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Elastic Vibration Analysis of Passenger Car Body in Bending and Torsional Modes*

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This paper presents a theoretical analysis of elastic vibrations of a passenger car body in its bending and torsional modes. The body structure is simulated as a three dimensional framework. Panels of the structure affect body vibrations restricting deformations of the structure. Fenders and doors are replaced by tension rigs and roof, floor pans, etc. are replaced by equivalent flexural stiffnesses added to the flexural stiffnesses of members surrounding the panels. The analysis has been accomplished with lumped mass system and matrix method. All input data for calculations are obtainable from drawings, so that we can estimate vibration characteristics of any particular body structure at the very first stage of basic design. Good approximations (over 90% accuracy) have been obtained between calculations (more than 200 degrees of freedom) and experiments in various cases of body structures.

1. Introduction

Passenger car bodies are complicated structures composed of thin panels and their elastic vibrations couple with vibrations of all other vehicle components. In order to analyse vehicle vibrations including body vibrations and in order to forecast body vibration characteristics of a new model for refining before it is manufactured if necessary, the elastic vibrations of a body structure must be analysed theoretically. For this purpose the author reported the theoretical analysis of body shakes of passenger cars about bending modes before⁽¹⁾, and here he will report the analysis about both bending and torsional modes in which a body structure is simulated as a three dimensional framework.

In lower frequency range there happen torsional modes more often than bending modes do. So it is very important to simulate the body structure as a three dimensional framework. Three dimensional analysis can explain theoretically not only torsional shakes but also bending shakes which two dimensional analysis explained before. Therefore, three dimensional analysis includes two dimensional analysis as for bending shakes and it explains all about body shakes analytically. This paper will report the analysis

including checks of the analysis by comparing calculations with experiments of three dimensional models and actual bodies up to complete white bodies. Good approximations have been obtained about every case of those above-mentioned. Though only the case of free-supports is reported in the paper, the analysis can be applied easily with a little change to other cases of various supports. Theoretical analyses of various types of actual bodies, forced vibrations, combined vibrations with panel vibrations, etc. will be induced in the near future.

The author will be very happy if the analysis is used by body designers and engineers to control body shakes in design processes and to analyse vehicle vibrations.

2. Theoretical analysis

2.1 Three dimensional framework for the analysis

In order to analyse both bending and torsional vibrations of a body the body structure is simulated as a three dimensional framework (shown in Fig. 1). Though the structure of a passenger car body is very complicated in general, the three dimensional framework should be as simple as possible to be analysed easily, if it can explain body shakes analytically. For analysis of shakes which are vibrations of whole body structures in lower frequency range below 50 or 60 Hz, the analytical model is

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supposed to be composed mainly of frames of the body structure like Fig. 2, and also even some panels are supposed to be added to the model simulating their shear stiffnesses which give great effects on body shakes. Fenders, doors, roof, floor pans, dashboard, fire wall, etc. are those important panels which should not be neglected for simulating body shakes. Fenders and doors are supposed to be replaced by tension rigs, which have only tensile stiffnesses and therefore affect in only tensile directions of rigs (see $\text{---}\text{W}\text{---}$ in Fig. 1). Other panels such as roof, floor pans, dashboard and fire wall are simulated as equivalent flexural stiffnesses which are added to those stiffnesses of members surrounding the panels in the plane of the panels.

In Fig. 1 the coordinates (FR-RH-UPR) about the body structure are shown, in which FR means front part, RH means right part, and UPR means upper part of the body. In the analysis as lumped mass system is used, mass particles and beams composing the framework are distributed. For convenience those beams of the framework are classified in three groups. Stringers mean beams distributed along FR axis, cross members mean beams along RH axis and pillars mean beams along UPR axis in Fig. 1. Four-door sedan, which is the most popular type of passenger car bodies in Japan, has three pillars. As for front pillars and rear pillars their upper parts are slant against their lower

Fig. 1 Three dimensional framework

Fig. 2 Fundamental framework

parts and those slant angles are very important to simulate body structures in order to obtain good simulations in body shakes.

2.2 Frequency equation about the framework

The framework like Fig. 2, which is composed of only frames excluding tension rigs from the structure shown in Fig. 1, is the basic model to be analysed in the analysis. The reason is that it is convenient to deal with only the framework at first, and then panel stiffnesses should be added to the analysis because panel stiffnesses affect additionally the vibrations of the framework.

In the analysis the elastic vibration system as Fig. 2, which has infinite degrees of freedom, is replaced by lumped mass system of finite degrees of freedom. So the system is composed of mass particles of concentrated masses and massless beams.

And the equations in the analysis are expressed in matrix expressions. In order to relate vibrations of the system analytically, three conditions which rule dynamic motions are necessary as you know. Each of three conditions is called condition of compatibility which is the relation between displacements of mass particles and end deflections of beams, elastic equation between end deflections of beams and internal forces of beams, and condition of equilibrium between internal forces of beams and external forces of mass particles. Degrees of freedom of the system like Fig. 1 should be quite a lot in order to analyse shakes with certain accuracy, and therefore the analysis must demand many core memories of a computer which calculates the analysis numerically. It is convenient that necessary core memories are as few as possible and so the author decided to use displacements of mass particles instead of end deflections of beams. Therefore, elastic equation in three dimensional analysis includes both condition of compatibility and elastic equation of two dimensional analysis presented before.

