Study on Annular Jets*

(3rd Report, Theoretical Analysis of the Main Stream Line and the Velocity Distribution)

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In this paper is described a theoretical analysis of the swirling jets issuing from an annular nozzle. An approximate calculation method for trajectory of the main stream line has been derived. And velocity distributions are obtained by solving momentum integral equation for the orthogonal curvilinear coordinates using the trajectory of the main stream line as coordinate. The values of coefficients in the equations are determined by experiments. And the theoretical predictions from the values of these constants agree with the experimental results.

1. Introduction

In previous papers⁽¹⁾⁽²⁾, experimental studies on annular unswirled jets and annular swirling jets were performed. In this paper, theoretical analysis on the main stream line and the velocity distribution in the annular swirling jet of semi-radial type is presented. An approximate calculation method for main stream line and velocity distribution in annular unswirled jet was already reported in the previous paper⁽¹⁾. Analyses of velocity distributions in round swirling jet have also been done by many investigators^{(3)~(6)}. Velocity distribution in radial annular jets was theoretically analyzed by Ullrich⁽⁷⁾. In the radial annular jet, there exists no region of subatmospheric pressure inside of the jet and the main stream lines are linear. However, in the semi-radial annular jets, the region of subatmospheric pressure exists near nozzle exit and the locus of a fluid particle becomes a complicated spatial curve. Hence, it seems to be difficult to obtain a theoretical solution for the velocity distribution in the semi-radial annular jet by solving the momentum equations which are derived by taking into account such a complicated static pressure distribution. Therefore, in this study, first a semi-theoretical equation of trajectory of main stream line is derived by using a simple flow model of a semi-radial annular jet, and secondly integral equations of momentum with orthogonal curvilinear coordinates whose main axis is identical with the main stream line of the jet are derived, and by solving the

equations, a general formula for the velocity distribution is obtained.

2. Notations

The following nomenclature is used in this paper.

X : nondimensional distance along x axis,
 x/R_{c1}

R : nondimensional radial distance, R_c/R_{c1}

u, v, w : velocity components corresponding to x ,
 y and θ directions

$\bar{u}, \bar{v}, \bar{w}$: temporal mean values of u, v and w

u', v', w' : fluctuation velocity components corresponding to x, y and θ directions

V_x, V_u, V_w : nondimensional velocities on main stream
line, $V_x = V_c/V_{c1}$, $V_u = \bar{u}_c/\bar{u}_{c1}$, $V_w = \bar{w}_c/\bar{w}_{c1}$

Y_B : nondimensional width of jet, y_b/y_{b1}

Subscripts

0: value at the nozzle exit

c : value on the main stream line

1: value at the end of potential core

Notations other than those defined above are the same as in the previous paper⁽²⁾.

3. An approximate equation of main stream line

3.1 Derivation of equations

We make the following assumptions taking into account the experimental results in the previous paper⁽²⁾.

(1) Fluid is incompressible and isothermal, and its jet is axial symmetric to z axis.

(2) The velocity vector distributed on the cross section of width b which is perpendicular to x axis is constant (Fig. 1). The velocity in recirculation region is negligible.

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(3) A force due to the static pressure difference acts in the direction perpendicular to x axis on the inner and the outer surfaces of jets with the width of b .

(4) A momentum J_m in the direction of main flow and an angular momentum J_θ are kept constant along the jet flow.

From the assumption (4), the conditions of conservation of momentum give the following equations. Since a momentum J_m on an arbitrary cross section of the jet is equal to J_{m0} on the nozzle exit section, we have

$$J_m = 2\pi\rho R_c b V_c^2 \cos \beta_c = J_{m0} = 2\pi\rho R_0 b_0 V_0^2 \cos \beta_0 \quad \dots\dots\dots(1)$$

The velocity V_0 at the nozzle exit is

$$V_0^2 = (2/\rho)(P_0 - P_\infty)$$

Substituting this into Eq. (1), we have

$$\rho b V^2 = 2(R_0/R_c)(\cos \beta_0/\cos \beta_c)(P_0 - P_\infty)b_0 \quad \dots\dots\dots(2)$$

