539.384.6

Strength of a Horizontal Reservoir Supported Partially by Equidistant Saddles*

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In a horizontal reservoir, partially supported by saddles, great stresses arise near the supports, and the magnitude of these stresses is closely related with the form of the saddle supports.

In this paper, we discussed the strength of such a reservoir and its relationship to the form of the saddle supports, utilizing the fundamental differential equation of cylindrical shells, and we pointed out the importance of the form of the saddle supports.

To decrease the magnitude of the stresses arising near the supports, we further made researches on the optimum form of the saddle supports.

From this study, we knew that the deformation of such a reservoir was very large, and so on.

1. Introduction

In a horizontal reservoir, partially supported by saddles, often used in chemical industry, great stresses arise near the supports, and the magnitude of these stresses is closely related with the form of the saddle supports. For example, if the saddles are made, as were used usually, so as to contact with the reservoir before deformation, the concentrated forces act at the edges of the saddles, therefore such saddles are unsuitable.

Up to now, reservoirs of this type have been discussed approximately by elementary theory and the like⁽¹⁾. But by these methods, it is impossible to make researches on the relationship between the form of the supports and the stress magnitude near them.

In this paper, we discuss the stresses and strains of such horizontal reservoirs and their relationship to the form of the saddle supports, utilizing the fundamental differential equation of cylindrical shells, and we point out the importance of the form of the saddle supports.

To decrease the magnitude of the stresses arising near the supports, we further make researches on the optimum form of the saddle supports.

2. Solution of problem

The reservoir we are discussing is shown in Fig. 1, and for the sake of simplicity, the following assumptions are made.

- (1) The liquid partially fills the reservoir.
- (2) The reservoir is long enough to have the same condition in each interval of supports.
- (3) No friction exists between the cylindrical shell and the supports.
- (4) The cylindrical shell contacts with the supports after deformation, while, except at the point B (Fig. 1), it does not always contact with them before deformation.
- (5) The supports are narrow enough.

The fundamental differential equation under radial load is as follows:

$$\left. \begin{aligned} & A \frac{\partial^8 w}{\partial x^8} + \frac{4B}{a^2} \frac{\partial^8 w}{\partial x^6 \partial \theta^2} + \frac{6C}{a^4} \frac{\partial^8 w}{\partial x^4 \partial \theta^4} + \frac{8-2\nu^2}{a^4} \\ & \times \frac{\partial^6 w}{\partial x^4 \partial \theta^2} + \frac{6L}{a^4} \frac{\partial^4 w}{\partial x^4} + \frac{1}{a^6} \left(4 \frac{\partial^4}{\partial x^2 \partial \theta^2} \right. \\ & \left. + \frac{1}{a^2} \frac{\partial^4}{\partial \theta^4} \right) \left(\frac{\partial^2}{\partial \theta^2} + 1 \right)^2 w \\ & = \frac{1}{D} \left(A \frac{\partial^4 p}{\partial x^4} + \frac{2U}{a^2} \frac{\partial^4 p}{\partial x^2 \partial \theta^2} + \frac{B}{a^4} \frac{\partial^4 p}{\partial \theta^4} \right) \end{aligned} \right\} \dots\dots\dots (1)$$

Fig. 1

* Received 7th October, 1969.

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Denoting by γ the density of liquid, and referring to Fig. 1 and ref. (2), we have the boundary conditions as follows;

$$\left. \begin{aligned} &\text{for } x = \pm l \\ &\frac{\partial w}{\partial x} = 0, \quad \frac{\partial v}{\partial x} = 0, \quad u \text{ is independent} \\ &\text{of } \theta \\ &\int_0^\pi T_x d\theta = a^2 \gamma \int_0^\pi \sin^2 \theta (\cos \alpha - \cos \theta) d\theta \\ &\quad = \frac{\pi}{2} \gamma a^2 f_t(\alpha) \\ &\text{where} \\ &f_t(\alpha) = \frac{1}{\pi} \left\{ (\pi - \alpha) \cos \alpha + \frac{\sin \alpha}{2} \right. \\ &\quad \left. + \frac{\sin 2\alpha \cos \alpha}{2} - \frac{\sin 3\alpha}{6} \right\} \end{aligned} \right\} \dots (2)$$

