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Analysis of Axi-Symmetric Extrusion*

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In the present paper, the axi-symmetric extrusion of the Tresca's solid is analysed by means of the semi-inverse method under the Haar-von Karman hypothesis. At first, the stress field is constructed by assuming the distributions of the pressure and the frictional stress on die surface. Then, the velocity distribution in the slip line field is calculated with the boundary conditions of the velocities of the rigid regions. Here, an arbitrary stream line is considered as the die surface, and the stress and the velocity fields obtained above become the solution of the problem of certain peculiar frictional condition, which is different from the given one. Several iterations of this procedure make the difference sufficiently small.

From the numerical calculations, the lower and the upper bounds of the extrusion forces are obtained, which are much better than the usual solutions, and the mean extrusion pressures are found to be 10% ~ 30% larger than those of the plane strain extrusions. Then the strain distributions are compared with the experimental results analysed by the moire and the gridwork methods.

1. Introduction

In the axi-symmetric problems of the solids which satisfy the Tresca's yield condition and the associated flow rule (Tresca's solid), the basic equations vary with the state of stress. Except the other cases where the equations are extremely simplified, two cases are important^{(6) (8)}. One is the case that the stress is represented by the vertex of the Tresca's yield hexagon, that is, the circumferential stress σ_θ is equal to one of the principal stresses in the r - z plane, which is known as the Haar-von Karman's hypothesis⁽³⁾. The other is the one that the stress is represented by the side of the Tresca's hexagon. Not so many problems, however, fall under the latter case because the flow rule requires the deformation to become plane strain deformation, that is, one of the principal strain rates in the r - z plane becomes zero. As the deformation of the axi-symmetric extrusion through the conical die seems to approach radial flow, the state of stress of the extrusion is naturally supposed to correspond to the former case.

Most problems of the metal processings are statically indeterminate because the boundary conditions on the surfaces contacting with tools are associated

not only with the stress but also with the velocity. The difficulty arising from the indetermination has been settled by tactful use of the hodograph in the plane strain problems, while, in the axi-symmetric problems, the effective methods corresponding to this are not yet found. Most of the usual analyses of the axi-symmetric extrusion are founded on the upper bound method and few are on the lower bound method. And now the difference between the upper bound load and the lower bound one is not so small.

In this paper, the force of the axi-symmetric extrusion is analysed. The Haar-von Karman hypothesis is assumed. For the settlement of the indetermination of the problem, the semi-inverse method is introduced as follows:

First, the distributions of the pressure and the frictional stress on the die surface are assumed, and the slip line field is constructed. Then, the velocity field is calculated under the boundary conditions of the velocities of the rigid regions located forward and backward of the deforming region respectively. Now imagine a fictitious die whose surface coincides with a certain stream line arbitrarily chosen, then the stress and the velocity field obtained as mentioned above are equivalent to the solution of this fictitious problem in which frictional stress distribution on the fictitious die surface is determined by the stress field later. Several trials of this procedure lead to the solution of the problem concerned which satisfies the given frictional condition sufficiently.

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The upper and the lower bounds of the forces of the extrusions with conical dies have been calculated numerically, which are found to be much better than usual ones. Moreover, qualitative agreement of the velocity fields of the upper bound solutions with the experimental results analysed by the moire method and the grid work method has been shown.

2. Basic equations

2.1 Equations of stress and velocity

The solid is a rigid, perfectly plastic Tresca's one. Let us assume the circumferential stress $\sigma_\theta (\equiv \sigma_3)$ is equal to the smaller principal stress in the r - z plane. When $\sigma_2 < \sigma_1$,

$\sigma_\theta = \sigma_2$ .....Haar-von Karman's hypothesis.

