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On the Selection of the Cutting Condition of Plain Milling Machines*

By Kiyoshi OGAWA**, Yoshiaki YOKOYAMA***, and Tsutomu HAGA****

We analyzed the process involving self-excited vibration of the plain milling machine and tried to select the stable cutting condition by the simulation with analog computer.

As the process involving self-excited vibration, a vibrating system of two directions of vertical and table feed was considered and a system diagram based on regenerative effect was described.

Then, the simulation circuit was composed on the analog computer using an equivalent vibrating system of the plain milling machine which was derived from the frequency response curve. The result coincided with the two kinds of self-excited vibrations at different frequencies caused in cutting experiment and with the stability diagram.

As a result, using the analog computer as a simulator, it was made possible to select the best cutting condition never accompanied by self-excited vibrations.

1. Introduction

The machine tool vibrations under cutting work deteriorate the surface roughness and accuracy, and shorten the life of the machine tool. Therefore, it is necessary to select the stable cutting conditions free from self-excited vibrations.

The machine tool vibrations are classified into forced or self-excited vibrations and the former is comparatively easy to analyze and to prevent comparatively, but the latter, which is often accompanied with large amplitude during cutting work, is not easy to analyze and many researches have been done about it. Using the regenerative effect pointed out by R. N. Arnold⁽¹⁾ in 1946, J. Tlustý, S. A. Tobias and the others researched about the cutting conditions in the case when the relative vibrations between tool and work became unstable, but they researched about the two-dimensional cutting where the nominal depth of cut was constant⁽²⁾. Tobias, J. Peters⁽³⁾ and H. Opitz⁽⁴⁾ researched about the self-excited vibrations in milling, but they disregarded the change of the depth of cut and the direction of the cutting force. The analysis of the self-excited vibrations in the plain milling, where this variation can not be disregarded, has been developed recently⁽⁵⁾, but the comparison of the results between the simulation and

the cutting experiment has not been discussed.

The authors represented the generating system of the self-excited vibrations in the plain milling work by a numerical and a block diagrams, and simulated the vibrating system by the analog computer and judged the conditions for stability of the cutting work and selected the best cutting conditions free from self-excited vibrations.

2. Nomenclature

The main symbols used in this report are:

- x, z : relative displacement in x and z direction between tool and work
- T, t_z : difference of cutting time
- h_0, h : nominal and actual depth of cut, respectively
- w : width of cut
- δ : relative displacement in the direction of the depth of cut between tool and work
- F, F_T, F_N : cutting force and those of tangential and normal directions, respectively
- F_x, F_z : resultant force components in the x and z directions
- k_T, k_N : specific cutting resistance in the tangential and normal directions, respectively
- K : transfer function between the depth of cut and the cutting force
- G : transfer function which represents the structural characteristics of machine
- z : number of cutting edges of milling cutter
- r : radius of milling cutter

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** Professor, Tokyo Institute of Technology, Meguro-ku, Tokyo.

*** Assistant, Tokyo Institute of Technology.

**** Engineer, Hitachi Seiki Co., Ltd.

- φ : rotational angle of milling cutter
 n, Q : rotational speed of milling cutter, rpm and rps, respectively
 s, s_z : feed and feed per tooth, respectively
 E : number of teeth in contact with the work
 μ, μ' : overlap factor and regenerative coefficient, respectively
 η : damping ratio (see Section 6.1)

3. Self-excited system of the plain milling work

3.1 Representation of the system by block diagram

Here is shown the outline of a self-excited system which is based on the regenerative effect and which has been adopted until recently.

Figure 1 shows a model of the cutting process; at an arbitrary cutting position, the relative deflection of the work and the cutter in the direction of the depth of cut during the previous T sec is $x(t-T)$, and at t time this deflection is $x(t)$. Considering the nominal depth of cut, the actual depth of cut can be represented as follows.

$$h(t) = h_0(t) - x(t) + \mu x(t-T) \quad (1)$$

where μ is the so called overlap factor⁽⁶⁾. Using the Laplace transforms, Eq. (1) is rewritten as follows.

$$H(s) = H_0(s) - X(s)(1 - \mu e^{-Ts}) \quad (2)$$

Making use of the transfer function $K(s)$, the cutting force F for h is written as follows.

$$F(s) = K(s)H(s) \quad (3)$$

In general, the structural characteristics G of the machine tool can be represented as follows.

$$G(s) = \frac{X(s)}{F(s)} \quad (4)$$

where F is the cutting force and x is the relative deflection. Figure 2 shows a block diagram of the relation of Eq. (2), (3) and (4).