$$\text{Elastic equation} \quad U = k \cdot D \cdots \cdots (1)$$

$$\text{Conditions of equilibrium} \quad F = b \cdot U \cdots \cdots (2)$$

where

D : vector of displacements of masses

U : vector of internal forces of beams

F : vector of external forces of masses

k : stiffness matrix

b : equilibrium matrix

As in the analysis only body structures freely supported are dealt with, external forces of mass particles are only their inertia forces. Then

$$F = I \cdot D \cdots \cdots (3)$$

where

I : inertia matrix

then frequency equation is obtained as follows.

$$|I - b \cdot k| = 0 \quad \dots\dots\dots(4)$$

Displacements of mass L_i are shown in Fig. 3 in their positive directions. In detail the vector of displacements of the system is

$$D = \{u_i, p_i, v_i, q_i, w_i, r_i\} \quad \dots\dots\dots(5)$$

External forces of mass L_i are also shown in Fig. 3 in their positive directions. Only three inertia forces are considered in directions of FR, RH and UPR, and no inertia moment is considered. According to several papers the effects of rotational inertias of beams which are a little longer are very small, less than two or four percent, and input data for calculations can be picked up easily in short time if rotational inertias are neglected. Those are the reasons why rotational inertias are considered negligible. Then vector of external forces of the system is

$$F = \{N_i, 0, V_i, 0, W_i, 0\} \quad \dots\dots\dots(6)$$

Internal forces of beam B_i are shown in Fig. 4 in their positive directions. Superscript L means the left end of the beam and R means the right end. Then vector of internal forces of the system is

$$U = \{M_i^L, Q_i^L, S_i^L, R_i^L, T_i^L, P_i^L, M_i^R, Q_i^R, S_i^R, R_i^R, T_i^R, P_i^R\} \quad \dots\dots\dots(7)$$

Figure 4 also shows stiffnesses of beam B_i . There are four stiffnesses such as two flexural stiffnesses EI and EH , one tensile stiffness EA and one torsional stiffness GJ . In the analysis those stiffnesses are used as $a_i, b_i, c_i, d_i, e_i, f_i$ as follows, where l_i is length of beam B_i .

$$\left. \begin{aligned} a_i &= \frac{6EI_i}{l_i^2} & b_i &= \frac{2EI_i}{l_i} & c_i &= \frac{6EH_i}{l_i^2} \\ d_i &= \frac{2EH_i}{l_i} & e_i &= \frac{GJ_i}{l_i} & f_i &= \frac{EA_i}{l_i} \end{aligned} \right\} \quad \dots\dots\dots(8)$$

$$\begin{pmatrix} M_i^L \\ Q_i^L \\ S_i^L \\ R_i^L \\ T_i^L \\ P_i^L \\ M_i^R \\ Q_i^R \\ S_i^R \\ R_i^R \\ T_i^R \\ P_i^R \end{pmatrix} = \begin{pmatrix} 0 & 0 & -a_i & 0 & 0 & -2b_i & 0 & 0 & a_i & 0 & 0 & -b_i \\ 0 & 0 & -\frac{2a_i}{l_i} & 0 & 0 & -a_i & 0 & 0 & \frac{2a_i}{l_i} & 0 & 0 & -a_i \\ 0 & 0 & 0 & -2d_i & c_i & 0 & 0 & 0 & 0 & -d_i & -c_i & 0 \\ 0 & 0 & 0 & c_i & -\frac{2c_i}{l_i} & 0 & 0 & 0 & 0 & c_i & \frac{2c_i}{l_i} & 0 \\ 0 & -e_i & 0 & 0 & 0 & 0 & 0 & 0 & e_i & 0 & 0 & 0 \\ -f_i & 0 & 0 & 0 & 0 & 0 & f_i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_i & 0 & 0 & b_i & 0 & 0 & -a_i & 0 & 0 & 2b_i \\ 0 & 0 & -\frac{2a_i}{l_i} & 0 & 0 & -a_i & 0 & 0 & \frac{2a_i}{l_i} & 0 & 0 & -a_i \\ 0 & 0 & 0 & d_i & -c_i & 0 & 0 & 0 & 0 & 2d_i & c_i & 0 \\ 0 & 0 & 0 & c_i & -\frac{2c_i}{l_i} & 0 & 0 & 0 & 0 & c_i & \frac{2c_i}{l_i} & 0 \\ 0 & -e_i & 0 & 0 & 0 & 0 & 0 & 0 & e_i & 0 & 0 & 0 \\ -f_i & 0 & 0 & 0 & 0 & 0 & f_i & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_i \\ p_i \\ v_i \\ q_i \\ w_i \\ r_i \\ u_{i+1} \\ p_{i+1} \\ v_{i+1} \\ q_{i+1} \\ w_{i+1} \\ r_{i+1} \end{pmatrix} \quad \dots\dots\dots(9)$$

Fig. 3 Displacements and external forces of mass particle

Fig. 4 Internal forces and stiffnesses of beam

2.2.1 Elastic equation

Elastic equation, the relation between displacements of masses and internal forces of beams is obtained by displacement method (the slope deflection theory) about each beam of the system. For beam B_i the relation is written in matrix expression as follows.

$$[U_i] = [k_i] \cdot [D_i]$$

Matrix k is called stiffness matrix.

(a) As for stringers which are named as B_i to B_{i+1} for beams (L_1 to L_{i+1} for masses) from the front end to the rear end of the system in order, elastic equation about beam B_i is shown as Eq. (9) in detail.

