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A New Mechanical Random Fatigue Testing Machine and Some Test Results*

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A new mechanical random fatigue testing machine was devised and manufactured which could give the random load to the specimen by using various sizes of steel balls mixed at an arbitrary ratio. The testing machine proved to be practical from a statistical point of view.

Some kinds of random fatigue tests were carried out with this testing machine, and each random load-time history was analyzed by three different load count methods, namely the peak count method, the range count method and the full wave count method. Fatigue lives were estimated from these analytical results by applying Miner's law, and they were compared with the actual lives obtained empirically in order to discuss the propriety of the load count method for service loads.

The evaluated lives by the full wave count method coincided to a considerable extent with the actual fatigue lives irrespective of the extent of the irregularity of the random load, which means that the full wave count method is most desirable for the random load counting.

1. Introduction

It is indispensable for rational designs to know the fatigue strength or life under the service load. The service load couldn't be, however, applied identically in a laboratory because of its peculiar irregularity in occurrence. The hypothesis must therefore be proposed that the random load, namely the service load, with the same statistical characteristics has the same fatigue strength. In investigating the effect of the random load on the fatigue properties, not only the load amplitude but also the mean load has to be taken account of. Furthermore, the auto-correlation function or the power spectral density has to be considered for the frequency analysis of a random load-time history.

Meanwhile as for the fatigue testing machines which can apply random loads, a few of those have been already designed with the aid of modern advanced electric or electro-servo techniques⁽¹⁾. They are, however, so expensive that they can't be installed easily. Moreover they

tend to have many drawbacks because they usually consist of many complicated electronic circuits. For these reasons the authors have devised a new mechanical random fatigue testing machine which applies random loads to the specimen by continuous collision of steel balls mixed at an arbitrary ratio.

In this paper was discussed first the practicality of the testing machine from various points of view and second, was made comparison between the estimated fatigue life by means of a digital procedure of the full wave count method⁽²⁾ and the actual fatigue life under random loading with the machine.

2. Description of the machine newly devised and constructed

Figure 1 indicates the schema of the principle of the random fatigue testing machine newly devised. One end of the specimen ① is fixed to a chuck ② and the other is coupled to a pair of levers ⑤ with a steel ball receiver ③ and a buffer ④. Various sizes of steel balls ⑦ mixed at an arbitrary ratio in a receptacle ⑥ are shared separately through conduits ⑧ and then made to fall to collide with receivers ③. Consequently, bending loads are given to a specimen through

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levers ⑤. Considering that the amount of bending load depends on the size of steel ball and that the collision sequence may be random, different random bending loads can be generated by selecting appropriately the sizes of steel balls, their mixing ratio, the fall of collision and so on. After colliding, steel balls are brought together into a container ⑨ and then are transferred again to the receptacle ⑥ by conveyers ⑩. A specimen will fail by continuous random loading of this kind. The mean level of a random load can be changed arbitrarily by proper selection of the ratio of the length of a pair of levers. The mechanical procedure of random loading of this sort has the merits both of low cost and of easy handling in comparison with the electric one. In this type of loading both bending moment and compressive load will be applied simultaneously to the specimen, but the latter may be negligible because of long levers ⑤.

Figure 2 gives the outline of an actually manufactured random fatigue testing machine. This corresponds to the case of removing one of the conduits shown in Fig. 1 which divide steel balls bilaterally. The lever is made of aluminum alloy to get high natural frequency

Fig. 1 Schematic representation of the principle of the testing machine

Fig. 2 The outline of the testing machine manufactured for trial

and is guided with vertical plates for the prevention of horizontal deflection. Moreover it is suspended with soft springs in order to exclude the effect of its self-weight. The collision distance of a steel ball can be changed by moving an adjustable block with a handle. The time to failure is measured with a timer connected to a limit-switch (or a stopper) shown in Fig. 2. The machine is halted by the stopper when the specimen breaks. The applied load is measured and calculated with a load-cell of wire strain gauges stuck on the surface of the adjustable block.

