ratio corresponding to the maximum thermal efficiency r_1 varies with a changing turbine inlet temperature T_c . These

relations are summarized as follows.

Append.-Table 1

$T_c^\circ\text{K}$	900	1000	1100	1200
r_1	1.45	1.35	1.28	1.24

It is also clear from above table that the higher the turbine inlet temperature T_c , the lower the first-stage pressure ratio r_1 becomes, and so the compressor outlet temperature rises.

(7) From the essential definition of the temperature effectiveness of the intercooler, the temperature T_A in the denominator of Eq. (1) should be the temperature of the cooling medium at the inlet

Append.-Fig. 1 Effect of moisture on the specific output

Append.-Fig. 2 Effect of moisture on the thermal efficiency

Append.-Fig. 3 Effect of turbine efficiency on the specific output of the intercooling type

Append.-Fig. 4 Effect of turbine efficiency on the thermal efficiency of the intercooling type

of the intercooler. In the CIMAC standard, however, the cooling-water temperature T_w is defined as equal to the atmospheric temperature T_A : $T_A = T_w = 15^\circ\text{C}$. But in order to avoid misunderstanding, Eq. (1) should be, as pointed out, rewritten in the following form

$$\eta_{ic} = \frac{T_{B1} - T_{A2}}{T_{B1} - T_w}$$

(8) In this paper, the authors consider that the temperature of the cooling medium at the inlet of the intercooler is equal to the temperature of the first-stage compressor inlet T_A following the CIMAC standard. In case of BS 3863, however, the temperature of the cooling medium at the inlet of the

intercooler is $T_w = 13^\circ\text{C}$, and the compressor inlet temperature is $T_A = 15^\circ\text{C}$. In this case also, $T_A = T_w$ may be found considering the cooling effectiveness.

(9) On the effect of the drain flowing out of the intercooler, it is evident from Append.-Figs. 1 and 2 that the humidity scarcely affects the thermal efficiency, but moderately affects the specific output at a higher temperature range ($t = 40^\circ, 50^\circ\text{C}$) with a decreasing relative humidity. Thus, it is impossible to neglect the effect of humidity in the range of higher temperatures and humidities. In this paper, however, such a consideration seems to be unnecessary.

(10) As will be pointed out, the authors intend hereafter to consider the mutual relation between the temperature effectiveness of the intercooler η_{ic} and

Append.-Fig. 5 Effect of turbine inlet temperature on the specific output of the intercooling type

Append.-Fig. 6 Effect of turbine inlet temperature on the thermal efficiency of the intercooling type

Append.-Table 2 Maximum specific output and thermal efficiency and their corresponding pressure ratios

Intercooler efficiency η_{ic}	Maximum specific output		Pressure ratio	
	$\left(\frac{IP_E}{G_a}\right)_{ic}$	$\frac{\left(\frac{IP_E}{G_a}\right)_{max}}{\left(\frac{IP_E}{G_a}\right)_0}$	$(r_{cmL})_{ic}$	$\frac{(r_{cmL})_{ic}}{(r_{cmL})_0}$
1.00	232	1.26	9.50	1.83
0.80	218	1.18	7.90	1.52
0.60	205	1.11	7.00	1.35
Simple type	184	1.00	5.20	1.00

Intercooler efficiency η_{ic}	Maximum thermal efficiency		Pressure ratio	
	$(\eta_{max})_{ic}$	$\frac{(\eta_{max})_{ic}}{(\eta_{max})_0}$	$(r_{cm\eta})_{ic}$	$\frac{(r_{cm\eta})_{ic}}{(r_{cm\eta})_0}$
1.00	0.283	1.00	11.20	1.41
0.80	0.279	0.985	12.70	1.26
0.60	0.276	0.975	11.30	1.12
Simple type	0.283	1.00	10.10	1.00

the pressure loss in the intercooler ΔP_{ic} in exact calculation.