From the condition of conservation of angular momentum, we have

$$J_\theta = 2\pi\rho R_c^2 b V_c w_c \cos \beta_c = 2\pi\rho R_0^2 b_0 V_0 w_0 \cos \beta_0 \quad \dots\dots\dots(3)$$

Substituting Eq. (2) and $w_c = V_c \sin \beta_c$ into Eq. (3), we have

$$R_c \sin \beta_c = R_0 \sin \beta_0 \quad \dots\dots\dots(4)$$

Now we consider a balance of forces acting on mass of $\rho R_c d\theta dx b$ shown in Fig. 1. Those forces consist of centrifugal force caused by the curved motion of fluid and the one caused by the static pressure difference which, from the assumption (3), is supposed to act perpendicular to the path of fluid particle. We have

$$\rho R_c d\theta dx b \left\{ \frac{(V_c \sin \beta_c)^2}{R_c} \cos \alpha_c + \frac{(V_c \cos \beta_c)^2}{r_{Rz}} \right\} = R_c d\theta dx (P_s - P_\infty) \quad \dots\dots\dots(5)$$

where the first term stands for the centrifugal force due to rotational motion around z axis, and the second term for that due to curved motion on R - z plane.

Substituting Eqs. (2) and (4) into Eq. (5), we have

Fig. 1

$$\frac{\{(R_0/R_c) \sin \beta_0\}^2 \cos \alpha_c}{R_c^2 \sqrt{1 - \{(R_0/R_c) \sin \beta_0\}^2}} + \frac{\sqrt{1 - \{(R_0/R_c) \sin \beta_0\}^2}}{R_c r_{Rz}} = \frac{(P_s - P_\infty)/(P_0 - P_\infty)}{2R_0 b_0 \cos \beta_0} \quad \dots\dots\dots(6)$$

Here, r_{Rz} and α_c are expressed by Eqs. (7) and (8) from geometrical relations.

$$\frac{r_{Rz}}{R_0} = \left\{ 1 + \left(\frac{dz_c'}{dR_c'} \right)^2 \right\}^{3/2} / \left(\frac{d^2 z_c'}{dR_c'^2} \right) \quad \dots\dots\dots(7)$$

$$\cos \alpha_c = \frac{1}{\sqrt{1 + \tan^2 \alpha_c}} = 1 / \sqrt{1 + \left\{ 1 / \left(\frac{dz_c'}{dR_c'} \right)^2 \right\}} \quad \dots\dots\dots(8)$$

where R_c' and z_c' are nondimensional coordinates

$$R_c' = R_c/R_0, \quad z_c' = z_c/R_0$$

Substituting Eqs. (7) and (8) into Eq. (6), we have the fundamental equation for main stream line.

$$\frac{\left(\frac{K_s}{R_c'} \right)^2}{R_c'^2 \sqrt{\left\{ 1 - \left(\frac{K_s}{R_c'} \right)^2 \right\} \left[1 + \left\{ 1 / \left(\frac{dz_c'}{dR_c'} \right)^2 \right\} \right]}} + \frac{\sqrt{1 - \left(\frac{K_s}{R_c'} \right)^2} \left(\frac{d^2 z_c'}{dR_c'^2} \right)}{R_c' \left\{ 1 + \left(\frac{dz_c'}{dR_c'} \right)^2 \right\}^{3/2}} = C \quad \dots\dots\dots(9)$$

where K_s and C are defined as

$$K_s = \sin \beta_0,$$

$$C = \{R_0(P_s - P_\infty)/(P_0 - P_\infty)\} / \{2b_0 \cos \beta_0\}$$

These values depend on the nozzle dimensions, that is, $2b_0/D$, α_0 and β_0 .

3.2 Numerical examples

Equation (9) was calculated numerically using the available data on distribution of the pressure coefficient C along z axis and using the following boundary conditions

$$z_c' = 0, \quad (dz_c'/dR_c') = \cot \alpha_c \text{ at } R_c' = 1$$

Fig. 2 Pressure constant

The calculation was performed by Runge-Kutta-Gill method using a computer.