3. Material tested, specimen and experimental procedure

The material tested was a kind of carbon steell plates of the thickness of 3.3 mm. Its chemical composition and mechanical properties are indicated in Tables 1 and 2 respectively. The material was machined into a specimen shown in Fig. 3. Specimens were tested either without heat treatment or with half an hour's vacuum annealing.

The experimental procedure of the random fatigue test on this machine is as follows: a random load generated mechanically is applied to a specimen and is detected with a load-cell through a dynamic strain meter. The out-put of the meter is then recorded with a pen-oscillograph or with a data-recorder. In the latter

Table 1 Chemical composition of the material used (%)

C	Mn	P	S	Cu	Cr	Al
0.03	0.28	0.008	0.008	0.03	0.03	0.001

Table 2 Static properties of the material used

Heat treatment	Upper yield point σ_{so} kg/mm ²	Lower yield point σ_{su} kg/mm ²	Ultimate tensile strength σ_B kg/mm ²	Elongation ϕ %
Without	33.5	31.6	37.0	38
Vacuum annealing at 900°C for half an hour	21.6	20.1	31.3	52

Fig. 3 Fatigue specimen (3 mm in thickness, $K_T=1.3$)

case it is digitalized for the calculation of the auto-correlation function or the power spectral density by a digital computer.

Random fatigue tests of specimens without heat treatment were carried out at three kinds of the mixing ratios, A, B and C. The sizes of steel balls and their mixing ratio tested are presented in each figure of the experimental

Fig. 4 Examples of recorded random load-time histories

Fig. 5 An example of auto-correlation function of a random load wave (Random test A)

Fig. 6 Power spectral density distributions of random load waves

results. Each random load was compared and discussed from a statistical point of view such as the load frequency distribution, the auto-correlation function and the power spectral density.

Annealed specimens were tested under three kinds of random loads, D, E and F. The mixing ratio of steel balls for the random load D was chosen to be 3 (for 25 mm ϕ): 7 (for 20 mm ϕ). That for E-7 : 3 and that for F-1 : 0. The method of fatigue life estimation under random loads was discussed by comparing each fatigue life obtained in such random load tests with those evaluated from the load count results by the full wave count method, the peak count method and the range count method, respectively.

4. Results and discussion

4.1 The random fatigue testing machine: manufactured for trial

In Fig. 4 are indicated some oscillograms recorded in the random tests at the chart speed of 30 mm/sec. The testing machine is found to have enough random characteristics from the figure. In order to get information on the randomness, the auto-correlation function and the power spectral density were then considered. The former, $R(\tau)$, and the latter, $S(f)$, in the random vibration $x(t)$ are generally defined as follows in the stationary ergodic random process⁽³⁾.

$$R(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) \cdot x(t+\tau) dt$$

$$S(f) = \int_{-\infty}^{\infty} 2R(\tau) e^{-i2\pi f\tau} d\tau$$

$$= 4 \int_0^{\infty} R(\tau) \cos 2\pi f\tau d\tau$$

$R(\tau)$ and $S(f)$ will be defined in the infinite time range, $t(-\infty, \infty)$, but they were dealt with in

Fig. 7 Load frequency distributions by the all peaks count method at different measuring time (Random test B)

practice approximately in the finite range of time. After digitalizing the random vibration $x(t)$ by an A-D converter, $R(\tau)$ was calculated at first with a digital computer and then $S(f)$ was done by Fourier's transformation of $2R(\tau)$. The details of treatment were as follows; sampling speed=114/sec, sampling time=about 20 seconds, total number of data=2 000, and total delay=1.75 seconds.