As mentioned in the assumption (4), the shell contacts with the supports after deformation and then, according to the assumption (3), undergoes radial reactions from the supports. Denoting by $2F_R(\theta)$ the reaction, the effective transverse shears at $x = \pm l$ are as follows;

$$\left(N_x + \frac{\partial \mathcal{M}}{a \partial \theta} \right)_{x=\pm l} = \begin{cases} 0 & \text{for } 0 \leq \theta < \pi - \beta \\ \pm F_R(\theta) & \text{for } \pi - \beta < \theta \leq \pi \end{cases} \dots (3)$$

Once $F_R(\theta)$ is determined, the solution of Eq. (1) in connection with the conditions (2) and (3) can be proved to be unique by means of the strain energy⁽²⁾. Therefore, the relationships of stresses and strains to the form of the supports can be discussed through $F_R(\theta)$.

Since

$$\begin{aligned} N_x + \frac{\partial \mathcal{M}}{a \partial \theta} &= \frac{\partial M_x}{\partial x} + \frac{2}{a} \frac{\partial \mathcal{M}}{\partial \theta} \\ &= D \left[\frac{\partial^3 w}{\partial x^3} + \frac{2-\nu}{a^2} \frac{\partial}{\partial x} \left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta} \right) \right] \end{aligned} \quad (3)$$

and considering the expression (2), we have

$$\left(N_x + \frac{\partial \mathcal{M}}{a \partial \theta} \right)_{x=\pm l} = D \left(\frac{\partial^3 w}{\partial x^3} \right)_{x=\pm l} \dots (4)$$

If $F_R(\theta)$ can be expressed by a known function $g(\theta)$ and an undetermined constant H as $F_R(\theta) = \gamma a^2 H g(\theta)$, Eq. (3) becomes, by expanding the right hand side in a Fourier series and by substituting Eq. (4) in the left hand side, as follows;

$$D \left(\frac{\partial^3 w}{\partial x^3} \right)_{x=\pm l} = \pm \gamma a^2 H \left(\frac{t_0}{2} + \sum_{n=1}^{\infty} t_n \cos n\theta \right) \dots (3)'$$

where, $t_0/2$ and t_n are Fourier coefficients.

The normal load p , due to fluid pressure, can be expressed in the following Fourier

series.

$$\left. \begin{aligned} p &= \gamma a \left\{ f_0(\alpha) - f_1(\alpha) \cos \theta - \frac{1}{\pi} \sum_{n=2}^{\infty} f_n(\alpha) \cos n\theta \right\} \\ \text{where} \\ f_0(\alpha) &= \frac{1}{\pi} \{ (\pi - \alpha) \cos \alpha + \sin \alpha \} \\ f_1(\alpha) &= \frac{1}{\pi} \left\{ \pi - \alpha + \frac{\sin 2\alpha}{2} \right\} \\ f_n(\alpha) &= \frac{2 \cos \alpha \sin n\alpha}{n} - \frac{\sin (n-1)\alpha}{n-1} \\ &\quad - \frac{\sin (n+1)\alpha}{n+1} \end{aligned} \right\} \dots (5)$$

We can find the solution of Eq. (1) for this normal load p to be⁽⁴⁾

$$\left. \begin{aligned} \frac{D}{\gamma a^4} \frac{w}{a} &= c + \Re A \mu_0^4 C_0 \cos \frac{\mu_0 x}{a} \\ &\quad + \left\{ -k f_1(\alpha) \left(\frac{x}{a} \right)^4 + A_1 \left(\frac{x}{a} \right)^2 + B_1 \right. \\ &\quad \left. + \Re \xi_1 C_1 \cos \frac{\mu_1 x}{a} \right\} \cos \theta \\ &\quad - \frac{B}{\pi} \sum_{n=2}^{\infty} \frac{f_n(\alpha)}{(n^2-1)^2} \cos n\theta \\ &\quad + \sum_{n=2}^{\infty} \Re \xi_i^n C_i^n \cos \frac{\mu_i^n x}{a} \cos n\theta \end{aligned} \right\} \dots (6)$$

where

$$k = \frac{(h/a)^2}{288(1-\nu^2)B}$$

In the expression (6), c , A_1 , B_1 , and C_0 , C_1 , C_i^n are real and complex arbitrary constants which must be determined by the boundary conditions. μ_0 and μ_1 are complex numbers defined by the following equations;