The values of C in the case of $\alpha_0 = 0^\circ$, $2b_0/D = 0.2$ are shown by the solid lines in Fig. 2. These values were determined based on the experimental results of static pressure distributions.

The values of C in the region of $z/R_0 < 2$ are determined using the static pressure difference ($P_{sR} - P_\infty$) at the location of y_a and y_b shown in Fig. 1, and those in the region of $z/R_0 > 2$ are determined using the static pressure P_{sz} on the z axis instead of P_{sR} . The calculated main stream lines in the cases of $2b_0/D = 0.2$, $\alpha_0 = 0^\circ$ for various values of β_0 are shown in Fig. 3. Generally, the calculated values agree approximately with the experimental results, but in the cases of $\beta_0 = 15^\circ, 60^\circ$ there is a little difference between them since the assumption (3) cannot be strictly applied. The value of the first term of Eq. (9) is always positive, but that of the second term can be either positive or negative according to the value of C . Therefore, as can be seen in Fig. 3, the main stream lines are complicated curves which have some deflection point.

4. Equation of velocity distributions

4.1 Fundamental equations

Now we are going to introduce equations of velocity distributions in jets provided that the main stream line of the jet is already known as mentioned above.

Equations of motion and continuity⁽⁸⁾ for incompressible, axial symmetric flow in orthogonal curvilinear coordinates (see Fig. 4), in which the main stream line, (i.e. x axis,) is used as an axis, are

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - \frac{\sin \alpha}{R} w^2 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu k_x \quad \dots\dots\dots(10)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + \frac{\sin \alpha}{R} uw + \frac{\cos \alpha}{R} vw = \nu k_r \quad \dots\dots\dots(11)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} - \frac{\cos \alpha}{R} w^2 = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu k_y \quad \dots\dots\dots(12)$$

Fig. 3 Main stream line

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\sin \alpha}{R} u + \frac{\cos \alpha}{R} v + u \frac{\partial \alpha}{\partial y} - v \frac{\partial \alpha}{\partial x} = 0 \quad \dots\dots\dots(13)$$

where the original point of x axis is set at the end of the potential core region where the velocity on the main stream line is equal to that at the nozzle exit section, and νk_x , νk_y and νk_z stand for friction forces due to viscosity.

Multiplying Eq. (13) by u , v , or w , substituting them into Eqs. (10), (11) and (12) respectively, and neglecting the following terms

$$\begin{aligned} \frac{\partial p}{\partial x} &\approx 0, & u \frac{\partial \alpha}{\partial y} &\approx 0, & \frac{\cos \alpha}{R} vw &\approx 0, \\ \frac{\cos \alpha}{R} v &\approx 0, & v \frac{\partial \alpha}{\partial x} &\approx 0 \end{aligned}$$

and substituting the relations for turbulent flow, $u = \bar{u} + u'$, $v = \bar{v} + v'$, $w = \bar{w} + w'$, $p = \bar{p} + p'$, $\bar{u}' = 0$, $\bar{v}' = 0$, $\bar{w}' = 0$, $\bar{p}' = 0$

we have

$$\begin{aligned} \frac{\partial \bar{u}}{\partial t} + \frac{\partial \bar{u}^2}{\partial x} + \frac{\partial \bar{u}\bar{v}}{\partial y} + \frac{\sin \alpha}{R} (\bar{u}^2 - \bar{w}^2) \\ = -\frac{\partial \bar{u}'^2}{\partial x} - \frac{\partial \bar{u}'v'}{\partial y} - \frac{\sin \alpha}{R} (\bar{u}'^2 - \bar{w}'^2) \quad \dots\dots\dots(14) \end{aligned}$$

$$\begin{aligned} \frac{\partial \bar{w}}{\partial t} + \frac{\partial \bar{u}\bar{w}}{\partial x} + \frac{\partial \bar{v}\bar{w}}{\partial y} + \frac{2 \sin \alpha}{R} \bar{u}\bar{w} \\ = -\frac{\partial \bar{u}'w'}{\partial x} - \frac{\partial \bar{v}'w'}{\partial y} - \frac{2 \sin \alpha}{R} \bar{u}'\bar{w}' \quad \dots\dots\dots(15) \end{aligned}$$

