A Simple Method to Estimate Blade Surface Pressure in Axial-Flow Turbomachines and Its Application

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A simple method is proposed to estimate the surface pressure distribution on a cascade airfoil in the retarded flow, is compared with theoretical and experimental results, and is discussed in detail. It is verified that the method is applicable to the design of axial-flow turbomachines considering the suction performance and efficiency.

There are two assumptions in this procedure: The first; the flow velocity on the surface may be obtained by superposition of the velocity on an isolated airfoil and the additional velocity induced by the cascade effect. The second; the induced velocity has a tangential component which decreases linearly along the chord line.

As an application, it is shown that a turbo-pump should be designed as follows, that is, a rather small flow coefficient when optimization of both suction performance and efficiency is desirable.

1. Introduction

Precise estimates of suction performance and Mach number limitation of axial-flow turbomachines have been obtained based on the experimental data of the pressure distribution on cascade airfoils. It is difficult to know the functional relation between performance characteristics i.e. head, flow rate, efficiency and cavitation or Mach number limitation, because the theoretical calculation is tedious and it may lead often to noticeable disagreement with experimental results on the cascade.

If some simple method for estimation utilizing the data on many different kinds of airfoil sections were available, it would be so convenient for the economical design of turbomachines.

There are some inspiring works in which R.D. Bowerman(1) has shown a method for calculating the pressure by multiplying the pressure coefficient of the single airfoil section by the dynamic head based on the mean velocity. K. Tabuse(2) has presented a method which takes an effect of the blade thickness into consideration. Therefore, the former may underestimate the cavitation parameter slightly upstream and near to the mid-chord position, while the latter only slightly improves the former.

We have tried to find a simple reasonable method to precalculate the suction performance with considerable accuracy without using the cascade data. The pressure distribution on the blade surface in a cascade is derived from applying isolated airfoil data(3), and the minimum pressure, considered as to the cavitation inception coefficient, can be formulated by means of the characteristics (flow and head rise coefficient) and the manometric efficiency.

There are two assumptions in this procedure: The first; the flow speed on the surface of the cascade airfoil may be obtained by superposition of the surface velocity on the isolated airfoil and the additional velocity induced as the cascade effect. The second; the induced velocity has a tangential component which decreases linearly along the chord line.

The pressure distributions using this method were compared with theoretical and experimental studies on the cascades in Germany and America, and also with our experimental results on the suction performance of axial-flow pump impellers.
It has been concluded that it is applicable to designing with acceptable accuracy in spite of the linear approximation.

As the application of the method, the relation between the suction performance and characteristics of a turbo-pump of usual type is estimated and it is made clear that a turbopump has the following tendency, that is, the manometric efficiency decreases according to the flow coefficient, while the suction performance gets better as the cavitation parameter decreases with the flow coefficient.

2. Nomenclature

The following nomenclature is used in this paper.

- $c$: pressure coefficient
- $c_{L}$: section lift coefficient
- $c_{Ld}$: design lift coefficient of standard
- $c_{Lw}$: lift coefficient of actual airfoil at design point
- $g$: acceleration of gravity
- $H$: total head
- $L_{e}$: effective shaft horse power
- $L_{d}$: disc friction loss
- $l$: chord length
- $p$: static pressure
- $P_{t}$: total pressure (absolute)
- $p_{t}$: total pressure (gauge)
- $P_{s}$: saturated vapour pressure (absolute)
- $Q$: volume rate of flow-rate
- $S^{*}$: speed coefficient on surface of airfoil
- $u$: circumferential speed of impeller
- $V$: standard speed of flow
- $v_{d}$: additional speed due to airfoil thickness
- $v_{o}$: meridional velocity of flow
- $v$: absolute velocity of flow
- $v_{t}$: tangential velocity of flow
- $w$: relative velocity of flow
- $x$: ordinate in direction of chord
- $\alpha$: angle of attack
- $\beta$: inlet flow angle
- $\beta'$: complementary angle of $\beta$
- $\gamma$: specific weight of water
- $A_{e}$: additional velocity of camber
- $\Delta v_{a}$: additional velocity due to angle of attack
- $\varepsilon$: drag-lift ratio of the blade element
- $\eta_{i}$: internal efficiency of pump
- $\eta_{Mk}$: manometric efficiency of blade system
- $\eta_{v}$: volumetric efficiency of pump
- $\xi$: radius ratio
- $\phi$: flow coefficient $= v_{d}/u$
- $\phi'$: head rise coefficient $= H/(u^{2}/2g)$
- $\tau$: cavitation parameter $= NPSH/(u^{2}/2g)$