(b) As for cross members which are named as B_i to B_{i+1} for beams from the left end to the right end of the system in order, elastic equation about beam B_i is shown as Eq. (10) in detail.

$$\begin{Bmatrix} M_i^L \\ Q_i^L \\ S_i^L \\ R_i^L \\ T_i^L \\ P_i^L \\ M_i^R \\ Q_i^R \\ S_i^R \\ R_i^R \\ T_i^R \\ P_i^R \end{Bmatrix} = \begin{bmatrix} 0 & 2b_i & -a_i & 0 & 0 & 0 & 0 & b_i & a_i & 0 & 0 & 0 \\ 0 & a_i & -\frac{2a_i}{l_i} & 0 & 0 & 0 & 0 & a_i & \frac{2a_i}{l_i} & 0 & 0 & 0 \\ -c_i & 0 & 0 & -2d_i & 0 & 0 & c_i & 0 & 0 & -d_i & 0 & 0 \\ \frac{2c_i}{l_i} & 0 & 0 & c_i & 0 & 0 & -\frac{2c_i}{l_i} & 0 & 0 & c_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -e_i & 0 & 0 & 0 & 0 & 0 & e_i \\ 0 & 0 & 0 & 0 & -f_i & 0 & 0 & 0 & 0 & 0 & f_i & 0 \\ 0 & -b_i & a_i & 0 & 0 & 0 & 0 & -2b_i & -a_i & 0 & 0 & 0 \\ 0 & a_i & -\frac{2a_i}{l_i} & 0 & 0 & 0 & 0 & a_i & \frac{2a_i}{l_i} & 0 & 0 & 0 \\ c_i & 0 & 0 & d_i & 0 & 0 & -c_i & 0 & 0 & 2d_i & 0 & 0 \\ \frac{2c_i}{l_i} & 0 & 0 & c_i & 0 & 0 & -\frac{2c_i}{l_i} & 0 & 0 & c_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -e_i & 0 & 0 & 0 & 0 & 0 & e_i \\ 0 & 0 & 0 & 0 & -f_i & 0 & 0 & 0 & 0 & 0 & f_i & 0 \end{bmatrix} \begin{Bmatrix} u_i \\ p_i \\ v_i \\ q_i \\ w_i \\ r_i \\ u_{i+1} \\ p_{i+1} \\ v_{i+1} \\ q_{i+1} \\ w_{i+1} \\ r_{i+1} \end{Bmatrix} \quad \text{.....(10)}$$

According to FR-RH-UPR coordinates cross members are considered as stringers rotated 90 degrees in FR-RH plane and that is the reason why Eq. (10) is different from Eq. (9).

(c) As for pillars which are named as B_i to B_{i+1} for beams (L_i to L_{i+1} for masses) from the bottom to the top of the system in order, elastic equation about beam B_i is shown as Eq. (11) in detail.

$$\begin{Bmatrix} M_i^L \\ Q_i^L \\ S_i^L \\ R_i^L \\ T_i^L \\ P_i^L \\ M_i^R \\ Q_i^R \\ S_i^R \\ R_i^R \\ T_i^R \\ P_i^R \end{Bmatrix} = \begin{bmatrix} -a_i & 0 & 0 & 0 & 0 & -2b_i & a_i & 0 & 0 & 0 & 0 & -b_i \\ -\frac{2a_i}{l_i} & 0 & 0 & 0 & 0 & -a_i & \frac{2a_i}{l_i} & 0 & 0 & 0 & 0 & -a_i \\ 0 & -2d_i & 0 & 0 & c_i & 0 & 0 & -d_i & 0 & 0 & -c_i & 0 \\ 0 & c_i & 0 & 0 & -\frac{2c_i}{l_i} & 0 & 0 & c_i & 0 & 0 & \frac{2c_i}{l_i} & 0 \\ 0 & 0 & 0 & e_i & 0 & 0 & 0 & 0 & 0 & -e_i & 0 & 0 \\ 0 & 0 & f_i & 0 & 0 & 0 & 0 & 0 & -f_i & 0 & 0 & 0 \\ a_i & 0 & 0 & 0 & 0 & b_i & -a_i & 0 & 0 & 0 & 0 & 2b_i \\ -\frac{2a_i}{l_i} & 0 & 0 & 0 & 0 & -a_i & \frac{2a_i}{l_i} & 0 & 0 & 0 & 0 & -a_i \\ 0 & d_i & 0 & 0 & -c_i & 0 & 0 & 2d_i & 0 & 0 & c_i & 0 \\ 0 & c_i & 0 & 0 & -\frac{2c_i}{l_i} & 0 & 0 & c_i & 0 & 0 & \frac{2c_i}{l_i} & 0 \\ 0 & 0 & 0 & e_i & 0 & 0 & 0 & 0 & 0 & -e_i & 0 & 0 \\ 0 & 0 & f_i & 0 & 0 & 0 & 0 & 0 & -f_i & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_i \\ p_i \\ v_i \\ q_i \\ w_i \\ r_i \\ u_{i+1} \\ p_{i+1} \\ v_{i+1} \\ q_{i+1} \\ w_{i+1} \\ r_{i+1} \end{Bmatrix} \quad \text{.....(11)}$$

(d) As for slant beams which are named for upper parts of front and rear pillars, displacements of masses adjacent to the beam should be changed from (u, p, v, q, w, r) to (u', p', v', q', w', r') which are identified by rotating the axes as the slant angle (see Fig. 8 in detail).