$$\begin{aligned} \frac{\partial \bar{v}}{\partial t} + \frac{\partial \bar{u}\bar{v}}{\partial x} + \frac{\partial \bar{v}^2}{\partial y} - \frac{\cos \alpha}{R} \bar{w}^2 + \frac{\sin \alpha}{R} \bar{u}\bar{v} \\ = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} - \frac{\partial \bar{u}'v'}{\partial x} - \frac{\partial \bar{v}'^2}{\partial y} + \frac{\cos \alpha}{R} \bar{w}'^2 \\ - \frac{\sin \alpha}{R} \bar{u}'\bar{v}' \quad \dots\dots\dots(16) \end{aligned}$$

where the friction forces due to viscosity can be neglected, since they are very small compared with the forces due to Reynolds stress.

Since the change of u' , v' , w' along x axis is negligible compared with that along y axis, we assume

$$\frac{\partial \bar{u}'^2}{\partial x} \approx 0, \quad \frac{\partial \bar{u}'w'}{\partial x} \approx 0, \quad \frac{\partial \bar{u}'v'}{\partial x} \approx 0$$

The conditions of steady flow are

$$\frac{\partial \bar{u}}{\partial t} = 0, \quad \frac{\partial \bar{w}}{\partial t} = 0, \quad \frac{\partial \bar{v}}{\partial t} = 0$$

Fig. 4

Substituting these relations into Eqs. (14), (15) and (16), we have the following fundamental equations of momentum

$$\frac{\partial \bar{u}^2}{\partial x} + \frac{\partial \bar{u}\bar{v}}{\partial y} + \frac{\sin \alpha}{R} (\bar{u}^2 - \bar{w}^2) = -\frac{\partial \bar{u}'\bar{v}'}{\partial y} \quad \dots\dots(17)$$

$$\frac{\partial \bar{u}\bar{w}}{\partial x} + \frac{\partial \bar{v}\bar{w}}{\partial y} + \frac{2 \sin \alpha}{R} \bar{u}\bar{w} = -\frac{\partial \bar{v}'\bar{w}'}{\partial y} \quad \dots\dots(18)$$

$$\frac{\partial \bar{u}\bar{v}}{\partial x} + \frac{\partial \bar{v}^2}{\partial y} - \frac{\cos \alpha}{R} \bar{w}^2 = -\frac{1}{\rho} \frac{\partial p}{\partial y} - \frac{\partial \bar{v}'^2}{\partial y} - \bar{u}'^2 \frac{\partial \alpha}{\partial x} \quad \dots\dots(19)$$

Rearranging Eq. (13) in the same way as described above, we have the equation of continuity

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\sin \alpha}{R} \bar{u} = 0 \quad \dots\dots(20)$$

4.2 Integral equation of momentum and equation of mass flow rate

The velocity profiles on a plane perpendicular to x axis are assumed as Eqs. (21) and (22) from the experimental results in the previous report⁽²⁾.

$$\bar{u}(x, y) = \bar{u}_c(x) \exp(-y^2/y_b^2) \quad \dots\dots(21)$$

$$\bar{w}(x, y) = \bar{w}_c(x) f(y/y_b) \quad \dots\dots(22)$$

where y_b is the value of y at the location of $\bar{u}/\bar{u}_c = 1/e$ and $f(y/y_b)$ is a function given by the distribution of w which is determined from the experimental result.