This means that the stress is represented by the vertex A of the hexagon in Fig. 1. A pair of right-handed curvilinear axes, α - and β -lines along the maximum shear lines is introduced and α - and β -lines are distinguished by the condition that the direction of the algebraically greater principal stress falls in the first and the third quadrants. Let the anti-clockwise angular rotation of the α -line from the r -axis be denoted by ϕ , the shearing yield stress by k and the mean of the normal stresses in the r - z plane by $2k\omega$. Then

$$\sigma_1 = 2k\omega + k, \quad \sigma_2 = 2k\omega - k$$

and the equations of equilibrium^(1b) are

$$\left. \begin{aligned} d\omega - d\phi + \frac{dr - dz}{2r} &= 0 \text{.....along } \alpha\text{-lines} \\ d\omega + d\phi + \frac{dr + dz}{2r} &= 0 \text{.....along } \beta\text{-lines} \end{aligned} \right\} \text{.....(1)}$$

Let the r - and z -components of the velocity be denoted by U and W respectively, then

$$\left. \begin{aligned} dU - Wd\phi + (U - W \tan \phi) \frac{dr}{2r} &= 0 \\ &\text{.....along } \alpha\text{-lines} \\ dW + Ud\phi + (U - W \tan \phi) \frac{dz}{2r} &= 0 \\ &\text{.....along } \beta\text{-lines} \end{aligned} \right\} \text{.....(2)}$$

Equations (1) and Eqs. (2) respectively correspond to the Hencky's equations and the Geiringer's equations, and the former equations coincide with the latter ones in which the third terms are added to the right-hand sides. Therefore numerical calculations

can be performed in a similar way.

Strain rates are

$$\left. \begin{aligned} \dot{\epsilon}_2 &= -\frac{u}{2r} - \frac{1}{2} \left[\frac{\partial W}{\partial S_\alpha} + U \frac{\partial \phi}{\partial S_\alpha} + \frac{\partial U}{\partial S_\beta} - W \frac{\partial \phi}{\partial S_\beta} \right] \\ \dot{\epsilon}_3 &= \frac{u}{r} \end{aligned} \right\} \text{.....(3)}$$

where

$$u = U \cos \phi - W \sin \phi$$

2.2 Equations expressed by difference quotients

The basic equations are expressed by difference quotients and the procedures of the calculation of the two fundamental problems are explained in the following.

(i) Cauchy's problem. The coordinates and ω and ϕ of the points 1, 2 are given in Fig. 3(a). From Eqs. (1),

$$\left. \begin{aligned} \omega_3 - \omega_1 - (\phi_3 - \phi_1) + \frac{(r_3 - r_1) - (z_3 - z_1)}{r_1 + r_3} &= 0 \\ \omega_3 - \omega_2 + (\phi_3 - \phi_2) + \frac{(r_3 - r_2) + (z_3 - z_2)}{r_2 + r_3} &= 0 \end{aligned} \right\} \text{.....(4)}$$

along the arcs $\widehat{12}$, $\widehat{23}$ respectively. Eliminating ω_3 , we obtain

$$\phi_3 = \frac{\phi_1 + \phi_2}{2} - \frac{\omega_1 - \omega_2}{2} + \frac{A_1 - A_2}{2} \text{.....(5)}$$

where A_1 , A_2 are the last terms of the left-hand sides of the first and the second ones of Eqs. (4) respectively. The coordinates of the point 3 are

$$\left. \begin{aligned} r_3 &= \frac{n(z_2 - z_1) + r_2 + mn r_1}{1 + mn} \\ z_3 &= z_1 + (r_3 - r_1)m \end{aligned} \right\} \text{.....(6)}$$

where

$$m = \tan \frac{\phi_1 + \phi_3}{2}, \quad n = \tan \frac{\phi_2 + \phi_3}{2} \text{.....(7)}$$

The procedure of the calculation is as follows. Substituting $m = \tan \phi_1$ and $n = \tan \phi_2$ instead of Eqs. (7) into Eqs. (6), we obtain the first approximations of r_3 , z_3 . Then ϕ_3 is determined by Eq. (5) and m , n are by Eqs. (7) successively. Substituting these values of m , n into Eqs. (6), we obtain better approximations of r_3 , z_3 ; ω_3 is determined by the first one of Eqs. (4).