Fig. 5 Stringer

Fig. 6 Cross member

$$\begin{pmatrix} u_i' \\ p_i' \\ v_i' \\ q_i' \\ w_i' \\ r_i' \end{pmatrix} = \begin{pmatrix} \sin \varphi_i & 0 & -\cos \varphi_i & 0 & 0 & 0 \\ 0 & \sin \varphi_i & 0 & -\cos \varphi_i & 0 & 0 \\ \cos \varphi_i & 0 & \sin \varphi_i & 0 & 0 & 0 \\ 0 & \cos \varphi_i & 0 & \sin \varphi_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} u_i \\ p_i \\ v_i \\ q_i \\ w_i \\ r_i \end{pmatrix} \quad (12)$$

$$\begin{pmatrix} U_1 \\ \vdots \\ U_i \\ \vdots \\ U_m \end{pmatrix} = \begin{pmatrix} k_1 & & \\ & \ddots & \\ & & k_i & \\ & & & \ddots \\ & & & & k_m \end{pmatrix} \cdot \begin{pmatrix} D_1 \\ \vdots \\ D_i \\ \vdots \\ D_n \end{pmatrix} \quad (13)$$

Then elastic equation of the system is induced as Eq. (13). Stiffness matrix of the system k is composed of small stiffness matrices about beams, which are distributed on the diagonal line of matrix k .

2.2.2 Condition of equilibrium

There are various mass particles connecting two beams, three beams or four beams. At each mass particle all internal forces of the beams and all external forces of the masses are in balance as equilibrium matrix shows. Equation (14) shows the simplest case of the mass connecting two beams.

$$\begin{pmatrix} N_i \\ 0 \\ V_i \\ 0 \\ W_i \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} M_{i-1}^R \\ Q_{i-1}^R \\ S_{i-1}^R \\ R_{i-1}^R \\ T_{i-1}^R \\ P_{i-1}^R \\ M_i^L \\ Q_i^L \\ S_i^L \\ R_i^L \\ T_i^L \\ P_i^L \end{pmatrix} \quad (14)$$

Though equilibrium matrices of masses connecting three or four beams become a little complicated, they can be obtained in the same way as Eq. (14). Equilibrium matrix of the system is composed of small equilibrium matrices about masses, which are distributed on the diagonal line of matrix b [see Eq. (15)].

Fig. 7 Piller

Fig. 8 Slant pillar

$$\begin{pmatrix} F_1 \\ \vdots \\ F_i \\ \vdots \\ F_n \end{pmatrix} = \begin{pmatrix} b_1 & & & \\ & \ddots & & \\ & & b_i & \\ & & & \ddots \\ & & & & b_n \end{pmatrix} \cdot \begin{pmatrix} U_1 \\ \vdots \\ U_i \\ \vdots \\ U_n \end{pmatrix} \quad \dots\dots\dots(15)$$

Inertia forces of mass particles are obtained using inertia matrix I , which is composed of small inertia matrices distributed on the diagonal line of matrix I [Eq. (17)]. Equation (16) is inertia matrix of mass L_i

$$\begin{pmatrix} N_i \\ 0 \\ V_i \\ 0 \\ W_i \\ 0 \end{pmatrix} = \begin{pmatrix} m_i \omega^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_i \omega^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_i \omega^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} u_i \\ p_i \\ v_i \\ q_i \\ w_i \\ r_i \end{pmatrix} \quad \dots\dots\dots(16)$$

$$\begin{pmatrix} F_1 \\ \vdots \\ F_i \\ \vdots \\ F_n \end{pmatrix} = \begin{pmatrix} I_1 & & & \\ & \ddots & & \\ & & I_i & \\ & & & \ddots \\ & & & & I_n \end{pmatrix} \cdot \begin{pmatrix} D_1 \\ \vdots \\ D_i \\ \vdots \\ D_n \end{pmatrix} \quad \dots\dots\dots(17)$$

where

ω : natural circular frequency

m_i : mass of mass particle L_i

Then finally frequency equation is obtained using Eq. (4), (13), (15) and (17).

2.3 Effect of shear panel on body shake (1)

Effects of shear panels on body shakes are classified in two groups. The one is tension rigs which connect two mass particles which have only tensile stiffnesses. The other is additional stiffnesses which are added to flexural stiffnesses of members surrounding the panels. The former is the simplest method to show shear stiffnesses of shear panels, and the latter is the method for the case in which tension rig cannot be used because it is very complicated to simulate with many cross-over rigs connecting two of mass particles distributed on the boundaries. Here, tension rigs are to be explained.

Fig. 9 Equilibrium of forces

2.3.1 About fenders

Tension rigs of fenders are shown as wave like line (—w—) in Fig. 1. As you know, fenders of passenger cars are built with thin panels which have rather small flanges stamped or welded in their boundaries and have small curvatures. Those fenders make cells for some utilities such as engine room, cabin, luggage compartment, with hood, roof, luggage compartment door and at the same time they reinforce overhangs of front body and rear body by their shear stiffnesses, not by their flexural stiffnesses. The fact is shown experimentally by the author that front body and rear body have rather little stiffnesses except fenders and therefore it becomes necessary to simulate the shear stiffnesses of fenders. It is easy to consider how fenders reinforce front and rear bodies. Figure 10 shows that one end of a fender is supposed to be fixed to a pillar and that its stiffness is simulated as a tensile stiffness of a rig between a mass of the pillar and a mass at the end or at the middle of the fender.

