Vibration of a Cylinder Caused by Wake Force*

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In this report experimental results on the vibration of a cylinder caused by sheeding vortices are presented. The experiments were undertaken at larger Reynolds number than in authors' previous report.

There are two experiments in this study, one being on an elastically supported cylinder and the other on a crank-excited cylinder. In the former, the exciting force coefficient is determined from a decrement and an amplitude of the cylinder. In the latter, the exciting force coefficient is found by an integration of pressures on the cylinder surface. These two results coincide with each other; the exciting force suddenly decreases at nearly the critical Reynolds number.

Moreover, at sub-critical Reynolds number a self-exciting force is induced just as in authors' previous report, but at super-critical Reynolds number the exciting force decreases with an increasing cylinder amplitude and at too large an amplitude the force is changed to a damping force.

1. Introduction

There are many reports on the vibration of a cylinder, elastically supported in the uniform flow, caused by cyclic shedding of vortices. In view of a fatigue failure of a tall stack and the fall of a Satan rocket on a launcher caused by wind excitation, more detailed investigations have come to be conducted. However, the exciting force and its mechanism are not yet clearly understood.

At present there are two theories, one of which states that it is a vibration like the resonance occurring at the flow velocity at which the vortex shedding frequency is nearly equal to the natural frequency of the cylinder. The other claims that it is random vibration occurring at a higher velocity.

Recently, a few studies from a point of view of the self-excitation are reported, they are based on the first theory and the authors mentioned also in the experimental study at a low Reynolds number in the previous report.

However, in the range of Reynolds numbers larger than the critical one the flow pattern differs, and it is not yet clear whether the excitation mechanism is a self-induced type just as the above type.

G.W. Jones conducted wind tunnel tests at very high Reynolds numbers, above the critical one, and it is noteworthy that he points out that the exciting force increases with cylinder amplitude just as in the low range below the critical one. Figure 1 shows the experimental range for the Reynolds number and the nondimensional frequency for various studies, namely of G.W. Jones and others in comparison with this report. It is regrettable that the Reynolds number of this report is one tenth that of G.W. Jones and near the critical one, so that the experimental results differ from those of Jones.

With the development of gas-cooled reactors the pressure of gas increased to more than 50
kg/cm² and conversely the nondimensional decrement 2mβ/βd² (where m: density of tube per unit length, β: logarithmic decrement of tubes, β: density of gas, d: tube diameter) decreased, and therefore it seems to be important to develop design methods for the prevention of tube-vibration.

The purpose of this study is to make clear the basic phenomenon of vibration of tube banks in heat exchangers.

2. Wind tunnel tests on elastically supported cylinders

2-1 The purpose of the experiment

Various up-to-date studies including model basin tests, cover low Reynolds numbers below the critical one. At high Reynolds numbers there are yet many unknown phenomena. Therefore, at first a wind tunnel test on a simple, elastically supported cylinder has been undertaken. Its objects are to reveal the outline of the phenomenon, to know the exciting force which is calculated from the amplitude and the decrement of the cylinder, and to explain the effect of the nondimensional frequency and the ratio of amplitude to cylinder diameter on the exciting force at high Reynolds numbers.

2-2 Experimental apparatus and method

In Fig. 2 the experimental apparatus is shown schematically. The model cylinder is 300 mm in diameter, 1000 mm long and it is made of aluminium sheet with a thickness 0.5 mm. It is supported at both ends by two cantilever beams to permit vibration in normal direction relative to the flow. The tunnel wall effect is negligible because the width of the tunnel is 3000 mm, against cylinder diameter 300 mm. There are three kinds of thickness of supporting beam, and their natural frequencies are 4.7, 9.3 and 15.2 c/sec respectively. Viscous type dampers are fixed at the ends of the beams near the cylinder, and a damper plate is inserted in the narrow space of the damper box which contains a mixture of glycerin and water. The linearity of decrement of the damper is sufficient. On the beam near its fixed end a wire strain gauge is affixed which serves to transmit the cylinder vibration to an oscillograph for recording.