Subscripts:
- $a$: blade tip
- $cz$: cavitation zone
- $G$: cascade
- $i$: isolated airfoil
- $s$: surface of airfoil
- $sp$: static pressure on surface of airfoil
- $th$: theoretical
- $tp$: total pressure on surface of airfoil
- $v$: value taking theoretical flow-rate into consideration
- $\infty$: geometrical mean
- $'$: specified by $(u^{2}/2g)$
- $1$: impeller inlet
- $2$: impeller outlet

3. Approximate calculation of surface pressure on cascade airfoil in retarded flow

Many theoretical methods of calculation of the pressure distribution have been proposed, because the cascade performance is so important. Most of them are based on the potential flow theory, and some works have been developed taking account of boundary layers. The calculations are rather tedious, and are difficult to apply in practical design procedures. The experimental data for the pressure distribution on cascade airfoils have been available, but are limited. In many practical designs, the airfoil and the solidity should be decided after consideration of the suction performance for pumps and of the Mach number limit for compressors. For the same purpose, we describe in this chapter our simple procedure.

3-1 Calculation of pressure coefficient on isolated airfoil

Speed coefficient $S^{*}$ of isolated airfoil is shown by I.H. Abbott and others as follows.

$$S^{*} = \frac{(v_{t}^{2})}{V} = \left[ \frac{v_{i} + c_{L0}}{V} \right] \sqrt{\frac{(c_{Lw} - c_{L0})}{c_{Lw}} \frac{dV}{V}}$$

Here, the plus or minus sign on the right hand side corresponds to suction side or pressure side of airfoil, respectively.

Putting relative inlet velocity $w_{1}$ in place of $V$, relative speed on the surface of airfoil $w_{i}$ in place of $v_{i}$ in Eq. (1), and applying Bernoulli's theorem between the impeller inlet and the blade surface, we get:

$$S_{i}^{*} = \left( \frac{w_{1}}{w_{1}} \right) \frac{2g(p_{i} - p_{1})}{\tau w_{1}^{2}}$$

$$c_{3;i} = \left( \frac{p_{i} - p_{1}}{\tau} \right) / \left( \frac{w_{1}^{2}}{2g} \right) = 1 - S_{i}^{*}$$
3.2 Calculation of pressure coefficient on a cascade airfoil in the retarded flow

The retarded flow in the cascade is complicated and its theoretical treatment is not easy. We simply assume that the velocity component \( v_\theta \) increases in proportion to \( z/l \). The assumption has been verified by our previous paper and by the study of D.A. Morelli and R.D. Bowerman. The other assumption is the possibility of the superposition of the velocities as above described. Therefore the speed coefficient of the cascade airfoil \( S_C^* \) can be derived approximately by superposing \( S_i^* \) and from retardation effect of the cascade as follows,

\[
S_C^* = \left( \frac{w_{xG}}{w_\infty} \right)^2 = \left( S_i^{*1/2} + \frac{w_{xG} - w_\infty}{w_\infty} \right)^2 = \left( S_i^{*1/2} - 1 + \frac{w_{xG}}{w_\infty} \right)^2
\]