$$\mu_0^4 + 12(1-\nu^2)(a/h)^2 = 0 \dots (7)$$

$$A \mu_1^4 + 4B \mu_1^2 + 12(1-\nu^2)B^2(a/h)^2 = 0 \dots (8)$$

μ_i^n is a complex root of the characteristic equation;

$$\begin{aligned} A \mu^3 + 4B n^2 \mu^6 + \{ 6C n^4 - (8-2\nu^2)n^2 + 6L \} \mu^4 \\ + 4n^2(n^2-1)^2 \mu^2 + n^4(n^2-1)^2 = 0 \end{aligned} \dots (9)$$

The suffix i means 1 or 2 which corresponds to the two roots of Eq. (9) not conjugate to each other, and in the expression (6) the terms with suffix i mean that two such groups for $i=1,2$ should be added. Equation (9) can be factored into two quadratics of $\mu^{2(5)}$. ξ_i^n is the same with ξ_i in ref. (6), and ξ_1 is ξ_i with λ_i/a replaced by μ_1 . The expression of u , v corresponding to w in the expression (6) can be obtained from the $u \sim w$ and $v \sim w$ relations⁽³⁾ as follows⁽⁷⁾;

$$\begin{aligned}
\frac{D}{\gamma a^4} \frac{v}{a} &= \left[k f_1(\alpha) \left\{ \left(\frac{x}{a} \right)^4 - 12\nu \left(\frac{x}{a} \right)^2 - \frac{G}{B} \right\} \right. \\
&\quad - A_1 \left\{ \left(\frac{x}{a} \right)^2 - 2\nu \right\} - B_1 \\
&\quad - \Re \eta_1 C_1 \cos \frac{\mu_1 x}{a} \left. \right] \sin \theta \\
&\quad + \frac{1}{\pi} \sum_{n=2}^{\infty} \frac{1 + (1/12)(h/a)^2 n^2}{n(n^2 - 1)^2} f_n(\alpha) \sin n\theta \\
&\quad - \sum_{n=2}^{\infty} \Re \eta_i^n C_i^n \cos \frac{\mu_i^n x}{a} \sin n\theta \\
\frac{D}{\gamma a^4} \frac{u}{a} &= \frac{1}{\nu} \left[\frac{1}{12} \left(\frac{h}{a} \right)^2 f_0(\alpha) - c \right] \left(\frac{x}{a} \right) \\
&\quad - \Re \nu A \mu_0^3 C_0 \sin \frac{\mu_0 x}{a} + \left[4k f_1(\alpha) \left\{ \left(\frac{x}{a} \right)^3 \right. \right. \\
&\quad \left. \left. + 6(2 + \nu) \left(\frac{x}{a} \right) \right\} - 2A_1 \left(\frac{x}{a} \right) \right. \\
&\quad \left. - \Re \kappa_1 C_1 \sin \frac{\mu_1 x}{a} \right] \cos \theta \\
&\quad - \sum_{n=2}^{\infty} \Re \kappa_i^n C_i^n \sin \frac{\mu_i^n x}{a} \cos n\theta
\end{aligned}
\tag{10}$$

In these expressions, η_i^n and κ_i^n are the same with η_i and f_i in ref. (6), and η_1 and κ_1 are the same with η_i and f_i having λ_i/a replaced by μ_1 .

Substituting the expressions (6) and (10) into the relations of stress-strain⁽³⁾, we have the expressions for the stress-resultants and couples. Substituting these expressions into the conditions (2) and (3), and solving them, we have

$$c = \frac{1}{24(1-\nu^2)} \left(\frac{h}{a} \right)^2 \{ 2f_0(\alpha) - \nu f_1(\alpha) \} \tag{11}$$

$$\begin{aligned}
C_1 &= - \frac{(h/a)^2}{12(1-\nu^2)B} \left(\frac{l}{a} \right) f_1(\alpha) \\
&\quad \times \frac{\bar{\mu}_1^5}{i(\mu_1 \bar{\mu}_1)^3 \sin(\mu_1 l/a) \Im \mu_1^2} \tag{12}
\end{aligned}$$