In order to obtain the tensile stiffness of a rig easily it is convenient to consider the fender as a shear field and to apply the theory of shear field⁽²⁾. The shear field is composed of tension members which take only axial loads and shear panels. In this case tension members are supposed to be flanges of the fender and shear panels are supposed to be fender panels which are also supposed to be flat. Figure 10 (a) shows a simple case of connecting the end of a fender to a pillar and (b) shows a case of connecting the middle of a fender to a pillar. After

Fig. 10 Tension rig of fender

shear flow is obtained considering equilibrium between shear flow and external load at the tension member, total strain energy of the shear field is obtained as Eq. (18) under the static load P . By using Eq. (18) and Castigliano's Theorem the displacement of the loaded point is obtained as Eq. (19), and then tensile stiffness of the fender rig EA' is obtained.

$$U = \int \frac{F^2}{2EA} dx + \int \int \int \frac{\tau^2}{2G} dxdydz \dots \dots \dots (18)$$

$$\delta = \frac{\partial U}{\partial P}, \quad \frac{EA'}{l'} = \frac{P}{\delta} \dots \dots \dots (19)$$

where

F : axial force

EA : tensile stiffness of tension member

q : τt shear flow

t : thickness of panel

EA' : tensile stiffness of tension rig

δ : elongation in the direction of load

G : shear modulus

l' : length of tension rig

In the case of Fig. 10 (b) it is very convenient to suppose some additional tension members which are not actual members and supposed to divide the shear panel. In this case the shear field becomes statically indeterminate of the second order and Eq. (20) is necessary to determine two shear flows, q_1 and q_2 .

$$\frac{\partial U}{\partial q_1} = 0, \quad \frac{\partial U}{\partial q_2} = 0 \dots \dots \dots (20)$$

2.3.2 About doors

Doors (front doors and rear doors) are supported on soft sponges, called door weather strips, around door flanges. And they are fixed to pillars at three points which are two door hinges and one door lock. So doors are supposed to be tension rigs which connect two pillars (see Fig. 1).

Door panels are simulated as shear fields having additional tension members as Fig. 11 and should be analysed by the same method as used for fenders. Shear panels of doors are composed of both outer panels and inner panels which are

Fig. 11 Tension rig of door

combined to make door rims at the boundaries. The shear fields of doors are statically indeterminate of the first order since additional members are assumed, and then Eq. (20) is required to determine q_1 or q_2 .

2.4 Effect of shear panel on body shake (2)

Another group of shear panels includes those which are surrounded by members of body framework as roof, floor pans, dashboard, fire wall as hatched in Fig. 12. It is not easy to simulate effects of those panels as tension rigs since a complicated network of tension rigs is necessary between two of mass particles. In those cases shear stiffnesses of panels are supposed to be estimated as additional flexural stiffnesses of members surrounding the panels in the plane of the panels. At first deflections of framework without shear panel, and with shear panel, should be estimated analytically, and then additional stiffnesses can be obtained by comparing those deflections. According to the estimations additional stiffnesses should be those of members multiplied by 10 to 10^2 . Using those additional stiffnesses the framework like Fig. 1 is sufficient to simulate body shakes. Therefore, it is not necessary to include panels as they are and then the analysis becomes very simple.

2.5 Body parts which are estimated by only their weights

Hood, luggage compartment door, etc. affect body shakes by only their weights. It is not necessary to simulate them as rigs or additional stiffnesses.

Fig. 12

Fig. 13 Four-door sedan

3. Input data for calculation

As it is the purpose of the analysis to forecast the characteristics of body shakes and to control them in their design stages, all input data for calculations must be picked up from drawings of body structures. In early stage of the design process input data are obtained roughly and in final stage they are obtained accurately. Accuracy of the calculation depends on accuracy of its input data. Input data mean mass of a mass particle, length of a beam, four

stiffnesses of a beam and tensile stiffness of a rig.

3.1 Mass of mass particle

In drawings projected to three planes mass particles should be distributed. All masses should be calculated in the space surrounded by three planes including the lines which equally divide the distance between two adjacent mass particles, and it is concentrated on the mass particle in the middle of the space.

3.2 Length of beam

It means the distance between two adjacent

Fig. 14 Three dimensional model with tension rig (I)

mass particles and it depends on the number of mass particles distributed in body structures. More mass particles should be distributed in complicated parts of body structures.

3.3. Stiffnesses of beam

Stiffnesses of a beam are assumed to be constant along the beam. They should be calculated about the section at the middle of two adjacent mass particles.

It is mentioned above how to obtain tensile stiffness of a tension rig.

4. Comparison between calculations and experiments

In order to prove the analysis three dimensional models like Fig. 1 and actual four-door

Table 1 Three dimensional model

Without tension rig	Calculation Hz	Experiment Hz
Bending with two nodes	23	24
Torsion with one node	31	31
Bending with three nodes	33	34
Bending of upper part	39	40
Torsion with three nodes	51	49
Bending in the plane view	52	
Torsion with three nodes	59	57
With tension rig	Calculation Hz	Experiment Hz
Torsion with one node	33	33
Bending with three nodes	37	39
Torsion with three nodes	57	56
Bending in the plane view	60	59
Bending with four nodes	63	65
Torsion with four nodes	65	
Bending of upper part	66	68

(e) The fifth mode

(f) The sixth mode

(g) The seventh mode

Fig. 14 Three dimensional model with tension rig (II)

sedan bodies of smaller car have been taken for both calculations and experiments. Calculations have been done by an electronic computer using 228 degrees of freedom and 38 mass particles distributed in the system. While experiments have been achieved by such way as models and actual bodies have been supported on four soft beds simulating conditions of free-supports and they have been excited by one electro-magnetic exciter at left ends of models or actual bodies.