From the velocity diagram in Fig. 1, \( w_{xG} \) and \( w_\infty \) become,

\[
w_\infty = \sqrt{w_{xG}^2 - (w_{xG} - w_\infty) \frac{\pi}{2}} \left[ \left( \frac{w_1 \cos \beta_1}{w_{xG}} \right)^2 + \left( \frac{w_1 \sin \beta_1 + w_1 \cos \beta_1 \tan \beta_2}{w_{xG}} \right)^2 \right] \]

\[
w_{xG} = \sqrt{w_{xG}^2 - (w - w_\infty) \frac{\pi}{2}} = w_1 \cos \beta_1 \]

Therefore

\[
\frac{w_{xG}}{w_\infty} = \sqrt{1 + \left( \tan \beta_1 + \left( \tan \beta_2 - \tan \beta_i \right) \frac{z}{l} \right)^2 / \left( \tan \beta_1 + \tan \beta_2 \right)^2} \]

The pressure coefficient on the cascade airfoil \( c_{pG} \) becomes from Eqs. (4) and (5),

\[
c_{pG} = \left( \frac{P_{xG} - P_i}{\gamma} \right) / \left( \frac{w_1^2}{2} \right) = 1 - \left( \frac{w_{xG}}{w_1} \right)^2 = 1 - S_C^* \times \left[ \left( \sin \beta_1 + \cos \beta_1 \tan \beta_2 \right)^2 + \cos^2 \beta_1 \right] \]

Fig. 1 Velocity diagram for an axial-flow impeller with axial inlet

| Table 1 | Values of pressure coefficient \( c_{pG} \) on the suction side for the airfoil NACA 0010 |
|----------------|-----------------|-----------------|-----------------|-----------------|
| \( \beta \) | \( \alpha \) | \( \epsilon_{LG} \) | \( \frac{w_{xG}}{w_\infty} \) |
| \( \alpha = 15^\circ \) | \( \alpha = 5^\circ \) | \( \alpha = 5^\circ \) | \( \alpha = 5^\circ \) |
| \( \epsilon_{LG} \) | 0.620 | 0.620 | 0.620 | 0.620 |
| \( \frac{w_{xG}}{w_\infty} \) | 0.2 | 0.4 | 0.6 | 0.2 | 0.4 | 0.6 |
| Calculated values | -0.768 | -0.445 | -0.231 | -0.488 | -0.322 | -0.187 |
| Theoretical data of L. Speidel | -0.705 | -0.420 | -0.220 | -0.515 | -0.340 | -0.240 |
| Experimental data of L. Speidel | -0.660 | -0.370 | -0.100 | -0.515 | -0.340 | -0.245 |
| \( \alpha = 15^\circ \) | \( \alpha = 5^\circ \) | \( \alpha = 5^\circ \) | \( \alpha = 5^\circ \) |
| \( \epsilon_{LG} \) | 0.492 | 0.492 | 0.492 | 0.492 |
| \( \frac{w_{xG}}{w_\infty} \) | 0.2 | 0.4 | 0.6 | 0.2 | 0.4 | 0.6 |
| Calculated values | -0.665 | -0.389 | -0.199 | -0.531 | -0.306 | -0.177 |
| Theoretical data of L. Speidel | -0.600 | -0.390 | -0.200 | -0.500 | -0.350 | -0.240 |
| Experimental data of L. Speidel | -0.660 | -0.380 | -0.215 | -0.500 | -0.350 | -0.250 |
| \( \alpha = 15^\circ \) | \( \alpha = 5^\circ \) | \( \alpha = 5^\circ \) | \( \alpha = 5^\circ \) |
| \( \epsilon_{LG} \) | 0.366 | 0.366 | 0.366 | 0.366 |
| \( \frac{w_{xG}}{w_\infty} \) | 0.2 | 0.4 | 0.6 | 0.2 | 0.4 | 0.6 |
| Calculated values | -0.565 | -0.334 | -0.166 | -0.420 | -0.284 | -0.164 |
| Theoretical data of L. Speidel | -0.590 | -0.380 | -0.220 | -0.470 | -0.355 | -0.230 |
| Experimental data of L. Speidel | -0.650 | -0.400 | -0.265 | -0.530 | -0.415 | -0.315 |
4. Comparison between results from approximate treatment and cascade data

The accuracy of the approximation method given in section 3-2 is verified by the results of studies on the cascade with several airfoils NACA 0010, 8410, 65-(12 A\text{al})10, 65-(12 A\text{al})10, 65-(4 A\text{al})10, 65-(12 A\text{al})10).