Substituting Eqs. (21) and (22) into Eq. (17) and integrating it from $-\infty$ to $+\infty$, we obtain an integral equation of linear momentum for x direction.

$$\begin{aligned} & \int_{-\infty}^{\infty} \frac{\partial}{\partial x} \left\{ \bar{u}_c^2 \exp\left(-\frac{2y^2}{y_b^2}\right) \right\} dy \\ & + \int_{-\infty}^{\infty} \eta \left\{ \bar{u}_c^2 \exp\left(-\frac{2y^2}{y_b^2}\right) - \bar{w}_c^2 f^2\left(\frac{y}{y_b}\right) \right\} dy \\ & = - \int_{-\infty}^{\infty} \frac{\partial \bar{u}'\bar{v}'}{\partial y} dy \quad \dots\dots(23) \end{aligned}$$

where $\eta = \sin \alpha/R$ and, with the assumption of $R \gg b$, $\eta = \sin \alpha_c/R$.

According to the assumption in the recirculation region that $\bar{u}=0$ and $\bar{u}'\bar{v}'=0$ at $y=-\infty$, Eq. (23) becomes

$$\sqrt{\frac{\pi}{2}} \frac{d}{dx} (\bar{u}_c^2 y_b) + \eta \left\{ \sqrt{\frac{\pi}{2}} \bar{u}_c^2 y_b - I \left(\frac{y}{y_b} \right) \bar{w}_c^2 y_b \right\} = 0 \quad \dots\dots(24)$$

where $I(y/y_b)$ is defined as

$$I(y/y_b) = \int_{-\infty}^{\infty} f^2(y/y_b) d(y/y_b)$$

The value I is a constant which can be determined by experiments and was found to be approximately 2.0 from the results in the previous report⁽²⁾.

Substituting Eqs. (21) and (22) into Eq. (18), integrating it from $-\infty$ to $+\infty$, and using the boundary conditions: $\bar{w}=0$, $\bar{v}'\bar{w}'=0$ at $y=\pm\infty$, we have the following integral equation of angular momentum:

$$\frac{d}{dx} (\bar{u}_c \bar{w}_c y_b) + 2\eta \bar{u}_c \bar{w}_c y_b = 0 \quad \dots\dots(25)$$

Substituting Eqs. (21) and (22) into Eq. (20) and integrating it from $-\infty$ to $+\infty$, we have the equation of continuity

$$\begin{aligned} & \int_{-\infty}^{\infty} \frac{\partial}{\partial x} \left\{ \bar{u}_c \exp\left(-\frac{y^2}{y_b^2}\right) \right\} dy \\ & + \int_{-\infty}^{\infty} \eta \bar{u}_c \exp\left(-\frac{y^2}{y_b^2}\right) dy = - \int_{-\infty}^{\infty} \frac{\partial}{\partial y} \bar{v} dy \quad \dots\dots(26) \end{aligned}$$

Now, if we use the notation $\mp v_\infty$ as the value of v at $y=\pm\infty$, the right term of Eq. (26) which gives an entrained flow rate can be expressed as follows:

$$- \int_{-\infty}^{\infty} \frac{\partial}{\partial y} \bar{v} dy = 2\bar{v}_\infty \quad \dots\dots(27)$$

According to Lee's investigation⁽⁵⁾ on round swirling jets we may assume that

$$\bar{v}_\infty = \lambda \bar{u}_c \quad \dots\dots(28)$$

where λ is a constant. The value of λ was found to be 0.065 by the experiment of the annular swirling jet in the previous report⁽²⁾.

Substituting Eqs. (27) and (28) into Eq. (26), we have the equation of flow rate

$$\sqrt{\frac{\pi}{2}} \left\{ \frac{d}{dx} (\bar{u}_c y_b) + \eta \bar{u}_c y_b \right\} = 2\lambda \bar{u}_c \quad \dots\dots(29)$$

When the following nondimensional quantities which are based on the values at the end of potential core are used, Eqs. (24), (25) and (28) become Eqs. (30), (31) and (32) respectively.