$$A_1 = \frac{f_1(\alpha)}{144(1-\nu^2)B} \left(\frac{h}{a} \right)^2 \left[\left(\frac{l}{a} \right)^2 + 12 \frac{R}{B} \right] \tag{13}$$

$$H = \frac{K}{t_1}, \quad K = -24k f_1(\alpha) \left(\frac{l}{a} \right) + \Re \mu_1^3 \xi_1 C_1 \sin \frac{\mu_1 l}{a} \tag{14}$$

$$C_0 = \frac{K}{A} \frac{t_0}{2t_1 i(\mu_0)^5 \sin(\mu_0 l/a) \Im \mu_0^2} \tag{15}$$

Equations (14) imply that the weight of the liquid is in equilibrium with the reaction from the supports.

C_i^n is a root of the following simultaneous equations.

$$\left. \begin{aligned}
\Re \mu_i^n \xi_i^n C_i^n \sin(\mu_i^n l/a) &= 0 \\
\Re \mu_i^n \eta_i^n C_i^n \sin(\mu_i^n l/a) &= 0 \\
\Re \kappa_i^n C_i^n \sin(\mu_i^n l/a) &= 0 \\
\Re(\mu_i^n)^3 \xi_i^n C_i^n \sin(\mu_i^n l/a) &= K t_n / t_1
\end{aligned} \right\} \tag{16}$$

B_1 means a quantity of translation of the

cylinder as a rigid body, and can be determined by the condition $w=0$ at $B(x=l, \theta=\pi)$ (Fig. 1).

Substituting these arbitrary constants thus obtained into the expressions of the displacements, stress-resultants and couples, we can obtain their quantities.

3. Effects of the circumferential shape of the supports

3.1 The case of the supports in contact with the shell

The supports used usually are in contact with the shell before deformation. In this case, the function $F_R(\theta)$ must be determined so as to satisfy the condition

$$w=0 \quad \text{for } x=\pm l, \quad \pi-\beta < \theta \leq \pi \tag{17}$$

In order to satisfy the condition (17), however, there should exist concentrated forces and concentrated moments at the edges A, A' (Fig. 1), which may be deduced from the fact that in the case of a beam shown in Fig. 2, a concentrated force $F=P$ and a concentrated moment $M=M_0+Pl$ must act to satisfy the condition $w=0$, for $-l' < x < 0$. Therefore it seems impossible to support the shell by saddle supports in such a manner to satisfy the condition (17). If the supports are brought in contact with the shell before deformation, the shell will deform as shown in Fig. 3, and the concentrated forces will act at the points A, A' , which may result in that the saddle supports should not be made so as to contact with the shell before deformation.

Fig. 2

Fig. 3

3.2 The case of the supports changed in the form merely near the edges

Now, let us consider the case of changing the form of the supports merely near the edges, not to cause the concentrated forces. Assuming the supports to be in contact with the shell in $\pi - \zeta < \theta < \pi$ (Fig. 1) before deformation, w and $F_R(\theta)$ may have the following properties;

$$\left. \begin{aligned} w &= 0 & \text{for } x = \pm l, \quad \pi - \zeta < \theta \leq \pi \\ F_R(\theta) &> 0 & \text{for } \pi - \beta < \theta \leq \pi \end{aligned} \right\} \dots (18)$$

By choosing a smooth function $F_R(\theta)$ to satisfy the condition (18), one form of the supports is determined. It is not only complicated but also unpractical to satisfy the first condition of the expression (18) mathematically completely from the point of view of construction of the supports. Accordingly we make it be satisfied approximately using the point matching method. To see the effect of variation of ζ on the stresses, let us consider the case of $F_R(\theta)$ expressed as follows;

$$\frac{F_R(\theta)}{\gamma a^2} = \sum_{j=0}^3 d_j H_j g_j(\theta) \dots (19)$$

$$\sum_{j=0}^3 d_j = 1 \quad (d_j \text{ is undetermined constant}) \dots (20)$$

$$\left. \begin{aligned} g_0(\theta) &= 1, \quad g_1(\theta) = \cos \frac{\pi(\pi - \theta)}{2\beta} \\ g_2(\theta) &= (\pi - \theta) \sin \frac{3\pi(\pi - \theta)}{2\beta} \\ g_3(\theta) &= (\pi - \theta)^3 \sin \frac{3\pi(\pi - \theta)}{2\beta} \end{aligned} \right\} \dots (21)$$