Table 1, the numerical quantities of the pressure coefficient \(c_{pA}\), calculated by Eq. (8) for the symmetric airfoil NACA 0010 compared with the theoretical and experimental data of L. Speidel and N. Scholz\(^{(10)}\). The cascade parameters are \(l/l=0.8, 1.0, 1.3, \beta_{1}=75^\circ, 85^\circ, \alpha_{1}=5^\circ, 15^\circ\). The values calculated by the simple procedure agree pretty well with the theoretical values of L. Speidel. Table 2 shows two examples for NACA 8410 that the blade setting is close to the design conditions. The minimum pressure and its location are close to the experimental results by L. Speidel.

From the above comparison, we can conclude that the above mentioned approximation is applicable in practical procedures in spite of its simplicity.

J.R. Erwin and others\(^{(11)}\) have offered data for compressor cascades with the united camber airfoils by systematic wind tunnel tests. Some of them are compared with the approximate method and shown in Fig. 2, and in Tables 3 and 4. Those are in good agreement and are considerable at such design conditions as indicated by a mark * on \(\alpha_{l}\) in the Tables, except for the case of \(l/l=1.5, \beta_{1}=60^\circ\).

As a whole, the approximation gives rather high pressure near the trailing edge. It may be understood that the mean outlet velocity \(w_{2}\) in our procedure is smaller than the actual velocity at the trailing edge and the mixing loss around the trailing edge is neglected. Although they affect a little the flow near the mid-chord, it is small because the location of the minimum pressure is far from the trailing edge. This effect will be discussed in a future report.

![Fig. 2 Effect of locations of maximum camber on pressure distributions on a cascade airfoil](image)

**Table 2 Values of pressure coefficient \(c_{pA}\) on the suction side for the airfoil NACA 8410**

<p>| (\beta_{1}) | (11^\circ) | (23^\circ) |</p>
<table>
<thead>
<tr>
<th>(\alpha_{1})</th>
<th>19°</th>
<th>7°</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_{pA})</td>
<td>0.976</td>
<td>0.800</td>
</tr>
<tr>
<td>(l/l=0.8)</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>(s/l)</td>
<td>( )</td>
<td>0.2</td>
</tr>
<tr>
<td>Calculated values</td>
<td>(0.13)</td>
<td>-0.416</td>
</tr>
<tr>
<td>Theoretical data of L. Speidel</td>
<td>(0.18)</td>
<td>-0.530</td>
</tr>
<tr>
<td>Experimental data of L. Speidel</td>
<td>(0.25)</td>
<td>-0.425</td>
</tr>
</tbody>
</table>

<p>| (\beta_{1}) | (10^\circ) | (24^\circ) |</p>
<table>
<thead>
<tr>
<th>(\alpha_{1})</th>
<th>20°</th>
<th>6°</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_{pA})</td>
<td>0.746</td>
<td>0.641</td>
</tr>
<tr>
<td>(l/l=1.0)</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>(s/l)</td>
<td>( )</td>
<td>0.2</td>
</tr>
<tr>
<td>Calculated values</td>
<td>(0.16)</td>
<td>-0.280</td>
</tr>
<tr>
<td>Theoretical data of L. Speidel</td>
<td>(0.20)</td>
<td>-0.555</td>
</tr>
<tr>
<td>Experimental data of L. Speidel</td>
<td>(0.20)</td>
<td>-0.390</td>
</tr>
</tbody>
</table>

Notice: In the above table, round bracket ( ) denotes the value of \(s/l\) at minimum pressure condition.
5. Total pressure described in terms of machine characteristics, and the cavitation zone parameter

To apply the method to the axial-flow pump in the previous paper, the total pressure coefficient is described in a form which included the characteristics explicitly. Furthermore, the value of the total pressure is compared with the cavitation zone parameter.