4.1 Cases of three dimensional models

One of three dimensional models has no tension rig like Fig. 2 and the other has tension

rigs like Fig. 1. Both of them are of 2 meter length, 50 centimeter height and 71.5kg weight, including tension rigs. Stringers, pillars and cross members are arc-welded, and tension rigs are bolted.

Results of calculations and experiments are compared in Table 1 and Fig. 14 about natural frequencies and vibration modes below 70 Hz. Table 1 shows cases both without rigs and with rigs, and Fig. 14 shows only the case with rigs assuming the displacement as 1 (unit disp.) at the point excited by an exciter. Though natural modes are named conventionally, those names seem to be improper for showing three dimensional

modes and it seems to be better to call them by orders as used in Fig. 14. According to Fig. 14 bending modes give only displacements of side members in the plane of FR-UPR. Meanwhile torsional modes give displacements both in the plane of FR-UPR and in the plane of FR-RH. RH and LH indicate right side member and left side member. CAL and EXP indicate calculation and experiment. Table 1 and Fig. 14 show that good approximations have been obtained between calculations and experiments about three dimensional models. In experiments only one of two natural frequencies, which are very close, has been picked up. In this case only calculated

Fig. 15 Actual built-in body

result is shown like Fig. 14 (f).

4.2 Cases of actual four-door sedan

To calculate body shakes of actual bodies it is the most difficult how to simulate actual bodies as the framework like Fig. 1, and the reason has been explained above. The analysis has been made of actual bodies shown in Table 2 in detail.

4.2.1 Tensile stiffnesses of fenders and doors

Tensile stiffnesses of fenders and doors have been calculated by the theory shown in 2.3 and then their results are compared in Table 3. They give sufficient approximations between calculations and experiments, and therefore tension rigs have been proved to be quite correct.

4.2.2 Additional flexural stiffnesses

In order to check additional flexural stiffnesses actual four-door sedan body has been experimented excluding its doors, hood, luggage compartment door, and fenders which have been cut off from the built-in body. As you can find in Table 4, natural frequencies and vibration modes coincide in spite of complexity of the actual body. Then additional flexural stiffnesses have been proved to be correct and profitable.

4.2.3 Actual built-in body

Built-in body means a body without doors, hood, luggage compartment door and front fender outer panels. Natural frequencies and modes calculated are compared with those experimented in Table 5 and Fig. 15. They coincide well in both natural frequencies and vibration modes.

Table 2 Four-door sedan body

Body length	3 970 mm	
Body width	1 550 mm	
Body height	1 135 mm	
Wheel base	2 420 mm	
Slant angle,	Front pillar	40 deg
	Rear pillar	25 deg
Fender (Length×Height),	Front	1 100 mm×550 mm
	Rear	850 mm×550 mm
Door (Length×Height),	Front	950 mm×550 mm
	Rear	900 mm×550 mm
Body weight	Calculated	Weighed
Built-in body	175 kg	183 kg
Body without door	224	233
Body with door	269	279

Table 3 Tensile stiffnesses of tension rigs, P/δ

	Calculation	Experiment
End of front fender (inner panel only)	$0.34 \times 10^3 \text{ kg/cm}$	$1.0 \times 10^3 \text{ kg/cm}$
End of rear render	0.43×10^4	0.14×10^4
Middle of rear fender	0.22×10^4	0.38×10^4
Front door	0.18×10^4	0.35×10^4

4.2.4 Actual body without door

This case means built-in body with hood, luggage compartment door and front fender outer panels. Natural frequencies are shown about both calculation and experiment in Table 6.

4.2.5 Actual body with door

Body with door means a complete white body including all panel parts of a body shell. Its natural frequencies are shown in Table 7 and its vibrations modes in Fig. 16. Calculated frequencies and modes coincide well with experimented ones, though accuracy tends to decrease according to the number of panel parts added to the body shell. All cases of body structures have been compared and they have shown coincidences good enough to prove that the analysis is correct and useful.

5. Conclusions

(1) Body structures can be replaced by three dimensional frameworks with tension rigs. The frameworks can be assumed as systems of finite degrees of freedom using lumped mass system

Table 4

	Calculation Hz	Experiment Hz
Bending with two nodes	36	33
Torsion with one node	39	
Bending with three nodes	46	43
Bending in the plane view	48	48
Torsion with two nodes	52	57

Table 5 Actual built-in body

	Calculation Hz	Experiment Hz
Torsion with one node	34	34
Bending in the plane view	40	
Bending with two nodes	41	43
Torsion with two nodes	55	60

Table 6 Actual body without door

	Calculation Hz	Experiment Hz
Torsion with one node	28	31
Bending in the plane view	33	
Bending with two nodes	36	36
		43
Torsion with two nodes	48	49
Torsion with three nodes	56	58

Table 7 Actual body with door

	Calculation Hz	Experiment Hz
Torsion with one node	28	29
Bending in the plane view	31	
Bending with two nodes	35	38
		45
Torsion with two nodes	47	50

and they can be analysed by matrix method to give body shakes in lower frequency range. The analysis has been proved to give good simulations about complicated four-door sedan with sufficient accuracy.

(2) Shear panels of body shells affect body shakes very much. Those panels are supposed to be classified in two groups, tension rigs and additional flexural stiffnesses.