(ii) Riemann's problem. The coordinates and ϕ of the points 1, 2, 3 are given. We have Eqs. (4) along the arcs $\widehat{13}$, $\widehat{23}$ and similar equations along the arcs $\widehat{01}$, $\widehat{02}$. By these four equations, ϕ_3

Fig. 1 Yield locus

Fig. 2 Cylindrical coordinate

(a) Cauchy's problem (b) Riemann's problem
Fig. 3 Construction of slip line fields

is calculated. The rest of the procedure is omitted, being similar to that of the problem mentioned above.

The velocity is determined in a similar way.

3. Solution

3.1 Principle of analysis and procedure of calculation

A semi-inverse method is applied. If the reductions of area are not very small, it seems that the slip line fields are expressed by such patterns as are shown typically in Fig. 4 (a), (b). Let us call these patterns "pattern I" and "pattern II" respectively.

(a) Construction of slip line field of pattern I

Assume the shape of the die surface $\overline{AB}$ and the distributions of the pressure and the frictional

stress on it. Then all of the slip line field ACBG can be constructed by the combination of the solutions of the two fundamental problems explained above. The velocities on the rigid regions of the left-hand side of $\widehat{AC}$ and of the right-hand side of $\widehat{BC}$ are inversely proportional to their cross sectional areas. The boundary conditions of these velocities determine the velocity field in the region ACBG. Here, the quantity of the solid flowing into the deforming region across $\widehat{AC}$ is equal to that out of the region across $\widehat{BC}$, which means that the points A and B are on the same stream line. The obtained stress and velocity fields as above mentioned are equivalent to the solutions of the extrusion whose die surface coincides with the stream line $\widehat{AB}$, where

the frictional stress is calculated. Several trials of such inverse method lead to the solution which satisfy the given boundary condition well enough.

(b) Construction of slip line field of pattern II

The boundary conditions of the shape of the die surface and the distributions of the pressure and the frictional stress concerning some part $\widehat{BF}$ of the die surface $\widehat{AB}$ are assumed. Then the slip line fields FEDBG, DEC, CEFH and HFA are constructed successively. And the velocity field is calculated next. The velocity field in the region AHF is determined by the velocity of the rigid region on the left-hand side of $\widehat{AHC}$ and the boundary conditions on the die surface $\widehat{AF}$. Further, the velocity fields in the regions FHCE, ECD are obtained. The boundary conditions of the velocities of the left hand side of $\widehat{DEF}$ and that of the rigid region on the right hand side of $\widehat{DB}$ determine the velocity field in the region FEDBG. Here, the points F and B are on the same stream line from the condition of the conservation of volume. The stream line is now assumed to be the die surface.

3.2 Lower bound

How the statically admissible stress fields are found in the wedge-shaped rigid regions in the neighborhood of the points A, B, C, D in Fig. 4 is discussed here⁽⁵⁾. In the neighborhood of the points C, D, the slip line fields can be extended at once.

Fig. 4 Slip line fields of axis-symmetric extrusions

In the neighborhood of the point A, statically admissible stress field exists if $\angle HAK \geq 1/2 \cdot \cos^{-1}(f/k)$, where f is the frictional stress specified on the container surface $\overline{AK}$ and k is the shearing yield stress. If $f=0$, $\angle HAK \geq 45^\circ$. In the case of the extrusion without the container, the reduction of area is required to be greater than the bulging limit^(1a). In the neighborhood of the point B, the stress field is found to be statically admissible with reference to the solution of the indentation of the flat punch into semi-infinite body⁽⁴⁾. Thus the solution obtained in the previous section is proved to be statically admissible which means the extrusion force is the lower bound.