5.1 Expression of the lift coefficient of a blade element in terms of the characteristics

It is assumed that the flow in the impeller is a free vortex type and that the velocity at inlet is parallel to the axis of rotation as shown in Fig. 1. We consider the power loss due to

Table 3 Values of pressure coefficient $c_{pld}$ on the suction side for the airfoil NACA 65-(4A1t=310)

<table>
<thead>
<tr>
<th>$\beta_1=30^\circ$</th>
<th>$l/l=1.0$</th>
<th>$l/l=1.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>2.7° *</td>
<td>5.5° *</td>
</tr>
<tr>
<td>$c_{pld}$</td>
<td>0.216</td>
<td>0.382</td>
</tr>
<tr>
<td>$z/l$</td>
<td>( )</td>
<td>( )</td>
</tr>
<tr>
<td>Calculated values</td>
<td>(0.44)</td>
<td>(0.26)</td>
</tr>
<tr>
<td>Experimental data of J.R. Erwin</td>
<td>-0.162</td>
<td>-0.263</td>
</tr>
<tr>
<td>$\beta_1=60^\circ$</td>
<td>$l/l=1.0$</td>
<td>$l/l=1.5$</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>3.4° *</td>
<td>5.4° *</td>
</tr>
<tr>
<td>$c_{pld}$</td>
<td>0.235</td>
<td>0.311</td>
</tr>
<tr>
<td>$z/l$</td>
<td>( )</td>
<td>( )</td>
</tr>
<tr>
<td>Calculated values</td>
<td>(0.50)</td>
<td>(0.43)</td>
</tr>
<tr>
<td>Experimental data of J.R. Erwin</td>
<td>-0.210</td>
<td>-0.344</td>
</tr>
</tbody>
</table>
| Notice: In the above table, round bracket ( ) denotes the value of $z/l$ at minimum pressure condition. And mark * denotes the value at design condition.

Table 4 Values of pressure coefficient $c_{pld}$ on the suction side for the airfoil NACA 65-(12A1t=310)

<table>
<thead>
<tr>
<th>$\beta_1$</th>
<th>30°</th>
<th>60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>10.6° *</td>
<td>10.9° *</td>
</tr>
<tr>
<td>$c_{pld}$</td>
<td>0.926</td>
<td>0.828</td>
</tr>
<tr>
<td>$z/l$</td>
<td>( )</td>
<td>( )</td>
</tr>
<tr>
<td>Calculated values</td>
<td>(0.53)</td>
<td>-0.336</td>
</tr>
<tr>
<td>Experimental data of J.R. Erwin</td>
<td>-0.181</td>
<td>-0.327</td>
</tr>
</tbody>
</table>
| Notice: In the above table, round bracket ( ) denotes the value of $z/l$ at minimum pressure condition. And mark * denotes the value at design condition.
the tip clearance as a leakage loss\(^{(13)-(14)}\), because it is difficult to divide the loss into two parts, the leakage loss and the hydraulic loss.