$$V_u = \bar{u}_c/\bar{u}_{c1}, \quad V_w = \bar{w}_c/\bar{w}_{c1}, \quad Y_B = y_b/y_{b1},$$

$$X = x/R_{c1}, \quad R = R_c/R_{c1}, \quad \eta' = \sin \alpha_c/R$$

$$\frac{d}{dX} (V_u^2 Y_B) + \eta' (V_u^2 Y_B) - \zeta \eta' (V_w^2 Y_B) = 0 \quad \dots\dots(30)$$

$$\frac{d}{dX} (V_u V_w Y_B) + 2\eta' (V_u V_w Y_B) = 0 \quad \dots\dots(31)$$

$$\frac{d}{dX} (V_u Y_B) + \eta' (V_u Y_B) = \epsilon V_u \quad \dots\dots(32)$$

where

$$\zeta = \sqrt{\frac{2}{\pi}} \left(\frac{\bar{w}_{c1}}{\bar{u}_{c1}} \right)^2 I, \quad \epsilon = 2\sqrt{\frac{2}{\pi}} \lambda \left(\frac{R_1}{y_{b1}} \right)$$

The values of ζ and ϵ are constants which are determined by the nozzle dimensions alone, and $\eta' = \sin \alpha_c/R_c$ is a function of X which is given in Section 3.2. Therefore, by solving Eqs. (30), (31) and (32), the velocities V_u , V_w and the width of jet Y_B can be calculated.

4.3 General equations for main stream velocity and width of jet

Integrating Eq. (31) with a boundary condition: $V_u V_w Y_B = 1$ at $X=0$, we have

$$V_u V_w Y_B = e^{-2\epsilon X} \quad \dots\dots(33)$$

where ξ is a function defined by the following equation

$$\xi = \int_0^X \eta' dX = \int_0^X (\sin \alpha_c/R) dX$$

Substituting Eq. (33) into Eq. (30), we have

$$\frac{d}{dX}(V_u^2 Y_B) + \eta'(V_u^2 Y_B) - \zeta \eta' \frac{1}{(V_u^2 Y_B)} e^{-4\epsilon} = 0 \quad \dots\dots\dots(34)$$

Solving Eq. (34) with the boundary condition: $(V_u^2 Y_B)=1$ at $X=0$, we have

$$(V_u^2 Y_B) = G(X) = e^{-\epsilon} (-\zeta e^{-2\epsilon} + \zeta + 1)^{1/2} \quad \dots\dots(35)$$

Substituting Eq. (35) into Eq. (34), we have

$$\frac{d}{dX} \frac{G}{V_u} + \eta' \frac{G}{V_u} = \epsilon V_u \quad \dots\dots\dots(36)$$

The velocity V_u can be expressed as follows, by solving Eq. (36) with the boundary condition: $V_u=1$, at $X=0$,

$$V_u = \frac{\bar{u}_c}{\bar{u}_{c1}} = e^{\epsilon G} / \sqrt{2\epsilon \int_0^X e^{2\epsilon G} dX + 1} \quad \dots\dots\dots(37)$$

The values of ζ and ϵ are constants which can be determined by the nozzle dimensions alone. Consequently, V_u can be numerically calculated from Eq. (37).

The value of Y_B can also be calculated from Eq. (35) with the value of V_u obtained above and so can the value of V_w from Eq. (33). The absolute velocity V can also be calculated by employing the

velocity components V_u and V_w as follows:

$$V = \sqrt{\bar{u}_c^2 + \bar{w}_c^2} \quad \dots\dots\dots(38)$$

4.4 Results of numerical calculation

The procedure of the numerical calculation is as follows. The origin of x axis is determined from Fig. 5, the values of ζ and ϵ are determined by the nozzle dimensions, the distribution of ξ along X axis which was obtained in Chapter 3 is numerically calculated, then the distribution of $\int_0^X e^{2\epsilon G} dX$ is numerically calculated and finally the value of V_u is obtained from Eq. (37). Consequently, V_w , Y_B and V can be calculated from Eqs. (33), (35) and (38).

The calculated and the experimental values of V_u , V_w and V_x are shown in Figs. 6 and 9 respectively. The calculated values agree well with the experimental ones when β_0 is within $30^\circ \sim 60^\circ$. If $\beta_0=15^\circ$, however, good agreement cannot be obtained, because the conversion of the annular jet into a single round jet near the nozzle exit makes the above-mentioned assumptions inapplicable.