(3) In the analysis the framework has been analysed by displacement method and large matrix method. While tension rigs have been analysed by theory of shear field.

(4) All input data for calculations should

be obtained from drawings of body structures with certain accuracy and objectivity. That is necessary to use the analysis for general uses even in early stage of body designs.

(5) In order to check the analysis calculated results and experimented ones have been compared about tensile stiffnesses of rigs, additional flexural stiffnesses of shear panels and natural vibrations of three dimensional models and actual four-door sedan bodies. According to those comparisons the theoretical analysis has been proved to be quite correct and accurate.

(6) The analysis has dealt with only natural

Fig. 16 Actual body with door

vibrations of frameworks freely supported. However, it seems to be quite easy to analyse cases of various supportings and various systems joined with other vibration systems, making a little change in the analysis mentioned above.

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Discussion

S. OHNO: (1) In the analysis the effect of a shear panel surrounded by beam members is represented by the increase in the flexural stiffness of the beam members. The writer would like to ask the procedure to estimate the increase in the flexural stiffness.

(2) The degree of approximation seems to be different in different parts. It may be close approximation that the upper pillar are slanted and the fenders are represented by the tension spring in the idealized structure. On the other hand the offset of the torque box, the kick-up of the frame and the transmission tunnel of the floor pan are neglected in the idealized structure. It is to be wished that the principle of approximation is explained.

S. YAMAKAWA: (3) How do you think about the relation between the number of lumped masses and the number of natural modes to be obtained about actual bodies?

(4) Please explain how to simulate joints of beams, especially joints of roof rails.

(5) Please explain the method to calculate additional flexural stiffnesses of shear panels.

H. KAGEYAMA: (6) Under the section 2.4, the paper says that flexural stiffnesses of members surrounding the panels should be multiplied by 10 to 10^2 . Please explain in detail how to calculate those multipliers accurately and objectively.

(7) What kinds of panels are the most important to obtain 90% accuracy for calculations?

(8) Isn't it necessary to give any corrections or changes in input data of the three dimensional model, for instance, mass, stiffness?

K. KIRIOKA: (9) Please present dimensions and furthermore, input data on the three dimensional model, so that the paper will be utilizable as a check program for vehicle vibrations and vibrations of general frameworks.

(10) The paper deals with body shakes in lower frequency range. What is your opinion on analyses of body vibrations in higher frequency range, including panel vibrations in future?

H. KAWAI: (11) Door lock seems to restrict deflections of pillars only in the direction of door opening and not in the fore-and-aft direction of a body structure. Are there any differences of calculations between (1) simulating doors as rigs and (2) simulating doors as only masses?

(12) How do you think about discrepancy between calculation and experiment of the first mode in Fig. 16 (a)?

(13) Please show experimental torsional mode (bending in the plane view) of Table 4 and its difference from the mode of Fig. 15 (a).

(14) How are front window-glass and rear window-glass dealt with? (According to our experiences there are two cases about torsional vibrations. In one case torsional vibrations are changeable with or without window-glasses and in the other case they are not changeable. Two cases seem to be classified by the difference of stiffnesses of front and rear window frames).

Author's closure

(1) Shear panels surrounded by members are simulated as additional flexural stiffnesses of the members, because in those cases it is quite complicated to use tension rigs between two of many mass particles distributed boundaries of the panels. Additional stiffnesses are estimated by multiplying stiffnesses of members by certain coefficients. To obtain the coefficients about two dimensional framework and shear panel shown in Append-Fig. 1, which are loaded by static force F , deflection of two dimensional framework δ_1 should be obtained by the equation as follows

$$\begin{pmatrix}
 0 & 0 & -2a_2/B & c_1+2a_2/B & a_2 & a_2 & 0 & 0 \\
 -c_2 & c_2+2a_1/H & 0 & 0 & 0 & -a_1 & 0 & 0 \\
 c_2+2a_3/H & -c_2 & 0 & 0 & -a_3 & 0 & 0 & 0 \\
 0 & 0 & c_3 & c_1 & 0 & 0 & 0 & 0 \\
 0 & a_1 & a_2 & -a_2 & -b_1 & -2(b_1+b_2) & 0 & 0 \\
 0 & a_1 & 0 & 0 & 0 & -b_1 & 0 & 1 \\
 a_3 & 0 & 0 & 0 & -b_3 & 0 & 1 & 0 \\
 a_3 & 0 & a_2 & -a_2 & -2(b_2+b_3) & -b_2 & 0 & 0
 \end{pmatrix}
 \begin{pmatrix}
 \delta_1 \\
 \delta_2 \\
 \varepsilon_1 \\
 \varepsilon_2 \\
 \theta_1 \\
 \theta_2 \\
 M_1 \\
 M_2
 \end{pmatrix}
 =
 \begin{pmatrix}
 0 \\
 0 \\
 F \\
 0 \\
 0 \\
 0 \\
 0 \\
 0
 \end{pmatrix}$$

where

$$a_i = \frac{6EI_i}{l_i^2} \quad b_i = \frac{2EI_i}{l_i} \quad c_i = \frac{EA_i}{l_i}$$

Meanwhile it is well-known to calculate deflection of the shear panel as follows.

$$\delta_p = \frac{1}{G} \frac{H}{tB} F \quad t: \text{thickness of panel}$$

Then coefficient C should be decided to obtain $\delta_1 = \delta_p$. This coefficient should be used to multiply only flexural stiffnesses of members EI , because EI is far effective from tensile stiffnesses EA for bending of the framework. So flexural stiffnesses of members become $(1+C)EI_i \doteq C \cdot EI_i$.