Fig. 3 Experimental result of elastically supported cylinder

Fig. 4 Experimental result of elastically supported cylinder

Fig. 5 Experimental result of elastically supported cylinder
In this experiment the oscillograph recording and the measuring of the flow velocity are performed at various flow speeds at which cylinder vibrations occur and with various combinations of the three kinds of supporting beams.

2-3 Experimental results

Experimental results are shown in Fig. 3 to 5. The cylinder amplitude plotted in these figures is a maximum value recorded in the oscillograph during about ten seconds, and it is shown as the ratio of the amplitude (a) to the cylinder diameter (d) against the nondimensional velocity \( V/f_d \) (where \( V \); flow velocity, \( f_d \); natural frequency of cylinder). Parameters are logarithmic decrement, measured by the oscillograph, due to the free vibration of the cylinder caused by a shock every time the damping liquid is replaced. The frequency of the cylinder vibration is nearly constant with its natural frequency. In each figure, the flow velocity and its Reynolds number at nondimensional velocity 6 are shown. Accordingly, Fig. 3 shows values lower than, Fig. 4 values near, and Fig. 5 values higher than the critical Reynolds number.

Now, in Fig. 3 the cylinder vibration begins at the nondimensional velocity 5, and at nearly 6, the peak of amplitude exists just as in the previous report\(^{(3)}\) since the flow velocity is 8.46 m/sec and Reynolds number is \( 1.70 \times 10^5 \) i.e. below the critical one. But the peak value decreases with increase of the decrement, and there is no peak at an extremely large decrement. Figure 4 shows the same tendency, but the maximum amplitude is one fifth that in Fig. 3.

Furthermore in Fig. 5, the tested velocity range is up to 6 in \( V/f_d \), and there is no peak. It is not evident whether or not the peak at \( V/f_d = 4 \) is the resonance peak. But it may be reasonable to say that the Strouhal number is nearly 0.3 at the super critical range for the narrow wake. It is, however, important that the amplitude is very small in comparison with Figs. 3 and 4.

Figure 6 shows the relationship between the exciting force coefficient and the ratio of amplitude to diameter. The exciting force coefficient is introduced by the calculation that the damping energy that is determined by the amplitude and decrement is equal to the exciting energy at resonance condition which is assumed to be a forced vibration. In the case of \( f_s = 4.7 \text{c/sec} \) in Fig. 6 self-induced vibration may be suspected to be the cause just as in previous report\(^{(3)}\) that is, at low Reynolds numbers. But in the case of \( f_s = 9.3 \text{c/sec} \) the exciting force coefficient is smaller than 0.1, and it is difficult to believe that the cause of vibration is self excitation only. Furthermore, in the case of \( f_s = 15.2 \text{c/sec} \) there is no resonance but the value for \( V/f_d = 5 \) is assumed to be a typical value. The exciting force coefficient is very small in this case. These vibrations should be treated as random vibrations.

3. Wind tunnel test on the vibrating cylinder excited by a crank

3-1 The purpose of the experiment

According to the result on the elastically supported cylinder stated in previous paragraphs it is not possible to explain the exciting mechanism because the amplitude is too small. Therefore, the purpose of this new test is to clarify the exciting mechanism by measurement of the pressure fluctuation on the surface of the oscillating cylinder.

3-2 Experimental apparatus and method

It is a known weak point of the test at the high frequencies that the long connecting rod incurs violent vibrations and buckling just as in previous report\(^{(3)}\) and Fung's\(^{(5)}\). But in Jone's tests excitation at high frequency became possible since he used individual oil hydraulic shakers at both ends of the cylinder. In this experiment, as shown in Fig. 7, for high frequency tests, the cylinder is supported by a stiff spring and on its opposite side it is pulled sinusoidally by a slender rod which is connected to a crank. By this method the cylinder vibration is correct and reliable because the tension of the rod decreases with an increasing frequency of the cylinder in the range below its natural frequency.

The cylinder is 300 mm in diameter, 1000 mm long and it is made of aluminium sheet of 1 mm thickness. It has crossheads inside both ends and can vibrate in direction normal to the flow along the duct wall. The slender rod is 10 mm in diameter, and it is held by stoppers to avoid a lateral vibration. The crank shaft has two large flywheels at both ends to permit the cylinder to vibrate sinusoidally, and there is a
stepless transmission between the shaft and the driving motor.