The auto-correlation function in the random test A is shown in Fig. 5. As clearly observed in the figure, $R(\tau)$ becomes small in the early stage and changes periodically after that, which means that the vibration consists both of periodic and of random one. The auto-correlation functions in the random tests B and C had almost a similar tendency, so that the figures are omitted here. Both periodic and random components were divided from the results respectively. The results showed that each periodic one was below almost 10~15% of a whole and that each random one reached above at least 85% of a whole, which means that the random load developed by the machine is random enough. The shape of periodically varying $R(\tau)$ may be regarded as sinusoidal, which will be thought to be due to the vibration of the system containing both the specimen and the lever.

The power spectral density distributions are next shown in Fig. 6. A peak is observed in each curve and this represents the existence of a dominant frequency in the random vibration. The dominant frequency changes a little according to the sizes of steel balls used and their mixing ratio, which will not disturb the utility of the testing machine because there is little effect of loading speed on the fatigue strength⁽⁴⁾.

Fig. 8 Comparison of load frequency distributions at different mixing ratios

Then the stability of the operation of the testing machine is discussed from the standpoint of load frequency distribution. Figure 7 gives the results of load counting in the random test B by applying the all peaks count method to some records of the random wave with a penoscillo. Little difference is observed between the load frequency distributions in each measuring time, and this means a stable operation of the machine. The number of all peaks in the figure represents the total number of both positive and negative peaks in each measuring time, and its mean value is about 1454. There has been a few errors in the counting results because of manual counting, but they could be called negligibly small. In the random test B, visible cracks appeared near the critical section of the specimen after 565 minutes from the beginning of the test and they kept on propagating rapidly until a failure occurred after 630 minutes. As the load frequency distributions after six and eight hours were almost changeless, they are omitted in the figure. Although a little change was recognized after crack initiation, it could be thought allowable because of the loading of constant moment type and of the crack initiation near failure that the load history before crack initiation was regarded as representative. Such representative frequency distributions in the random tests, A and C, are shown in Fig. 8. In the random test C which was carried out on considerably large sizes of steel balls and was discontinued

(a) Detection and classification of a full wave to be taken out

(b) Rearrangement of peaks after the taken-out full wave without changing their orders

Fig. 9 Fundamental idea of digital procedure of the full wave count method

midway because of large plastic deformation of the specimen before failure, the load frequency distribution was observed to have a little difference from others. As shown in the figure, the load frequency distribution is found to depend on the sizes of steel balls used and their mixing ratio.

The facts mentioned above have revealed the practicality of the mechanical type random fatigue testing machine.

4.2 Random fatigue tests and evaluation of lives by the full wave count method

4.2.1 On the digital procedure of the full wave count method

The load count method is of great importance for the evaluation of lives under random loads. The authors had already reported on the full wave count method which analyzed a service load wave into individual proper full waves with mean loads⁽²⁾. As much labour is naturally required in applying this method manually, the following digital procedure of this method has been devised. Figure 9 shows its basic idea. First the full wave of the minimum amplitude AMIN is detected among the N peaks arranged orderly and then is taken out. The mean value of the full wave XSM is calculated simultaneously, and after that the full wave is classified as a wave both with mean value of XSM and with amplitude of AMIN. After taking out a full wave in this way, the number of arranged peaks decreases by two. In Fig. 9(a), for example, both the I -th and the $(I+1)$ -th peak cease to exist. As shown in Fig. 9(b), all peaks after the taken-out one have to be therefore rearranged from the I -th position without changing their orders. Consequently, for example, the original N -th peak will lie in the $(N-2)$ -th position. The digital treatment of the full wave count method is thought to be fundamentally possible by operating successively in this way. Figure 10 gives a concrete flow chart of such digital procedure. Such data are provided first as peak value $P(I)$, the total number of peaks N and the division width both of amplitude and of mean value (DA and DM respectively), and then for reference in the life estimation are calculated both the maximum and the minimum peak value, the mean value among all peaks, the mean crossing rate and so forth. The judging levels both of amplitude and of mean, $SAL(J)$ and $SML(K)$ respectively, are next reckoned, and every initial value of the number of full waves at each classification position $NFW(J, K)$ is settled to zero. As the number of residual peaks after taking

out a full wave decreases by two, both the total number of the taken-out full waves NEX and the number of residual peaks $NRES$ have to be taken account of. Their initial values are also settled properly. Then the minimum amplitude AMIN between the adjacent two peaks is detected and the corresponding position is determined. For this purpose the initial value of AMIN has to be made larger than every value of amplitude in the original random wave. After AMIN has been determined, such operation has only to be done as shown previously in Fig. 9. When the position has been judged where a taken-out full