Therefore, when the nominal depth of cut $H_0(s)$ is constant, the presence of self-excited vibrations

Fig. 1 Explanation of cutting system

can easily be studied through the stability of the loop in Fig. 2 and when the values of K and G are given, a stability test of the automatic control circuit can be adopted easily.

Using the cartesian coordinates (x, y, z) to describe the behaviour of the machine tool, x is taken along the table in the feed direction, y in the fore and the back, and z in the vertical direction.

In case of milling with a straight tooth cutter, the relative deflections in y direction can be neglected, the vibration phenomena can be checked more accurately by considering the deflection in x and z directions, then by considering them only in z direction as adopted in the past.

Moreover, for small values of E , the depth of cut changes periodically if we consider both the cutting system and the force caused by the variation of the depth of cut can not be neglected in the discussion of the stability.

Figure 3 shows a block diagram of the self-excited system in the plain milling taking in consideration the two points mentioned above. The nominal depth of cut H_0 is a function of time and changes periodically. H is the relative deflection between the edge of the milling cutter and the work in the direction of the depth of cut, and the actual depth of cut is denoted by $H = H_0 - \Delta H$. K_x and K_z are the transfer functions by which the cutting forces in the direction of x and z are to be obtained, and they are not constant because the direction of cut changes. G_x and G_z are the transfer functions representing the structural characteristics of the plain milling machine in x and z directions, respectively and the structural characteristics are assumed to be independent of each other. $\mu'(1 - e^{-Ts})$ represents the regenerative effect, and T , the difference in the time of cut between the cutting edges during the milling operation.

Fig. 2 Block diagram of vibrating system in one direction (Block diagram to show regenerative effect)

Fig. 3 Block diagram in plain milling

Then, after computing $\mu' \Delta X$ and $\mu' \Delta Z$, and changing them by G_{hx} and G_{hz} to the relative displacement H in the direction of the depth of cut, ΔH is fed back and the block diagram forms a closed loop.

3.2 Regenerative coefficient μ'

Hitherto, the overlap factor μ' has been used to represent the influence of the previous surface of cut to the next one, and its further meanings will be explained hereinafter⁽⁷⁾. That is, the displacements of the relative vibration between the cutting edges of the tool and the work do not remain completely as the "Chatter mark" on the surface of the work with the same form of the vibration, but the periodic shape on the surface of cut is disturbed by the roundness of the cutting edges, by the interference of the surface of the relief angle of the tool, and by the vibration in the direction other than that of the depth of cut. And, when the surface is cut, the relative vibrations caused by the cut of last time influence that of this time by μ times the regenerative effect.

Using Eq. (1) with the overlap factor $\mu < 1$, the mean actual depth of cut h_{mean} becomes $h_{\text{mean}} < h_0$, but this is unreasonable in the plain milling where $\mu = 1$ and $h_{\text{mean}} = h_0$. Therefore, in the plain milling where μ must be given as small value considering the deviation of the cutter, the regenerative effect must be taken approximately as $\mu' \{x(t) - x(t-T)\}$ as shown in Fig. 3, where μ' is called the "regenerative coefficient" to be distinguished from the overlap factor.

3.3 Numerical expression and block diagram of the self-excited system in plain milling

The locus of each edge of the plain milling cutter is a trochoid curve. Each edge of cutter cuts some parts of the work included in the neighbouring trochoid curves drawn at the interval s_z and the depth of cut is shown as the thickness in the direction normal to that of cutting. In the general cutting

conditions in which the table feed s is small enough compared with the cutting speed V , the geometric nominal depth of cut h_0 , in case of up-cutting, is represented approximately by the next equation,

$$h_0 = s_z \sin \varphi \quad \dots\dots\dots (5)$$

where φ is the angle of rotation of the cutter.

Now, if there is no deviation of the cutter, the actual depth of cut h is considered as the relative deflection between tool and work. Using the relative displacement δ in the direction of the depth of cut,

$$h = s_z \sin \varphi - \{\delta(t) - \delta(t-t_z)\} \quad \dots\dots\dots (6)$$

where, t_z becomes approximately $(1-s/2\pi r)/z\Omega$, but, in general cutting conditions $s/r \ll 1$, it is sufficient to use $t_z = 1/z\Omega$.