Fig. 5 Length of potential core region

Fig. 7

Fig. 6

Fig. 8

Fig. 9

The decreasing rate of the absolute velocity V_x becomes greater in proportion to the increase of β_0 as shown in Fig. 8. This can be explained by the fact that the spread angle of jet becomes wider in proportion to the increase of β_0 and that the surface area for turbulent diffusion becomes larger. On the other hand, the decreasing rate of the velocity V_u becomes smaller with the increase of β_0 , as shown in Fig. 6, because the characteristics of the decreasing rate depend not only on those of V_u but also on the variation of β_c . In general, when X is in the same location V_u is larger than V_x and V_w is smaller than V_x .

The calculated and the experimental values of the width of jets Y_B are shown in Fig. 10. As can be seen from it, the calculated values agree with the experimental values if $\beta_0 = 30^\circ \sim 60^\circ$, but the two do not agree if $\beta_0 = 15^\circ$ for the same reason as mentioned above. Y_B increases linearly with the increase of X , as shown in Fig. 10. This means that the spread angle of jet along x -direction is constant in each case.

Fig. 10

5. Conclusions

The theoretical analysis of the main stream lines and the velocity distributions in annular swirling jets with the semi-radial flow pattern have been presented. The general equations of main stream lines, the distribution of each velocity component and the width of jets have been derived. The constants used in these equations were given from the experimental values in the previous papers⁽²⁾.

It has been proved by comparing the calculated values with the experimental values that this calculation method is applicable to the semi-radial jets of $\beta_0 = 30^\circ \sim 60^\circ$.

References

- (1) K. Akagawa and M. Taga: *Trans. Japan Soc. Mech. Engrs.*, Vol. 31, No. 221 (1965-1), p. 105.
- (2) M. Taga and K. Akagawa: *This Bulletin*, p. 1294.
- (3) L. G. Loitsyanskii: *Prikladnaya Matematika i Mekhanika*, Vol. 17 (1953), p. 3.
- (4) H. Gortler: *Revista Matematica Hispano Americanas*, Ser. 4, Vol. 14 (1945), p. 143.
- (5) Shao-Lin Lee: *Trans. ASME*, Ser. E, Vol. 32, No. 2 (1965-6), p. 258.
- (6) N. A. Chigier and A. Chervinsky: *11th Int. Symp. Comb.*, (1967), p. 489.
- (7) H. Ullrich: *Forsh. Ing.-Wesen.*, Bd. 25, Nr. 6 (1959), S. 165; Bd. 26, Nr. 1 (1960), S. 19.
- (8) K. Tabushi: *Bulletin of University of Osaka Prefecture*, Ser. A, Vol. 3 (1955), p. 9; Vol. 4 (1956), p. 7.

Discussion

Editorial Director: It is necessary to know the pressure constants C in order to calculate the main stream line. In the 3rd report, only the values of C for $\alpha_0 = 0^\circ$, $2b_0/D = 0.2$ are shown and not for the other cases. It was stated that P_{sR} should be used as the values of P_s in the region of $z/R_0 < 2$. But in the 2nd report, the values of C_s which are based on the values of P_{sR} are shown. It may be

desirable that the values of C for the other nozzle dimensions be shown in order to generalize the method of determining the main stream lines by using Eq. (9).

Author's closure

The values of C for various β_0 are shown in Fig. 2 in this paper as those for typical nozzle dimen-

Append.-Fig. 1 Pressure constant (influence of $2b_0/D$)

sions. In addition, the values of C for various $2b_0/D$ when $\alpha_0=0^\circ$ and $\beta_0=45^\circ$ are shown in Append.-Fig. 1, and those for various α_0 at $2b_0/D=0.2$, $\beta_0=45^\circ$ in Append.-Fig. 2. Solid lines and broken ones in the figure show values of C used in the calculation of Eq. (9). These values were determined by

Append.-Fig. 2 Pressure constant (influence of α_0)

using the experimental values of P_{sz} in the region of $z/R_0 > 2$ and that of P_{sR} in the region of $z/R_0 < 2$.

In general, the values of C are mainly affected by β_0 and only slightly by α_0 and $2b_0/D$