The internal efficiency \(\eta_i\), the theoretical flow rate in an impeller \(Q_t\), and the effective power to an impeller \(L_e\) are written as follows,

\[
\eta_i = \frac{\gamma Q H}{L_t}, \quad Q_t = \frac{Q}{\eta_i}, \quad L_e = \gamma Q H_{th} + L_d \quad (9)
\]

As \(\gamma Q H_{th}\) is much greater than \(L_d\), \(L_d\) is neglected in Eq. (9), and the theoretical head \(H_{th}\) becomes

\[
H_{th} = \frac{L_t}{\gamma Q_t} \quad \cdots \cdots \cdots \cdots \cdots (10)
\]

Using cascade theory, we may obtain the following equations:

\[
c_{LG} \left( \frac{l}{l} \right) = \frac{\phi_s}{\phi_s} = \frac{2(w_{st} - w_{st})}{w_{st}} \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (11)
\]

\[
H_{th} = \frac{c_{LG}}{2gt} \left\{ \frac{v_{st}}{2} + \left( u - \frac{g H_{th}}{2u} \right)^{1/2} \right\}
\]

\[
 \times \left( 1 + \frac{1}{v_{st}} \left( u - \frac{g H_{th}}{2u} \right) \right) \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (12)
\]

From Eq. (11), the following non-dimensional expression is obtained

Table 5 Main characteristics and lift coefficient of the impeller type GI

<table>
<thead>
<tr>
<th>Flow coefficient (\phi_h)</th>
<th>0.280</th>
<th>0.260</th>
<th>0.200</th>
<th>0.160</th>
</tr>
</thead>
<tbody>
<tr>
<td>Characteristics</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Head rise coefficient (\phi_h)</td>
<td>0.180</td>
<td>0.258</td>
<td>0.292</td>
<td></td>
</tr>
<tr>
<td>Internal efficiency (\eta_e)</td>
<td>0.650</td>
<td>0.650</td>
<td>0.560</td>
<td></td>
</tr>
<tr>
<td>Lift coefficient (c_{LG})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>0.436</td>
<td>0.576</td>
<td>0.765</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>0.517</td>
<td>0.700</td>
<td>0.942</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>0.640</td>
<td>0.892</td>
<td>1.223</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>0.807</td>
<td>1.166</td>
<td>1.642</td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>0.937</td>
<td>1.389</td>
<td>2.000</td>
<td></td>
</tr>
</tbody>
</table>

Notice: (1) In the above table, the lift coefficients \(c_{LG}\) on a cascade airfoil shows the value calculated by Eq. (13).

(2) The airfoil Clark Y is used for the type CY1, but in calculating the lift coefficient the data on the airfoil NACA 4412 are used in place of the airfoil Clark Y-12%. Because the data on the airfoil Clark Y are deficient and also both of them closely resemble each other.

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Fig. 3 Pressure distributions on a cascade airfoil for the operation conditions of the impeller type NA1

---

\[
\phi_s = 0.280, \quad \eta_e = 0.194
\]

---

\[
\phi_s = 0.240, \quad \eta_e = 0.305
\]

---

\[
\phi_s = 0.200, \quad \eta_e = 0.330
\]
where, $\phi_{as}$, $\phi_{cs}$ are the head rise and flow coefficient, respectively. Both $\phi_{as}$ and $\phi_{cs}$ are nondimensionalized based on the impeller tip speed $u_s$, $\xi$ is the radius ratio $r/r_s$.

The lift coefficient can be determined from experimental results on the head rise and flow coefficients assuming values for volumetric efficiency $\eta_s$, and the drag-lift ratio $\epsilon$.

Tables 5 and 6 show the main pump characteristics and the lift coefficients of the sections at five radial positions $a$, $b$, ..., $e$ of the tested impellers type GI, CY1, NA1. $\epsilon$ is assumed to be 0.04. $\eta_s$ is assumed to be 0.96 for the type GI, but 0.97 for the type CY1 and NA1.