3.3 Upper bound

If the solution obtained in section 3.1 does not satisfy the flow rule, the upper bound of the extrusion force can be calculated as the product of the velocity field u_i and the associated stress field σ_{ij} which are so chosen as to satisfy the flow rule

$$(\sigma_{ij} - \sigma_{ij}^0) \dot{\epsilon}_{ij} \geq 0,$$

$$\text{where } \dot{\epsilon}_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i})$$

and σ_{ij}^0 : arbitrary, statically admissible stress

.....(8)

Let us express the strain rates by two parameters λ and μ as follows:

$$(\dot{\epsilon}_1, \dot{\epsilon}_2, \dot{\epsilon}_3) = (\lambda + \mu, -\lambda, -\mu)$$

Then the flow rule which is kept on each vertex

Table 1 Correspondence of strain rate to stress
(Letters in the column "stress" are referred to Fig. 1)

Values of λ, μ	Stress	$w/2k$
$\lambda \geq 0, \mu \geq 0$	A	$\lambda + \mu$
$\lambda < 0, \mu \geq 0, \lambda \leq \mu$	B	μ
$\lambda < 0, \mu \geq 0, \lambda > \mu$	C	$-\lambda$
$\lambda < 0, \mu < 0$	D	$-(\lambda + \mu)$
$\lambda \geq 0, \mu < 0, \lambda < \mu $	E	$-\mu$
$\lambda \geq 0, \mu < 0, \lambda \geq \mu $	F	λ

Fig. 5 Mean extrusion stress ($\mu=0$)

and side of the hexagonal in Fig. 1 is represented as shown in Table 1. It is assumed at the beginning of the analysis that the state of the stress is expressed by the point A in Fig. 1. Therefore, if $\lambda, \mu \geq 0$ this is satisfied with reference to the first row of Table 1, and that solution becomes an exact one. When the flow rule is not satisfied, the upper bound of the extrusion force is calculated by the following expression

$$\dot{w}/2k = (\sigma_1 \dot{\epsilon}_1 + \sigma_2 \dot{\epsilon}_2 + \sigma_3 \dot{\epsilon}_3)/2k \dots \dots \dots (9)$$

where $\sigma_1, \sigma_2, \sigma_3$ are chosen with reference to Table 1 so as to satisfy Eq. (8). The values of Eq. (9) are listed up in the third column in Table 1.

4. Upper and lower bounds of extrusion force

4.1 Extrusion of solid bar

(a) Extrusion through smooth conical die

The extrusions through the smooth conical dies (α : semi-angle of the die) are treated here. It is known from our experience that the solution satisfying the given boundary condition well enough is obtained by three or four trials of the calculations* of the stress and the velocity fields on the assumption of the distribution of the pressure varying linearly along the die surface. Some examples of the constructed slip line fields are shown in Fig. 4, where the pressure distributions on the die surfaces are also illustrated. The networks are shown by fine lines along which numerical calculations are carried out. It has been found that the solutions do not satisfy the flow rule in some parts of regions, and the upper bounds of the extrusion forces are calculated with the procedure explained in section 3.3. The upper and the lower bounds of the mean extrusion forces are shown in Fig. 5.

(b) Errors in analysis

(i) Boundary condition. As for the deviations δ of the inversely obtained die surfaces from the given ones, $\delta/\overline{AB} < 1/100$ and the ratios μ of the frictional stresses to the pressures on the die surfaces are 0.015 at most, which sufficiently approximate the condition $\mu=0$, in all but several problems of the pattern II. The errors of the extrusion forces arising from these errors of the boundary condition seem to be negligible because the extrusion forces little vary in the repetition of the calculations.

(ii) Error in integration. The lower bound of the mean extrusion force is calculated by two integrations: one is along $\overline{AC}$ and the other along $\overline{AB}$. The difference of these values suggests the order of the error in integration, which is shown by the symbol ϕ in Figs. 5 ~ 7.

* It takes 10 min. for the calculation of the slip line field of the pattern I and 18 min. for that of the pattern II with the computer FACOM 222.

(iii) Condition on the central axis. Because of the singularity of the central axis, the slip line field cannot be constructed so far as the central axis is reached. But the error of the extrusion force caused by this incompleteness looks negligible, as the extrusion force of tube analysed later in Fig. 9 does not vary when the inner diameter is small.