Electrocapacitance type pressure gauges are attached at a symmetrical point at the midspan inside the cylinder, and the pressure is recorded on the oscillograph. The amplitudes of the cylinder are 7.5 mm, 15 mm and 30 mm; the flow velocities are 7, 14 and 21 m/sec; and the frequency is varied gradually from 3 to 9 of the nondimensional velocity. Set angles of pressure gauges are 0°, 30°, 60° and 90° from the stagnation point down-stream at the upper side and −90°, −120°, −150°, −180° at the lower side, as shown in Figs. 9 and 10. In addition, the mark of the top dead center is recorded simultaneously, on the oscillograph by electro-magnetic pickup.

3.3 Experimental results

As shown in Fig. 8, for example, the pressure recorded on the oscillogram is distorted, and it includes a noise of high frequency. Only the fundamental component of the pressure is in relation to the cylinder vibration. Furthermore, the fundamental component has to be divided into the exciting or damping component in phase with the vibrating velocity and the inertia or restoring component in phase with the vibrating deviation. These components are obtained by harmonic analyser. They are shown in Figs. 9 and 10 in the form of nondimensional pressure coefficients $C_p$ which define the ratio of the alternating pressure to the velocity head.

In these figures the cylinder amplitude is 30 mm and 15 mm respectively, and the upper side is a component in phase with the cylinder velocity or sine component, and the lower one is in phase with the deviation or cosine component. In each figure the left hand side refers to the flow velocity of 21 m/sec and the right hand side to that of 14 m/sec. (a), (b), (c) and (d) correspond to the pressure measuring positions of 90°, 60°, 30° and 0° from the stagnation point in downstream direction respectively, as shown by these figures where the cross marks refer to the upper side and the point marks to −90°, −120°, −150° and −180° (stagnation point in upstream direction) respectively.

Now, in Fig. 9 (a) for example pressure coefficients of sine component are large in the range from 5 to 6 of nondimensional velocity $V/\nu d$, in which cross marks are positive and point marks are negative. From these results it is concluded that the cylinder receives the hydraulic force in phase with its vibration velocity, in other words, the cylinder receives the vibration energy from the fluid. Furthermore, at higher frequency or at small velocity cosine components are positive in point mark and negative in cross mark, from which we conclude that the hydraulic force is an inertia effect because the upward force is applied to the cylinder when the latter is in the top dead centre. In addition, it is not possible to explain why values of $V/\nu d$ at maximum pressure amplitude at lateral positions on the cylinder are not identical, in spite of the fact that these are measured at symmetrical position.

Fig. 7 Experimental apparatus of crank excited cylinder

![Fig. 7 Experimental apparatus of crank excited cylinder](image)

Fig. 8 Oscillogram of pressure, oscillating cylinder

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Fig. 9 Pressure amplitude at cylinder surface (cylinder amplitude; 30 mm, phase base; top dead centre)

Fig. 10 Pressure amplitude at cylinder surface (cylinder amplitude; 15 mm, phase base; top dead centre)
In (b) of Fig. 9, that is at 60°, the point marks of sine components are positive in comparison with (a), which causes a damping force to rapidly increase at low V/fad. But the cross marks are still positive. The cosine component acts as inertia force just as (a). In (c) point marks are positive, and so this component acts as damping force, but the lateral components are small. In (d) which is at the stagnation point both components are very small as can be seen easily.

Figure 10 shows the result of tests with a cylinder amplitude of 15 mm, and in (a) of this figure the value of V/fad of maximum amplitude decreases in comparison with Fig. 9 (a). The maximum value of 14 m/sec is considerably greater than that of 21 m/sec. It is impossible to explain why the values of V/fad at the maximum pressure amplitude at lateral positions are not identical. In other cases, i.e., (b), (c) and (d) of Fig. 10 there are the same tendencies as shown generally in Fig. 9.

Moreover, Fig. 11 shows an example of the pressure oscillogram at 90° recorded while the cylinder was at rest. As shown, the pressure amplitude is very small and irregular, in comparison with Fig. 8. This is an example of smallest irregularity, and the Strouhal number derived from this oscillogram is 0.286.