5.2 Total pressure coefficient represented by characteristics

As was done for the static pressure coefficient of Eq. (8), we may define the total pressure coefficient $c_{1pG'}$ as follows using $u$ instead of $w_1$,

$$c_{1pG'} = \frac{p_{1u} - p_{1l}}{\frac{u^2}{2g} - 1} \left( \frac{w_{1u}}{u} \right)^2$$

$$= 1 - S_0^2 \left( \frac{w_{1u}}{u} \right)^2$$

$$= 1 - \left( \frac{S_0^{4/2} - 1}{u - 1} \right) \left( \frac{u + w_{1u}}{u} \right)^2 \quad (14')$$

The relation between $c_{1pG}$ and $c_{1pG'}$ is

$$c_{1pG'} = 1 - (1 - c_{1pG}) \left( \frac{w_{1u}}{u} \right)^2 \quad (15)$$

The relative velocity becomes

$$w_{1u} = u \left[ \phi_{sa}^2 + \left( 1 - \left( \frac{\phi_{sa}}{4} \right) \right)^{1/2} \right] \quad (5')$$

$$w_{1G} = u \left[ \phi_{sa}^2 + \left( 1 - \phi_{sa} \frac{x}{2l} \right)^{1/2} \right] \quad (6')$$

Therefore,

$$c_{1pG'} = 1 - \left( \frac{S_0^{4/2} - 1}{u - 1} \right) \left( \phi_{sa}^2 + \left( 1 - \frac{\phi_{sa}}{4} \right)^{1/2} \right)$$

$$+ \left( \phi_{sa}^2 + \left( 1 - \phi_{sa} \frac{x}{2l} \right)^{1/2} \right)^2 \quad (14')$$

Figure 3 shows the total pressure coefficient at the operation conditions of the type NA1 given in Table 6.

5.3 Comparison of the pressure coefficient with the cavitation zone parameter

The physical meaning of the cavitation zone parameter is as follows: Cavitation occurs on the blade surface when the NPSH of the turbo-pump decreases and the pressure at the minimum pressure point drops to the vapor pressure. With further decrease of the NPSH, cavitation develops and covers a part of the blade surface. The cavitation zone parameter has been defined as the parameter that indicates a condition of the cavitation inception or development.

It is known that the blade surface pressure on the cavitating conditions is very complicated. We have observed that the rear end of the cavitation zone fluctuates violently. The chordwise positions of the front and mean rear end of the cavitation zone have been measured and are successively plotted for various cavitation zone parameters in Fig. 4 with a broken line, i.e. the cavitation zone line.

The cavitation zone parameter is compared

\[ \text{Fig. 4 Comparison of total pressure coefficients } c_{1p} \text{ and cavitation zone parameters } \tau_{cs} \text{ for the impeller type NA1} \]
with the total pressure coefficient of Eq. (14). The curves resemble each other closely in shape. But values of \( \tau_{re} \) at the rearward end of the cavity are smaller than the calculated total pressure coefficient. This means that the pressure recovery in the cavitation zone seems to delay as compared with that under non-cavitation conditions. The minimum pressure however has to agree with the maximum value of \( \tau_{re} \) because this is equivalent to the condition that the cavitation occurs at the vapor pressure.

It is shown in Fig. 4 that the maximum \( \tau_{re} \) is very close to the minimum value of \( c_{tp} \), especially when the minimum pressure points are located slightly downstream the leading edge. The chain line shows \( c_{tp} \), i.e. the pressure coefficient on the isolated airfoil with same lift coefficient as \( c_{L0} \) in Table 6. The absolute value of \( c_{tp} \) is compared with \( \tau_{re} \). It is evident that \( c_{tp} \) is higher in absolute value than the experimental cavitation parameter.

In the light of these considerations we may conclude that our method gives reasonable estimates for the cavitation limit.