Considering the regenerative coefficient μ' , next equation must be used instead of Eq. (6).

$$h = s_z \sin \varphi - \mu' \{\delta(t) - \delta(t-t_z)\} \quad \dots\dots\dots (7)$$

Next, consider the cutting force for h . Divide the cutting force on one of the cutting edges into the tangential component F_T and the normal component F_N , and let k_T and k_N be the specific cutting resistances,

$$\left. \begin{aligned} F_T &= k_T wh = k_T w \{s_z \sin \varphi - \mu' \Delta \delta(t)\} \\ F_N &= k_N wh = k_N w \{s_z \sin \varphi - \mu' \Delta \delta(t)\} \end{aligned} \right\} \quad \dots\dots\dots (8)$$

where,

w : width of cut

$$\Delta \delta(t) = \delta(t) - \delta(t-t_z)$$

F_T and F_N change their magnitudes and directions. They are functions of φ . Changing them into the relative forces F_x and F_z which act in the x and z directions of the milling machine (see Fig. 4),

$$\left. \begin{aligned} F_x &= F_N \sin \varphi + F_T \cos \varphi = wh(k_N \sin \varphi + k_T \cos \varphi) \\ F_z &= F_N \cos \varphi - F_T \sin \varphi = wh(k_N \cos \varphi - k_T \sin \varphi) \end{aligned} \right\} \quad \dots\dots\dots (9)$$

The relative deflection δ in the direction of the depth of cut is caused by F_x and F_z and the relative deflection x and z between tool and work are shown in the next equations using the structural characteristics G_x and G_z .

$$\left. \begin{aligned} X(s) &= G_x F_x \\ Z(s) &= G_z F_z \end{aligned} \right\} \quad \dots\dots\dots (10)$$

Comparing with Fig. 5, the relative deflection δ

Fig. 4 Cutting force

Fig. 5 Relative deflection between tool and work

is shown by the next equation.

$$\delta(t) = x(t) \sin \varphi + z(t) \cos \varphi \dots \dots \dots (11)$$

Equations (6) ~ (11) are those of the self-excited system in case of cutting with one edge of the cutter, and in case of the number of the teeth in contact with the work E being less than or equal to 1. F_x and F_z in case of including $E < 1$ are explained hereinafter.

Number the edges of the cutter in the direction opposite to that of the rotation, and set $\varphi = 0$ at the beginning of the cutting by the first edge of the cutter. Then induce the equations which show the cutting period and the cutting force in case of $E \leq 1$, $1 < E \leq 2$ and in the general case where $p-1 < E \leq p$. The equations of the self-excited system of plain milling are as follows.

$$\left. \begin{aligned} X(s) &= G(s)F_x(s), & Z(s) &= G_z(s)F_z(s) \\ F_x(t) &= \sum F_{xi}(\varphi_i), & F_z(t) &= \sum F_{zi}(\varphi_i) \\ F_{xi}(t) &= h_i K_x(\varphi_i), & F_{zi}(t) &= h_i K_z(\varphi_i) \\ K_x(\varphi_i) &= w(k_N \sin \varphi_i + k_T \cos \varphi_i) \\ K_z(\varphi_i) &= w(h_N \cos \varphi_i - k_T \sin \varphi_i) \\ h_i &= s_i \sin \varphi_i - \mu' \Delta s_i(t) \\ \Delta \delta_i(t) &= \delta_i(t) - \delta_i(t - t_z) \\ \delta_i(t) &= x(t) \sin \varphi_i + z(t) \cos \varphi_i \\ \varphi_i &= \varphi - \frac{2\pi}{z} (i-1) = 2\pi \Omega \left(t - \frac{i-1}{z\Omega} \right) \\ t_z &= \frac{1}{z\Omega} \left(1 - \frac{s}{2\pi r} \right) \end{aligned} \right\} \dots \dots \dots (12)$$

4. Simulation circuit on analog computer

Constructing a block diagram of the self-excited system of a plain milling machine as shown in Fig. 6 on the analog circuit, the behavior of each vibrating element, the depth of cut and the cutting force can be computed by giving to each potentiometer, suitable values which represent the kind of the work, the size of the cutter, the speed of rotation of the spindle, the depth of cut and the feed.

Fig. 6 Block diagram in case of $(p-1) < E \leq p$

Given the cutting conditions, the behavior of the milling machine during operating can be found directly as the result of the computation on the analog computer without doing actual machining. Therefore, the analog computer can be used as simulator of the plain milling machine.