Fig. 10 A flow chart of digital procedure of the full wave count method

wave should be classified, one count at the position is taken.

$$\text{NFW}(J, K) = \text{NFW}(J, K) + 1$$

Every time a full wave is taken out, the following counts are taken.

$$\text{NEX} = \text{NEX} + 1$$

$$\text{NRES} = \text{N} - \text{NEX} * 2$$

After rearrangement of residual peaks, the operation to detect the next minimum amplitude is repeated in the same way. NRES decreases according to the repetition. If NRES becomes smaller than two, no more full waves can't be taken out. The operation is therefore stopped in this case and the count results have only to be printed. In the flow chart shown here, no especial distinction is drawn between the positive and the negative stress range, which means that Bauschinger's effect is left out of consideration.

The digital procedure stated above is thought to be useful because of its easiness in applying the full wave count method to random loads. The peak values are easily distinguished from the data digitalized by an A-D converter, which is not here referred to.

Table 3 Statistic information on random loads ($\Delta\sigma = 2.0 \text{ kg/mm}^2$)

	Random load D	Random load E	Random load F
Mean value	$2.0 \Delta\sigma$	$2.7 \Delta\sigma$	$2.8 \Delta\sigma$
Standard deviation	$5.5 \Delta\sigma$	$5.5 \Delta\sigma$	$6.1 \Delta\sigma$
Mean deviation	$4.8 \Delta\sigma$	$4.8 \Delta\sigma$	$5.4 \Delta\sigma$
Mean crossing rate	0.88	0.90	0.92
Zero crossing rate	0.83	0.83	0.81

Fig. 11 An example of statistical information on random vibration (Random load D)

Fig. 12 Variation of mean crossing rate

4.2.2 On the fatigue life evaluation of a random load by the full wave count method

Three kinds of random fatigue tests D, E and F were carried out and then the life evaluation under each random load was done by a few load count methods.

Successive 1200 peaks were first selected for each random load. Twelve sections were then

Fig. 13 Load frequency distribution of random load D by the full wave count method

Fig. 14 Load frequency distribution of random load E by the full wave count method

Fig. 15 Load frequency distribution of random load F by the full wave count method

chosen whose width involved respectively 100, 200, ..., 1100 and 1200 peaks from the first one. In each section were computed the mean value as a whole, the standard deviation and the mean deviation. One of the results is indicated in Fig. 11, from which is clearly observed that these change little irrespective of the sectional width. The mean crossing rates in Fig. 12 obtained in the same way would have a similar tendency except in a few sections of small width. In Table 3 are indicated these values in the section of the maximum width.

Figures 13~15, in which a mean stress is selected as a parameter, give the count results obtained by applying the digital procedure of the full wave count method to the above-mentioned 1200 peaks selected in each random load. In applying this procedure, DM and DA

were made respectively to be $2\Delta\sigma$ and $\Delta\sigma$, where $\Delta\sigma$ equals 2.0 kg/mm^2 . When a value to be judged lay between the two adjacent judging levels, it was classified as the nearer one. If it lay just in the middle between them, it was done as the larger one. Full waves with zero amplitude mean therefore those with the amplitude of smaller value than half a division width. From these figures is found that each random load has a load distribution observed usually. In Figs. 16~18 are shown the count results obtained by applying both the all peaks and the range count method to the same data. In these cases the division width was selected $\Delta\sigma$, and each mean value as a whole was done as the basic level. The positive and the negative amplitude in these figures correspond to the loading and the unloading stress range, respectively. It is natural that the frequency of a high stress by the all peaks count method is larger than that by the range count method, which was clearly observed especially in the random load D.