(c) Approximate expression

Figure 5, where the reduction of area r on the abscissa is converted into the extrusion ratio $R[=1/(1-r)]$, is inserted again in Fig. 6. The mean extrusion stress is approximated by the following expression :

$$\left. \begin{aligned} p/2k &= A + B \log_e R \\ A &= 1.2 \left(\frac{\alpha^\circ}{90^\circ} \right) \\ B &= 0.74 + 0.22 \left(\frac{\alpha^\circ}{90^\circ} \right) \end{aligned} \right\} \dots\dots\dots (10)$$

Fig. 6 $p/2k \sim \log_{10} R$ diagram ($\mu=0$)

Fig. 7 $p/2k \sim \log_{10} R$ diagram ($\mu=0.05, 0.1$)

The redundant work — the deviation of the extrusion stress from the energy consumption of the ideal deformation $\frac{p}{2k} = \log_e R$ (one dotted lines in Fig. 5 and Fig. 6) — decreases with the increasing extrusion ratio, which is observed distinctly when the die angle is small.

(d) Extrusion through rough conical die

Similar procedures were applied to the analyses of the extrusions through rough dies* whose frictional coefficients $\mu=0.05$ or 0.1 . The mean extrusion forces are graphed in Fig. 7, where one and two dotted lines respectively correspond to $\mu=0.05, 0.1$. The curves are almost straight and the gradients are steeper than those of the fine solid line corresponding to $\mu=0$.

(e) Comparison of the solution with that for Mises' solid

The upper bound of the forces of the extrusions through smooth conical dies which B. Avitzur⁽⁹⁾ obtained for the Mises' solid**, using the velocity field of two spherical rigid-plastic boundaries and radial flow, are shown by broken lines in Fig. 8. Solid lines correspond to the solutions in this paper for the Tresca's solid. Here, the tensile yield stress Y of the Mises' solid and the shearing yield stress k of the Tresca's solid are so related to each other that the Tresca's hexagonal inscribes the Mises' circle in the π -plane, that is, $Y=2k$. The upper

Fig. 8 Comparison with the Avitzur's solution ($\mu=0$)

* The terms "upper bound" and "lower bound" are also used conventionally in the case of $\mu \neq 0$.

** The solid which satisfies the Mises' yield condition and the associated flow rule.

and the lower bounds for Mises' solid* obtained from the upper and the lower bounds in this paper for the Tresca's solid are shown by the upper and the lower ends of the arrows respectively in Fig. 8. The Avitzur's solution falls out of the limits obtained in this paper when the die angle is large.

The expression

$$\frac{p}{2k} = 0.8 + 1.5 \log_e R$$

which was extracted experimentally from the extrusions of lead through the rectangular dies by W. Johnson⁽⁷⁾, is shown by the broken line in Fig. 7. It coincides with the solution in this paper if the frictional coefficient in the experiment is a little bit greater than 0.05, in consideration of the fact that the extrusion force for Mises' solid is larger than the extrusion force for Tresca's solid in this paper as is shown in Fig. 8.

In this paper the restriction of the velocity field is much released under the Haar-von Karman's hypothesis which is suitable for the extrusion, while, in the usual solution with simple velocity field, it is not sufficiently released. It is the reason why the upper bound of this paper becomes lower than the usual one in spite of the disadvantage of the conversion of the solution for the Tresca's solid into the solution for the Mises' solid.

4.2 Plug-extrusion of tube

The plug-extrusions of tubes with variable inner diameters are analysed for the purpose of the comparison of the force of axis-symmetric extrusion with that of the plane strain extrusion. The result is shown in Fig. 9. When the inner diameter is large, the extrusion force decreases with the increasing diameter, but when the inner diameter is small, the

Fig. 9 Plug-extrusion of tube ($\alpha=60^\circ$, $r=0.5$)

* If L_u and L_l are respectively the upper and the lower bounds for the Tresca's solid, $2/\sqrt{3} L_u$ and L_l respectively become the upper and the lower bounds for the Mises' solid.

extrusion force does not vary.

Now the forces of the axis-symmetric extrusions of solid bars are compared with those of the plane strain extrusions which coincide with the extrusions of tubes in the limiting condition of $r_c/r_B=1$. The results are shown in Fig. 10, where figures mean the reductions of area. From this comparison it is concluded that the forces of the axis-symmetric extrusions are 10% ~ 30% larger than those of the plane strain extrusions.