The generating circuits of $\sin \varphi_i$ and $\cos \varphi_i$ for the intermittent saw tooth wave φ_i must be made. Here, $\sin \varphi_i$ and $\cos \varphi_i$ are computed using φ_i which is obtained from the simple generating circuit⁽⁸⁾.

As the value of φ_i is small in general, there is no difference in the results of computation, whichever is used, the function generator or the equation of approximate expansion by the multiplier.

Then, $x(t-t_z)$ and $z(t-t_z)$ must be computed but, the dead time circuit by the transfer function approximation can not be used, because it causes a large phase difference and a response error of the step function. Therefore, we used the hybrid computer.

The simulating circuit is shown in Fig. 7. By this circuit, the simulation, in so far as $E \leq 2$, can be done. The physical meaning of the output of each element is known by the figure and the diodes in the circuit eliminate the negative depth of cut which has no meanings.

5. Experiment

5.1 Measurement of frequency response of plain milling machine

In establishing the analog computer circuit for the simulation dealing with the choice of the cutting conditions, the structural characteristics G_x and G_z of the machine must be determined. The structural characteristics can be described by suitable transfer functions determining the relative displacements of the arbour and the table, due to the cutting force, or by suitable differential equations. Here, we have derived them by approximating each element to an equivalent one degree of freedom vibrating system in one direction x or z . This approximation can be done with the frequency response curve⁽⁹⁾.

By exciting the arbour of the knee type milling machine (arbour 1 in. diameter, collar 35 mm external diameter) with a magnetic vibrator fixed on the table, the frequency response curves were obtained by measuring the absolute displacements in x and z directions of the elements with the aid of a capacitance detector device and a moving coil type vibrometer. Figure 8 shows the results of the recording.

The natural frequency of the equivalent vibrating system can not just be approximated

from the frequency response curve where some of the maximum values are contained in x and z directions. Therefore, observing the free vibration caused by hitting the table and the arbour, and comparing it with the results of the frequency analysis, the natural frequency that might lead to a self-excited vibration must be estimated before the cutting operation.

Figure 8 shows the maximum amplitude of the arbour in x direction at the frequency of 30 c/sec,

Fig. 7 Simulating circuit in case of $p=2$

Fig. 8 Frequency response curve

and this is the frequency of the rocking mode at which the arbour and table vibrate in same phase. This can also be known through the shock test. The next maximum comes at 145 c/sec and this is the vibration of the overarm system which produces the nodal line to the column. There are no other frequencies during the frequency analysis besides 30 c/sec and 145 c/sec.

Besides the rocking mode frequency, the resonant amplitude of the table in x direction was recorded at 71.5 c/sec, while the frequency obtained from the free vibration which was caused by the shock test was 71 c/sec. No other appreciable frequency does appear simultaneously during the frequency response test and the shock test.

On the response curve, besides the rocking frequency, the maximum amplitudes of the arbour in z direction at 175 c/sec and 500 c/sec are noticed, while 550 c/sec, 200 c/sec and 62 c/sec can be read from the record of the shock test. 175 c/sec is the resonance frequency of the overarm, and 550 c/sec is the frequency of the deflecting vibration of the arbour itself. With a static force applied between the arbour and the table at 200 mm from the column, it has been found that the deflection of the arbour in z direction is about three times that of the overarm.

In general, the response curve is complicated and the amplitudes are small for the table which vibrates in z direction and which has a relatively high stiffness due to the knee. From the free vibration test, 190 c/sec and 400 c/sec are obtained as the resonant frequencies.

Summing up the results, the natural frequencies for the equivalent vibrating systems can be assumed as follows.

Arbour	x -direction	145 c/sec
	z -direction	550 c/sec
Table	x -direction	71.5 c/sec
	z -direction	190 c/sec or 400 c/sec

5.2 Vibration during machining

During machining operation, the vibrations of the arbour and the table in x and z directions have been measured by the same technique as used in the measurement of the frequency response test, but for the table vibration in z direction, the moving coil type pick up fixed on the table was used (Fig. 9). The position of the cutter is the same as used in the frequency response test, and the cutting operation was carried out as follows.