The fatigue life estimation was then done from the above-mentioned count results. Figure 19 gives the S-N diagram of the material under the mean stress of zero. The effect of mean

Fig. 16 Load frequency distributions of random load D both by the all peaks count method and by the range count method

Fig. 17 Load frequency distributions of random load E both by the all peaks count method and by the range count method

Fig. 18 Load frequency distributions of random load F both by the all peaks count method and by the range count method

Fig. 19 S-N diagram of the material used in case of $\sigma_m = 0$

stress must be appraised rationally in some way for the fatigue life estimation. It is thought to be most desirable for this purpose to know empirically S - N diagrams under various mean stresses, but now modified Goodman's diagrams⁽⁵⁾ have been adopted.

The procedure of the life estimation is as follows. In the first place the count results by each count method during a certain measuring time t are converted into a load frequency distribution of completely reversed loads in such a way as mentioned above. Cumulative cycle ratio $(\sum n/N)_t$ is then calculated by using the S - N diagram in Fig. 19. In this case the number of cycles to failure of under stresses is assumed infinite, namely $N=\infty$, and furthermore both smaller positive and larger negative peaks than the mean stress as a whole must be eliminated from the count results by the all peaks count method. As failure must occur at $\sum n/N=1$ under the assumption of Miner's law, the theoretically estimated life may result as follows: $N_{th}=n_s/(\sum n/N)_t$, where n_s means the total number of cycles of repeated loads during the measuring time t .

In Table 4 are given both the theoretical life N_{th} estimated in this way and the actual

Table 4 Estimated fatigue life by each load count method

		Random load D	Random load E	Random load F
	N_f	3.84×10^5	3.29×10^5	1.30×10^5
N_{th}	FWCM	3.82	2.46	0.99
	PCM	2.94	2.10	0.65
	RCM	14.16	7.39	1.38

Symbols in the table mean the following:

N_f : actual fatigue life

N_{th} : theoretically estimated life

FWCM: the full wave count method

PCM: the peak count method

RCM: the range count method

Fig. 20 Effect of the extent of the irregularity of a wave on each load count result

life N_f obtained in the test under each random load. The propriety of the life estimation was compared in Fig. 20 where ordinate and abscissa indicated respectively the fatigue life ratio N_{th}/N_f and the mean crossing rate which was regarded to represent the irregularity of the wave. In case of a random wave with considerable irregularity, the evaluated life by the range count method will naturally be considered to be much different from the actual life, while that by the peak count method didn't show so much change and was about 50~80% of the actual life. That by the full wave count method also was not affected by the irregularity of the wave, and what was better, it was closer to the actual life than that by the peak count method. Considering, further, that it is on the safe side, this may imply that the full wave count method will be the best way to estimate the fatigue lives under random loads. The similar trend in Fig. 20 was already reported in other papers⁽⁶⁾⁽⁷⁾, but the full wave count method may be said to be more and more important according to the increase of the irregularity of a service load.

5. Conclusions

A new mechanical and simple random fatigue testing machine was devised and manufactured. Further, the discussion was done on the propriety of the life estimation under random loads. The main conclusions obtained are as follows.

(1) The random fatigue testing machine proved to be sufficiently practical which gave the random load mechanically by various sizes of steel balls.

(2) The random load obtained by this machine had a randomness of more than 85% and the most dominant frequency lay between 7 c/sec and 12 c/sec on account of the mechanism.

(3) The full wave count method which takes account of mean stresses is naturally more complicated than other count methods, but it can easily be performed by a digital computer.

(4) The estimated life by the full wave count method lay between 75% and 100% of the actual life and was not affected by the extent of the irregularity of a wave. And what was better, it was closer to the actual life than the estimated life by the peak count method. Considering, further, that it is on the safe side, the full wave count method may be called the best way for the life evaluation under service loads.

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