6. Estimation of suction performance using performance characteristics

Utilizing the view point expressed in 5-3 and assuming that the cavitation parameter equals to the total pressure coefficient (with minus sign), from Eq. (14) we obtain

\[
\tau = -c_{tp} = \left( \frac{p_{11} - p_{10}}{\gamma} \right) \left( \frac{u^2}{2g} \right) = \left( S_{t}^{1/2} - 1 \right) \left( \frac{\phi_2^2}{\phi_1} + \left( 1 - \frac{\phi_2}{4} \right) \right) \left( \frac{1}{2} \right)^{1/2} + \left( \frac{\phi_2^2}{\phi_1} + \left( 1 - \frac{\phi_2}{2} \right) \right) \left( \frac{1}{2} \right)^{1/2} - 1 \]

(Fig. 5) shows the cavitation parameter calculated using Eq. (16) for various values of \( \phi_2 \) and \( \phi_1 \). In actual design, as \( \phi_2 \) is related to \( S_{t}^{*} \) through the lift coefficient, the designer has to iteratively calculate values of \( S_{t}^{*} \) and \( l/l \) until he obtains the required cavitation parameter \( \tau \). Ordinarily \( S_{t}^{*} \) ranges from 1.2 to 1.4 at the tip section and from 1.5 to 1.8 at the root section. The maximum value of \( S_{t}^{*} \) appears at \( z/l = 0.2 \) to 0.3 for an airfoil which has the maximum thickness on the upstream side of the chord, and at \( z/l = 0.5 \) to 0.6 for a laminar flow airfoil. The \( S_{t}^{*} \) increases steeply with an increasing angle of attack, while for the airfoil in a cascade, the maximum \( S_{t}^{*} \) becomes smaller than the maximum \( S_{t}^{*} \), and the position of minimum pressure shifts upstream due to the
The retarding effect \((w_{0\theta} - w_{\infty})/w_{\infty}\) if the distributions of \(S_{i}^*\) and \((w_{0\theta} - w_{\infty})/w_{\infty}\) are decided, we can determine the distribution of \(S_{i}^*\), thus obtaining the cavitation parameter. Using the maximum absolute value of \(S_{i}^*\) in Eq. (16), the maximum value of \(\tau\) becomes the estimated cavitation inception coefficient.

The manometric efficiency \(H/H_{0}\) is one of the important characteristics in the design of the turbomachine. T. Ikui has described the following relation\(^{(15)}\), for the non-prerotation type.

\[
\eta_{M_{b}e} = 1 - \varepsilon \left( \frac{\phi_{r}^2 + (1 - \phi_{r}/4)^2}{\phi_{r}^2 + (1 - \phi_{r}/4)^2} + \mu \frac{\phi_{r}^2 + \phi_{r}^*}{16} \right)
\]

(17)

The manometric efficiencies \(\eta_{M_{b}e}\) and \(\eta_{M_{b}e}/\tau\) are shown in Fig. 6 as functions of the flow coefficient. Figure 6 is an example for the case of \(S_{i}^*=1.2\), and \(z/l=0.3\), but we can expect the same tendency from other conditions by reasoning through analogy.

It is recognizable that the maximum value of \(\eta_{M_{b}e}/\tau\) is obtained at a rather small value of \(\phi_{e}\), which is the recommended value for pump impellers that require both superior suction performance and high efficiency. For example, \(\phi_{e}\) at maximum \(\eta_{M_{b}e}/\tau\) is near 0.1 when \(S_{i}^*=1.2\).

7. Conclusions

A simple method is proposed to estimate the surface pressure on a cascade airfoil in the retarded flow. The method is compared with theoretical and experimental results, and is discussed in detail.

It is verified that the method is applicable to the design of axial-flow turbomachinedes when optimization of suction performance and efficiency is desired.

The main results are as follows:

1. The surface pressure on an airfoil in cascade in retarded flow can be estimated approximately by modifying isolated airfoil data.

2. Applicability of the method for the design of axial-flow turbomachines is certified by comparison with the two-dimensional wind tunnel test and theoretical results.

3. The estimated pressure coefficient agrees well with the cavitation zone parameter of the test pump impellers, especially of the impeller with superior suction performance.

4. A rather small flow coefficient is recommended for designing turbomachines when optimization of both suction performance and efficiency is desired.

The method may be applicable to estimation of the choking limit of compressors.

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