i) Cutting condition (up cut milling in dry condition)

spindle speed $n=51 \sim 1130$ rpm (6 stages)

feed $s=11 \sim 149.6$ mm/min
(6 stages)

width of cut $w=12$ mm

depth of cut $d=0.2 \sim 1.6$ mm (S 20 C)
 $d=1.0 \sim 4.0$ mm (brass)

ii) Cutter

material SKH 9

rake angle $\alpha=5^\circ$

angle of relief $\beta=6^\circ$

helix angle $\lambda=0^\circ$

diameter $D=100$ mm

width $L=14$ mm

number of teeth $z=26$

iii) Work

mild steel (S 20 C)

brass (BsBM 2)

According to the records, besides the rocking mode, the frequency of the arbour in x direction has a range of $120 \sim 150$ c/sec and $500 \sim 800$ c/sec; in z direction, $500 \sim 800$ c/sec. The frequency of the table in x direction is in the range of $70 \sim 80$ c/sec and $130 \sim 150$ c/sec and the amplitudes of the table in z direction are too small and irregular to have a constant frequency. The frequency range of $500 \sim 800$ c/sec detected in the vibration of arbour in x direction and that of $130 \sim 150$ c/sec of the table in x direction were caused by the arbour during self-excited vibration in z and x direction, respectively.

Fig. 9 Experiment of cutting

($d=0.8$ mm, $n=95$ rpm, $s_z=21.0 \mu$)
(i) Wave by deflection of arbour

($d=0.8$ mm, $n=95$ rpm, $s_z=20 \mu$)
(ii) Wave by simulation
(a)

($d=1.0$ mm, $n=95$ rpm, $s_z=13.4 \mu$)
(i) Wave by deflection

(1) $\mu'=0$ $\mu'=0.5$ $\mu'=1.0$

($d=1.0$ mm, $n=95$ rpm, $s_z=10 \mu$)
(ii) Wave by simulation
(b)

Fig. 10 Example of vibrating wave by experiment and simulation

6. Simulation of the vibration of the milling machine on analog computer

6.1 Representation of structural characteristics

The following frequencies were chosen as the natural frequencies of the equivalent systems.

Arbour	x-direction	$f_{Ax}=145$ c/sec
	z-direction	$f_{Az}=550$ c/sec
Table	x-direction	$f_{Tx}=71.5$ c/sec
	z-direction	—

Fig. 11 Vibrating wave in case of work, S20 C

Because the vibration of the table in z direction has small amplitude and is not stationary, the system can not be regarded as a self-excited vibration one. By measuring the values of characteristics from the amplitudes near the resonant point of the frequency response curve, the structural characteristics are described by the differential equations as follows (unit is kg and mm).

$$\left. \begin{aligned} x &= x_A - x_T \\ \ddot{x}_A + 5.69 \times 10^7 \eta \dot{x}_A + 8.29 \times 10^5 x_A &= 7.75 \times 10^2 F_x \\ \ddot{x}_T + 2.49 \times 10^7 \eta \dot{x}_T + 2.02 \times 10^5 x_T &= -1.19 \times 10^2 F_x \\ \ddot{z} + 4.80 \times 10^2 \eta \dot{z} + 1.19 \times 10^7 z &= 6.10 \times 10^3 F_z \end{aligned} \right\} \dots (13)$$

In Eq. (13), the damping of the system during the test of frequency response is assumed to increase during the cutting operation⁽¹⁰⁾, and the increasing ratio is represented by η .

Equation (13) describing the structural characteristics of the milling machine can easily be constructed on the circuit of the analog computer.

6.2 Determination of each constant in the simulation circuit

To simulate using a low speed analog computer which is superior to that of repetition type, the equations must be time scaled ($\tau = 200t$) and they are represented as follows.

$$\left. \begin{aligned} x &= x_A - x_T \\ x_A'' + 0.285 \eta x_A' + 20.75 x_A &= 232 \sum h_i (k_N \sin \varphi_i + k_T \cos \varphi_i) \\ x_T'' + 0.125 \eta x_T' + 5.05 x_T &= -35.8 \sum h_i (k_N \sin \varphi_i + k_T \cos \varphi_i) \\ z'' + 2.40 \eta z' + 298 z &= 1830 \sum h_i (k_N \cos \varphi - k_T \sin \varphi_i) \\ h_i &= s_z \sin \varphi_i - \mu' \Delta \delta_i(\tau) \\ \Delta \delta_i(\tau) &= \Delta x(\tau) \sin \varphi_i + \Delta z(\tau) \cos \varphi_i \\ \varphi_i &= \frac{\pi \Omega}{100} \tau, \quad \tau = 200t, \quad z = \frac{200}{z \Omega} \left(1 - \frac{s}{2\pi r} \right) \end{aligned} \right\} \dots (14)$$

The equations are transformed so that the length may be in microns and the weight in grams.