3-4 Exciting force calculated by integration of pressure

The exciting force coefficient Ce is calculated by the following equation from pressures on each position of the cylinder surface:

\[ Ce = \frac{\pi}{12} (C_{peo} - C_{p-60}) + \frac{\pi}{6} (C_{p60} \sin 30° + C_{p60} \sin 60° - C_{p-120} \sin 120° - C_{p-150} \sin 150°) \]

As previously stated, pressures are measured on a half side only except for lateral positions, in which case it is assumed that the pressures at the symmetrical positions are the same. The distribution of the pressure along the surface within ±15° from a measuring position is assumed to be uniform. The integration of the lateral component of these pressures is shown by the form of the exciting force coefficient Ce with V/fad. The restoring force coefficient Cr is defined in the same manner as the cosine components.

Figure 12 shows the calculated result for the data in the previous paragraph, using the above method. In this figure (a) and (b) show the exciting force coefficient Ce on the upper side and the restoring force coefficient Cr on the lower side at the cylinder amplitude of 30 mm; (c) and (d) are taken at the cylinder amplitude of 15 mm.

Comparing these exciting components, in (a) in which the cylinder amplitude is 30 mm and the velocity is 21 m/sec, Ce is negative, i.e.
damping force in all range of \( V/f \). However in (c) for amplitude of 15 mm and velocity of 21 m/sec, the exciting force is presented at \( V/f \) 5.5, but the maximum \( C \) is very small compared with that in the range of low Reynolds numbers. This coincides with the fact that there is a small exciting force at a small amplitude of the cylinder, as shown in Fig. 6. This is a quite difference from that at low Reynolds numbers.

(b) and (d) show the reasonable results for the intermediate state, between the lower and super critical Reynolds number. Namely, the maximum value of \( C \) is 0.05 at \( V/f \) of 5 to 6 in (b) and 0.17 at \( V/f \) of 4.5 to 7 in (d), in spite of the cylinder amplitude decreasing to half. This is nearly equal to the \( C \) for a frequency of 9.3 c/sec, as shown in Fig. 6, and it corresponds to the fact that the vibration is not very large. In addition, the restoring force coefficient \( C \) is a negative large value at low \( V/f \) and this acts as the inertia force. In other studies on the model basin previously reported\(^{12} \) it is conjectured that it is due to this inertia effect that the frequency of the cylinder is a little lower than its natural frequency.

4. Conclusions

In this study the exciting force coefficient and the exciting mechanism on the vibration of the cylinder caused by the wake force are investigated by wind tunnel test, with an elastically supported and a crank-excited cylinder. Conclusions of these studies are as follows:

1. The exciting force suddenly becomes small as the critical Reynolds number is approached.

2. At low Reynolds numbers, lower than the critical one, the exciting force is produced over a considerably wide range of the cylinder amplitude, and there is a maximum value in the middle of the range, but the maximum value decreases near the critical Reynolds number, and in this condition the cylinder amplitude is not very large. It is further believed that at too large an amplitude the hydraulic force acts as a damping force.

3. In the experiment with the crank excited cylinder the exciting force, that is in phase with the vibration velocity of the cylinder, is produced at \( V/f \) of 5 to 6, and the force appears at lateral positions of the cylinder surface, but at other positions forces are damping.

4. The exciting force coefficient calculated by integration of the lateral component of the pressure is nearly equal to that of the elastically supported cylinder.

In spite of the complexity and irregularity in the test result, conclusions as above may be drawn. Especially, it is concluded that the exciting force is very small near the critical Reynolds number. It is desirable that further investigations be performed as there is only one study of super critical Reynolds number, the one by G.W. Jones, in which it is shown that there is a large excitation in the sub-critical range as well, probably due to the regular shedding of vortices.

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References


Discussion

H. Ōta (Nagoya University):

1. Does the frequency of the cylinder vibration, that is caused at the resonant condition, coincide with the natural frequency of the cylinder? Because the coincidence is basis to a judgement whether the vibration is a self-induced vibration, such as the one described in this report, or a forced vibration caused by a cyclic shedding of vortices. It would be useful to show the measured frequency in Fig. 3 to 5, for \( V/f \) of 4 to 8.