5. Discussion of velocity fields of upper bound solutions and comparison of them with experimental results

In the case of the deformation such as the extrusion where comparatively larger part of the boundary is restrained with tools, the velocity field of the upper bound solution, it seems, approximates the true one. In this chapter, the distributions of the strain rate and the equivalent strain obtained as above are explained and further compared with the experimental results.

Several representatives of our solutions are shown in Fig. 11 ~ Fig. 13 and the experimental results are in Fig. 14 ~ Fig. 17. Here, the radius of the billet is taken as the unit of length and the extruded length of the billet is taken as the unit of time. The photographs of the moire patterns are taken after the extrusion of infinitesimal quantity, where the grids for the moire method are printed on the longitudinal sections deforming in the steady state in advance⁽²⁾. The moire or the grid work method is applied for the analyses.

Figure 11 (a) is a standard example of theoretically obtained slip line fields in which both of the die angle and the reduction of area are rather small. Three typical modes of deformation are observed in

Fig. 10 Comparison of axis-symmetric extrusion with plane strain extrusion

the deforming region*. The first mode is seen in the triangle AFB contacting with the die surface, where the strain rates in the r - z plane practically vanish and the deformation is akin to that of the uniaxial compression**. The second is in the fan-like regions with the centers of the singular points

A, B, where the strain rates in r - z plane increase greatly. The third is at the point of intersection C of the lines of discontinuity, $\widehat{AC}$, $\widehat{BC}$, where the strain rates are high. These specific characteristics of the three modes of the deformation coincide with those observed in the experimental results—the

Fig. 14 Experimental result, $\alpha=10^\circ$, $r=0.2$; 56S-Al (44.7 ϕ)

* Similar features are also observed in the plane strain extrusion, which has been discussed already⁽²⁾.

** In this region, the stream lines diverge, which means the condition $\dot{\epsilon}_2, \dot{\epsilon}_3 \leq 0$ corresponding to the Haar-von Karman hypothesis is not satisfied.

photographs of the moire patterns and the arranged results, Fig. 14 and Fig. 15. According to the distribution of the equivalent strain in the cross section of the extruded portion, Fig. 11 (c) and Fig. 15

also coincide with each other.

In the case of the large die angle, Fig. 16, the solid is almost sticking to the die surface, because the pressure on it is heavy. If the frictional stress were taken into account in the theoretical analysis, the fan-like region with the center B of Fig. 12 would spread and a concentration of the strain rate would be developed.

When the reduction of area is large, Fig. 13 and Fig. 17 are obtained. They coincide with each other rather well.*

Lastly, the theoretical solution of the axis-symmetric extrusion is compared with that of the plane strain extrusion. This comparison may be useful for the deduction of the informations of the former from those of the latter, though it has no physical meaning. Comparing Fig. 11 with Fig. 18, we know that the distribution of the equivalent stress of the axis-symmetric extrusion and that of the plane strain extrusion are alike and that its variation in the cross section of the extruded portion of the former is smaller than that of the latter. In this respect, it has been already discussed experimentally⁽²⁾. The coincidence of the theoretical solution of this paper with the experimental results is not so perfect as in the case of the plane strain extrusion.

6. Conclusions

In this paper, the axis-symmetric extrusion of the Tresca's solid is analysed by the semi-inverse method under the Haar-von Karman hypothesis. The conclusions are:

- (1) Excellent upper and lower bounds of the extrusion force are obtained and the extrusion force is shown by a simple approximate expression Eq. (10).
- (2) It is found that the mean extrusion stress of the axis-symmetric extrusion is 10% ~ 30% larger than that of the plane strain extrusion.
- (3) The velocity field of the upper bound solution in this paper coincides qualitatively with the experimental results.
- (4) The variation of the distribution of the strain of the axis-symmetric extrusion is a little

* When the die angle is small, the moire fringes composed of the vertical grid lines are akin to co-centric circles in the spherically symmetric flow, to which the flow of Fig. 17 corresponds.

smaller than that of the plane strain extrusion.

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