(2) As you point out, the framework of the analysis is a little different from the actual body. The reason is that the framework should be simplified as much as possible if the analysis can give 90% accuracy. Therefore, these parts which are important to obtain 90% accuracy should be simulated accurately and even analysed theoretically. Meanwhile these parts which are not so important should be dealt with coarse accuracy. In detail about those you pointed out:

Append.-Fig. 1

(a) Slant upper pillar must be considered to simulate body shakes accurately.

(b) Tension rigs of fenders should be considered to simulate bending and torsional shakes, especially bending shakes. As they are so important that the analysis cannot be achieved without them, they have been analysed theoretically.

(c) Offsets between sidesill and front frame or rear frame may be neglected to deal with shakes simply. However, torque boxes were analysed in another analysis, and then the author found out that torsional stiffnesses of torque boxes could be considered as flexural stiffnesses of sidesills. Also the author analysed kick-ups and found out that they could be neglected to obtain 90% accuracy, though they were important for bending shakes.

(d) The author considers according to his various experiments that flexural stiffnesses of transmission tunnel is not so effective in dynamics and they can be neglected in the analysis. In the analysis stiffnesses of these parts which are not connected to the main framework should be neglected in order to analyse simply.

(3) A system of 38 masses give more than 4 or 6 natural vibrations. However, in actual body vibrations it is very difficult to obtain shakes above 60 Hz experimentally and in that high frequency range panel vibrations are more important than shakes. That is the reason why in the analysis only 4 or 6 modes are picked up. Of course in order to obtain fine results of only 4 or 6 modes, less than 38 masses are sufficient, if those masses should be distributed carefully.

(4) In the analysis all joints of beams are considered as connected rigidly. Even for cases in which stiffnesses of beams are not continuous,

Apped.-Fig. 2

joints should be considered rigid and not rigid near the joints instead.

(5) See the author's closure (1).

(6) See the author's closure (1).

(7) For bending shakes tension rigs of fenders and doors are the most important, especially the rig connecting end mass of front fender to front pillar. In detail the frequency of bending mode with two nodes is calculated without the rig 30% less than experimental one and the frequency of torsional mode with two nodes is calculated 30% less. Without door rigs the frequency of torsional mode is calculated 6% less.

Append.-Table 1 Stiffnesses of three dimensional model

Beam	l_i cm	EI_i $\times 10^8$ kgcm ²	EH_i $\times 10^8$ kgcm ²	GJ_i $\times 10^7$ kgcm ²	EA_i $\times 10^8$ kg
25	0.0480	0.442	1.12	0.215
25	0.0480	0.442	1.12	0.215
25	0.0384	0.354	0.893	0.172
25	0.0384	0.354	0.893	0.172
25	0.0109	0.273	0.246	0.132
29	0.0134	0.335	0.300	0.161
29	0.0088	0.218	0.196	0.105
32.6	0.0088	0.218	0.196	0.105
21	0.0088	0.218	0.196	0.105
35	0.0384	0.354	0.893	0.172
35	0.218	0.0088	0.196	0.105
35	0.0088	0.218	0.196	0.105
35	0.218	0.0088	0.196	0.105

Append.-Fig. 3

For torsional shakes additional flexural stiffnesses of panels are the most important. Without those stiffnesses frequencies of torsional modes are calculated 30~45% less than experimental ones. Without additional stiffnesses of dash board and fire wall calculated frequencies are 5% less than experimental ones.

(8) Calculated masses of the model coincided with actually weighed ones. However, stiffnesses of several beams were corrected and they are a little different from calculated ones, because their actual lengths are different from distances between two adjacent masses as some beams are folded and arc-welded. See the author's closure (9) in detail.

(9) The author shows the three dimensional model in Append.-Fig. 3 and its input data in Append.-Table 1. Beams are folded and arc-welded. Tension rigs of dia. 10 mm are fixed by bolts. Weight of framework is 71.5 kg including the weight of rigs 1.5 kg.

(10) In higher frequency range (above 60 Hz) panel vibrations are more important than shakes in actual body vibrations, and shakes give only boundary conditions of panel vibrations. It seems to be convenient and reasonable that analyses of panel vibrations should be achieved for individual problems as local vibrations at first, and then elastic vibrations of whole body structure should be analysed.

(11) Differences of frequencies between the case of door as tension rig and the case of door as mass are about 6% as shown below. It

Append.-Fig. 4

cannot be explained except door rigs that frequencies of bending modes with doors are always higher than those without doors. Frictions between strikers and locks of doors seem to be more than stiffnesses of door panels.

	Door as rig	Door as mass
Torsion with one node	28 Hz	28 Hz
Bending in the plane view	31	31
Bending with two nodes	35	33
Torsion with two nodes	47	46

(12) The author considers that discrepancy between calculation and experiment in Fig. 16 (a) depends on coarse accuracy of experiment. He achieved many cases of experiments which

included similar cases and checked the accuracy of each experiment. About Fig. 16 (d) the author considers that the true frequency comes between 45 Hz and 50 Hz, if panel vibrations don't disturb the shake mode.

(13) See Append.-Fig. 4 which is the fourth mode of Table 4. You will find clearly that it is different from Fig. 15 (a)

(14) As you point out, body shakes with window-glasses are probably different from those without window-glasses. In the analysis window-glasses are not included. The author recommends that window-glasses should be simulated as they are mounted with springs to window frames.