2. It is not possible to consider that the exciting force \( P \) in Fig. 5 is very small because \( P \) in Fig. 5 is about ten times as large as that in Fig. 3, as is shown in the following equation

\[
P = a k \delta / \pi = 4 \pi a k \delta / M
\]
where $K$: the spring constant, $M$: equivalent mass of the cylinder.

The conclusion (1) in this report in which it is stated that the exciting force decreases sharply at the velocity close to the critical Reynolds number seems to derived as the result of the exciting force being expressed with $a/d$.

(3) In many previous reports it is stated that the Karman vortex is shed alternately without relation to the cylinder vibration. In this study it is assumed that the cylinder vibration is caused by the exciting force which is produced by the vortex shedding, and that the cylinder vibration in turn induces the vortex shedding.

J. INOUYE (Kyushu Institute of Technology):

(4) The authors state in paragraph 2.3 “These vibrations should be treated as random vibration.” Is it right to say that the self exciting effect is negligible because the exciting force at $V/f_d=5$ in Fig. 5 is random and small as well, as is shown in Fig. 11, in which the cylinder is stationary?

(5) What is the reason for selecting the maximum value in the oscillogram recorded during ten seconds?

(6) Show the typical pressure distribution on the cylinder, surface, neglecting higher harmonics, when the cylinder passes the neutral position of vibration.

Authors’ closure

(1) In this experiment a distinct difference of the frequency near the resonance condition is not detected from the oscillogram. An exception is the model basin test, in which the inertia component is not negligible for the large hydraulics force, and then the frequency becomes large with increase of the flow velocity. But in a wind tunnel test such as this one, the frequency is kept constant over a wide range close to the resonance condition.

(2) The exciting force calculated from test results, considering the logarithmic decrement with the equation pointed out in question (2), is concluded as shown in Fig. 6. Especially, it is believed that the exciting force is changed to a damping force in Fig. 5 at super critical Reynolds numbers, referring to Fig. 12.

(3) In this report and in the previous one a self excited vibration is suggested, in which the cylinder vibration induces the vortex shedding, but there are many reports in which it is stated that there is an exciting force on the stationary cylinder. It is not possible to explain this with the self excitation theory for a stationary cylinder only. Thus it is believed

a stationary cylinder only. Thus it is believed that the theory mentioned in question (3) is more correct. The authors would like to mention a complementary test conducted to solve these problems. The experimental apparatus is constructed of three cylinders in line, with the central cylinder supported rigidly at both ends by sensitive load cells which are inserted in both end cylinders, as shown in Append.-Fig. 1. The exciting force is measured in a wind tunnel test. In Append.-Fig. 2 experimental results are shown for duct lengths of 100 mm and 200 mm. In the former, there is no exciting force, and in the latter a considerably large force is induced. In experiments on the clearance between cylinders of 0.2, 2 and 5 mm, the exciting force is large at small clearance.

From these results it is concluded that, the end effect of the model cylinder, and especially the effect of the duct length is remarkable, and it is conjectured that there was such a large end effect in the experiment of this report and in two dimensional experiments that the theory as mentioned in question (3) may be reasonable.

(4) In the condition of Fig. 5 the flow velocity is 27.3 m/sec and the Reynolds number of $5.5 \times 10^5$ is super critical. In this condition the steady Magnus force is very small and/or
negative, as shown is Swanson's report\(^{(3)}\) that was quoted in the previous report\(^{(3)}\). When the cylinder vibrates it is assumed that the exciting force suddenly decreases and that then the effect of the self excitation is negligible.

(5) Of course, the maximum amplitude for unlimited duration cannot be recorded during only ten seconds; but it is admitted that the near-maximum value can be recorded. The reason for adapting the maximum value is for the sake of practicality.

(6) Pressures on the cylinder surface recorded on an oscillograph are as shown in Fig. 8. These pressures are divided into a component in phase with the velocity, and a component in phase with the deviation of the cylinder; and then these components are shown in Figs. 9 and 10 for the sine component and the cosine component.

The pressure distribution when the cylinder passes the neutral position of the vibration corresponds to the component in phase with the velocity of the cylinder vibration.