(11) First, in this paper, to look into the special character of the intercooling, the authors discussed the case without regeneration. The reason why the combination $\eta_c = \eta_t = 0.85$ is chosen, is to know the general tendency. Furthermore the values of η_c , η_t , η_{ic} should be appropriately corrected when actually applied. Append.-Figs. 3 and 4 and Append.-Table 2 show the calculated result of the combination of $\eta_c = 0.85$ and $\eta_t = 0.90$ at $T_c = 1000^\circ\text{K}$. Also in this case, the thermal efficiency of the intercooling type is lower than that of the simple type.

(12) Concerning this point, the authors have already finished the study. From the 2nd report on they intend to deal with this.

(13) In this paper, the comparison is made for $T_c = 1000^\circ\text{K}$ and 1200°K . This is one example to

see the effect of turbine inlet temperature T_c . Append.-Figs. 5 and 6 show the result of the specific output and the thermal efficiency for $T_c = 1400^\circ\text{K}$ as pointed out, assuming $\eta_c = \eta_t = 0.85$ and $\eta_{ic} = 0.80$. Like other types of the gas turbine cycle, both the specific output and the thermal efficiency increase with a rising turbine inlet temperature T_c .

(14) Append.-Figs. 7 and 8 show the comparison of the results when precisely and approximately calculated, according to Figs. 12 and 14 respectively.

1) Specific output

The approximately calculated value is smaller than precisely calculated one for all pressure ratios. This is considered to be mainly the effect of the specific heat due to the temperature variation. At the condition of maximum specific output, the compressor pressure ratio of the approximately calculated value is also smaller than the precisely calculated

Append.-Fig. 7 Specific output of the intercooling type by the precise calculation method

Append.-Fig. 8 Thermal efficiency of the intercooling type by the precise calculation method

Append.-Table 3 Comparison of the values of specific output ($T_c = 1000^\circ\text{K}$)

Intercooler efficiency η_{ic}	Approximately calculated value		Exactly calculated value		Ratio	
	① $\left(\frac{HP}{G_a}\right)_{\max}$	② r_{cmL}	③ $\left(\frac{HP}{G_a}\right)_{\max}$	④ r_{cmL}	①/③	②/④
1.00	185.3	8.59	216.5	10.80	0.855	0.795
0.80	172.0	7.19	199.3	8.30	0.864	0.866
0.60	160.5	6.25	185.5	7.05	0.865	0.886

Append.-Table 4 Comparison of the values of thermal efficiency ($T_c = 1000^\circ\text{K}$)

Intercooler efficiency η_{ic}	Approximately calculated value		Exactly calculated value		Ratio	
	① $\eta_{\max}$	② $r_{cm\eta}$	③ $\eta_{\max}$	④ $r_{cm\eta}$	①/③	②/④
1.00	22.35	11.47	23.51	14.10	0.950	0.813
0.80	21.65	10.10	22.75	12.55	0.954	0.805
0.60	21.05	8.97	22.00	11.20	0.957	0.800

one.

2) Thermal efficiency

The approximately calculated amount of the heat supplied is estimated to be smaller than the precisely calculated one. On the other hand, the specific output of the approximately calculated value is estimated to be too small. Therefore, these two effects cancel each other and the difference between the approximate and precise calculation methods is not so large as for the case of specific output. Furthermore, the pressure ratio corresponding to the maximum thermal efficiency η_{cm} of the precisely calculated value is larger than the approximate one. From the above mentioned comparison, either the specific output or the thermal efficiency is estimated to be too small in the approximate calculation method,

but the general tendency of the result of the precise calculation method differs but little from that of the approximate one.

Both pressure ratios corresponding to the maximum thermal efficiency and the maximum specific output, however, become higher when precisely calculated.

(15) Following this paper, the authors are analyzing various types of gas turbine, and intend ultimately to make a general calculation chart covering the various types of gas turbines. Moreover, they intend to elaborate a generalized formula, and with the help of a digital computer, make up a number of tables and graphs which will help designers to find the optimal design parameters easily.