Even if the form of the supports may be made as mentioned above, pretty large reactions are likely to be caused near the edges of the supports. This is the reason why $F_R(\theta)$ is approximated by such simple smooth functions expressed as Eqs. (21). Equation (19) means to take the sum of the solution for $d_j \gamma$, a part of the density γ of the liquid. Once the solution of the case of $d_j = 1$ is obtained, we can obtain the solution for $F_R(\theta)$ leaving d_j undetermined, the term of a quantity of translation B_1 being dealt with separately. For example, denoting by w_j the normal displacement for $d_j \gamma$ without involving the term of translation, w can be expressed as follows;

$$\frac{D}{a^4 \gamma} \frac{w}{a} = \frac{D}{a^4 \gamma} \frac{1}{a} \sum_{j=0}^3 d_j w_j + B_1 \cos \theta \dots (22)$$

Since the differences between the values of $w_j (j=0, 1, 2, 3)$ become greater as the distance of the point concerned from the point B becomes greater, we choose four points, B, D and two other points which are chosen so as to divide DB in ratios 1:2:3, and we can determine d_j and B_1 from the condition $w=0$ at these points and

by virtue of Eq. (20). Thus this problem is solved.

The results of the numerical calculation worked out by the above procedure on the case $l/a=2$, $a/h=50$, $\nu=0.3$ are presented in the following with the dimensionless values expressed by Eqs. (23).

$$\left. \begin{aligned} w' &= \frac{D}{\gamma a^4} \frac{w}{a}, \quad n_{ex} = \frac{N_x + \partial W / \partial \theta}{\gamma a^2} \\ t_x &= \frac{T_x}{\gamma a^2}, \quad t_\theta = \frac{T_\theta}{\gamma a^2} \\ m_x &= 6 \left(\frac{a}{h} \right) \frac{M_x}{\gamma a^3}, \quad m_\theta = 6 \left(\frac{a}{h} \right) \frac{M_\theta}{\gamma a^3} \end{aligned} \right\} \dots (23)$$

Figures 4, 5 represent the values of w' , n_{ex} , m_x and m_θ . The meaning of I, II, ... VI is shown in Table 1. It can be seen from Fig. 4 that the condition (18) is almost satisfied in the case of I, II, III. From Figs. 4, 5, many characters can be observed, but we confine ourselves to say that the displacement w is very large, and m_x and m_θ become very great near the edges of the supports. For instance, in the case II, w at $\theta=0$ is 4 000 times the maximum deflection calculated as if

Table 1

	β rad.	ξ deg	x/l	α rad
I	$\pi/2$	70	1	0
II	0.7	30	1	0
III	0.7	10	1	0
IV	$\pi/2$	70	0	0
V	0.7	30	0	0
VI	0.7	10	0	0

Fig. 4 Variations of w' and n_{ex} at the cross sections of $x=0$ and $x=l$

the reservoir were a beam having length $2l$ and being fixed at both ends. Then let us discuss the relationships of stress and strain to ζ . It is seen from the result of the calculation that it is in the region of $\zeta < 3\beta/4$ that the condition (18) can be satisfied with practically sufficient accuracy by using the functions shown in expressions (21); even in the region of $\zeta > 3\beta/4$, however, the stress and strain can be shown approximately by the same expressions. For small ζ , the condition (18) can be satisfied sufficiently using only smooth functions such as g_0 , g_1 , without necessity to use the functions

Fig. 5 Variations of m_x and m_θ at the cross sections of $x=0$ and $x=l$

Fig. 6 Variations of $(m_x)_{\max}$, $(m_\theta)_{\max}$, $(\sigma_x')_{\max}$, $(\sigma_\theta')_{\max}$ and $w_{x=l, \theta=\pi-\beta}$ versus ζ

such as g_2 , g_3 . In this case the solution becomes similar to that given in section 3.3.