The specific cutting forces are obtained in the first approximation from the data⁽¹¹⁾ which have been used in general:

mild steel (S20 C)	$k_T = 0.5$ g/ μ^2 ,
	$k_N = 0.3$ g/ μ^2
brass	$k_T = 0.25$ g/ μ^2 ,
	$k_N = 0.15$ g/ μ^2

The coefficients η and μ' in Eq. (14) were determined by comparing the records of the simulation and the experiment of which examples were shown in Fig. 10, and the

values of η and μ' were given as follows.

$$\eta=2, \mu'=0.5$$

6.3 Examination and comparison of the results with the simulation and experiment

Figures 11 and 12 show some examples of data which show if there are the self-excited vibrations and the examples of the wave obtained by the simulation. In the figure, the data of the experiment are shown in (i) in which the scale of x_A and z_A is 20μ per one division and (S) is stable and (I) is unstable.

In the experiment, the instability is caused by the self-excited vibrations in z direction of the arbour

in case of S 20 C, and in x direction of the arbour in case of the brass. In the simulation, the amplitudes of the arbour in z and x directions diverge in case of S 20 C and brass, respectively, and the result of the simulation coincided with that of the experiment.

But, in the cutting of S 20 C at $n=51$ and 95 rpm, the instability was caused by the rocking vibrations and this instability was denoted by the symbol (I)'.

Figures 13 and 14 show the comparison of the results of the experiment and the simulation on the stability chart. In the figures, the symbols $\circ$, $\times$ and $\langle \circ \rangle$ mean (S), (I) and (I)', respectively,

Fig. 12 Vibrating wave in case of work, brass

and the symbol Δ means that it could not be distinguished between (I) and (S). The hatching parts in the figures show the unstable parts which were desired from the results of the simulation.

There is fairly good agreement between the experimental results and the simulation in case of S20C and in the high speed part of the spindle, and the variation of the chart following the variation of the depth of cut d is in good agreement with that of the brass.

It seems the error in high speed part of the spindle in the cutting of S20C is caused by the error; when a saw tooth wave is generated, the frequency of cutting is high in the simulating circuit, and by the error in setting the frequency in the circuit as the natural frequency of the arbour is high in z direction.

7. Conclusions

The milling is often accompanied by a vibration, but it is not easy to estimate the condition how the vibration is caused in selecting the cutting conditions and the stability of the vibration is not yet considered

as the standard in selecting the cutting conditions. The authors analysed the system of self-excited vibrations in a plain milling machine with a straight edged milling cutter and used the analog computer to select the cutting conditions.

The conclusions derived from the paper are summarized as follows:

(1) In the system of self-excited vibrations in a plain milling machine with a straight edged milling cutter, the directions and the magnitudes of the cutting forces following the vibration of the depth of cut are changed periodically and, taking in consideration the above mentioned, the system was clarified by analyzing the two directions of the machine, i.e., vertical and feed of the table.

(2) Constructing the system of self-excited vibrations which are shown by the block diagram on the circuit of the analog computer, the cutting conditions of the milling machine were changed continuously and over a wide region on the computing circuit and made easy to select the most desirable cutting condition seeing directly the vibrations caused by each condition.

Fig. 13 Stability chart (S20C)

Fig. 14 Stability chart (brass)

(3) The structural characteristics were obtained from the frequency response curve. The arbour and the table, in x direction, were represented by the equivalent vibrating system with a one degree of freedom, respectively. In this case, the natural frequencies which might cause self-excited vibrations were given by the discussion about the static stiffness of the arbour and the table, and the free vibration by the shock test.

(4) The stability charts which were obtained by the computing circuit agreed with the experimental results and could give the cutting conditions never accompanied by self-excited vibrations.

Therefore, for a plain milling machine, of which the structural characteristics are decided from the frequency response curve, if the work material, the kind of tool, and the width of cut are given, the stability chart can be obtained on the analog computer circuit using as parameters the rotational speed of the spindle, the depth of cut, and the feed per

edge of the cutter.

Then, the generating conditions of the self-excited vibrations are obtained and can be selected the most desirable cutting conditions never accompanied by self-excited vibrations.

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