The maximum values $(m_x)_{\max}$ and $(m_\theta)_{\max}$ of m_x and m_θ against ζ are shown in Fig. 6. The maxima of absolute values of the dimensionless skin stresses σ_x' , σ_θ' expressed by Eqs. (24) designated as $(\sigma_x')_{\max}$, $(\sigma_\theta')_{\max}$ respectively are also shown in Fig. 6 against the same abscissa ζ .

$$\left. \begin{aligned} \sigma_x' &= (\sigma_x)_s \cdot h / \gamma a^2 = t_x \pm m_x \\ \sigma_\theta' &= (\sigma_\theta)_s \cdot h / \gamma a^2 = t_\theta \pm m_\theta \end{aligned} \right\} \dots\dots\dots (24)$$

where $(\sigma_x)_s$ and $(\sigma_\theta)_s$ are skin stresses.

For the case of $\beta=0.7$, the curves of $(\sigma_x')_{\max}$ and $(\sigma_\theta')_{\max}$ are nearly equal. In Fig. 6, w' at $x=l$, $\theta=\pi-\beta$ is also shown, the curves of which give the measures of the necessary shape-alternation of the supports at their edges. It can be seen from Fig. 6 that to decrease the maximum values of the stresses and the moments, the form of supports must be changed over a wide range.

3.3 The form of the supports to make the stresses minimum

It is shown in section 3.1 and section 3.2 that the stresses in the shell become great partially near the edges of the supports. Here we look for the form of the supports which can make the magnitude of the stresses decrease, and this is performed by finding a function $F_R(\theta)$ which does not cause large stresses in the shell.

In general, $F_R(\theta)$ can be expressed as a Fourier series. The terms of higher frequency, however, cause partial magnification of resultant stresses and resultant moments, hence we take only the first two smooth terms of a Fourier series for $F_R(\theta)$.

$$\frac{F_R(\theta)}{\gamma a^2} = d_0 H_0 g_0(\theta) + d_1 H_1 g_1(\theta) \dots\dots\dots (25)$$

where

$$g_0(\theta) = 1, \quad g_1(\theta) = \cos \frac{\pi(\pi-\theta)}{2\beta} \dots\dots\dots (26)$$

$$d_0 + d_1 = 1 \dots\dots\dots (27)$$

The determined constants d_0 and d_1 should be such as to minimize the maximum of absolute values of stresses arising in the shell. We denote d_0 thus determined by d_{0p} . To find the values of d_{0p} means the determination of $F_R(\theta)$, which results in the solution of the problem. B_1 can be obtained from the condition $w_{x=l, \theta=\pi} = 0$.

In the following, we mention the result of the numerical calculation worked out on the same case as in section 3.2, i.e. $a/h=50$, $l/a=2$, $\nu=0.3$. As the maximum values of the transverse and tangential shear stresses are less than

about $1/5$ times the maximum values of the normal stresses, we discuss mainly about the absolute values of the normal stresses. In the following, the term of "maximum value" means the maximum or the minimum of absolute value. And, since it is seen from the result of calculation that the maximum stress arises in the case of the reservoir completely filled i.e. $\alpha=0$, we discuss this case.

Using the same notation as used in section 3.2, and using d_0 as a parameter of expressing $F_R(\theta)$, $(\sigma_x')_{\max}$ and $(\sigma_\theta')_{\max}$ are shown in Fig. 7 against d_0 for the case of $\beta=0.7, \pi/2$, as examples,

Fig. 7 Variations of $(\sigma_x')_{\max}$ and $(\sigma_\theta')_{\max}$ versus d_0

Fig. 8 Variations of d_{0p} , $(\sigma_x')_{\max}$, $(\sigma_\theta')_{\max}$ and $\Delta a'$ versus β

from which d_{0p} is found as marked by a vertical segment. The value of d_{0p} against β is shown in Fig. 8. The values of the maximum stresses for $d_0=d_{0p}$ are also shown in Fig. 8. These curves show the minimum values, to which the maximum stresses can be decreased by changing the circumferential shape of supports. This curve shows, natural it may be, that it is advantageous to take β as large as possible.

Using the same notations as used in section 3.2, w' , n_{ex} , m_x and m_θ for $d_0=d_{0p}$ at $x=l$ are shown in Figs. 9, 10, as examples. By comparing these figures with the figures in section 3.2, it can be said that, in the present case, unnecessary

Fig. 9 Variations of w' and n_{ex} at the cross section of $x=l$

Fig. 10 Variations of m_x and m_θ at the cross section of $x=l$

increase of the reaction is avoided, and the maximum values of stresses are decreased by permitting the shell to deform to a certain degree, and the plate of the shell is used effectively. The displacement w in this case is greater than in the case of section 3.2. For example, in the case of $\beta=0.7$, the deflection of the upper part of the shell is about 6500 times the maximum deflection calculated as if the reservoir were a beam having a length $2l$ and being fixed at both ends. The curve of w for $\pi-\beta < \theta \leq \pi$ in Fig. 9 corresponds to the form of the supports, and by finding a suitable constant $\Delta a'$, this curve agrees with the curve of $\Delta a'(1+\cos\theta)$ approximately as shown in Fig. 9 (in Fig. 9 the values of $\Delta a'(1+\cos\theta)$ are marked ". . ."), which shows that if the supports are made such as to have radius $a+\Delta a$, the resultant stresses in the shell become almost equal to that in the case of $d_0=d_{0p}$, provided

$$\Delta a = \Delta a' \cdot \gamma a^5 / D$$

Fig. 11 Variations of σ_x' and σ_θ' at the cross section of $x=l$

Fig. 12 Variations of σ_x' and σ_θ' along generatrix

The value of $\Delta a'$ against β is also shown in Fig. 8. In Fig. 11, the values of σ_x' and σ_θ' in the case of $d_0=d_{0p}$ for $x=l$ where the maximum stress arise are shown and in Fig. 12 those for the angle where the maximum stress arise are shown.

Lastly, we add a remark that there arises no additional trouble for a partially filled reservoir on the supports so constructed as mentioned above.

4. Effects of the width of the supports

Hitherto, we have assumed that the width of the supports is zero. For the case of the supports with non zero width, we can obtain the solution by superposing or integrating the solution for the case of zero width. We briefly discuss two simple cases obtained by superposing.

Let the width of the supports be $2s$. If the reaction from the support is constant along the x direction, we call this support a uniform support, and if the reaction acts only at both ends of the support and the shell is assumed to be held on two zero-width supports having an interval of $2s$, we call this a two-lines support. Superposing the curves in Fig. 12 for these two cases, we can see the following.

(1) In the case of the supports with non zero width, the part of the shell at some distance from the supports is in about the same condition as in the case of zero width supports.

(2) Comparing σ_x' with σ_θ' in Fig. 12, we see σ_x' varies more remarkably than σ_θ' near the supports. Therefore the maximum value of σ_x' decreases rapidly with an increasing width of supports. Though we see $(\sigma_x')_{\max}$ becomes greater than $(\sigma_\theta')_{\max}$ for β approaching $\pi/2$, in Fig. 8, σ_θ also must receive attention.

(3) In the case of $s/a < 0.15$, the two-lines support is rather advantageous than the uniform support.

(4) To find the most suitable form of support for the axial direction, we must use the same method as used in section 3.3 for the circumferential direction; in the case of $s/a < 0.1$, however, the two-lines support may be considered as nearly optimum, because in this range of s/a the stresses scarcely change in the interval of $2s$.

5. Conclusions

As mentioned above, the stresses arising in the reservoir are closely related to the form of the supports. This can be said generally for

cylindrical shells supported partially. Usually, the form of the supports is easily determined, so it may be supposed that there arise great stresses partially. These high stresses, probably, cause only partially non-elastic deformation and seem to cause no serious trouble. In some uses of the reservoir, however, there may occur such troubles as the decrease in corrosion-resistance, fatigue limit and the reduction of strength for some materials.

The supports should be made in the most suitable forms as mentioned or in the forms which are ascertained to cause only safe stresses in shell. This needs accurate workmanship, but in practice it is difficult to construct supports so accurately. If supports are constructed not so accurately, we must take care of the magnitude of the stresses.

In a partially supported reservoir mentioned

here, the deformation is very large, and the deflection of upper part of the reservoir reaches 4 000~6 500 times the maximum deflection calculated as if the reservoir were a beam having the length $2l$ and being fixed at both ends as discussed in section 3.2 and section 3.3, becoming greater as β decreases. For practical use of partially supported reservoir, we should pay attention to such magnitude of the deformation.

References

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