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Natural Convection Heat Transfer from Vertical Rectangular-Fin Arrays*

(Part 4, Heat-Transfer Characteristics of Nonisothermal-Fin Arrays)

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A method is presented for predicting the characteristics of practical nonisothermal-fin arrays from the results of the previous reports on isothermal ones. According to this method, the effects of the fin-geometry, thermal conductivity, and temperatures on the heat-transfer characteristics have been quantitatively clarified. A heat-transfer experiment on a fin array with the fin efficiency of about 80% and the hemispherical emissivity of 0.67 has verified the validity of the present method of prediction.

1. Introduction

In practical fin-array design, thin fins are usually adopted so as to dissipate the greatest amount of heat with the smallest weight, as far as their mechanical strength is not lost; accordingly, the ordinary isothermal condition of heat-transfer surfaces can be hardly attained. Since the heat-transfer characteristics of such nonisothermal-fin arrays are a complicated, three-dimensional problem depending on nine or more factors, studies of predicting those characteristics can be found in only a few papers; e.g., by Bartas and Shinn⁽¹⁾ and the author⁽²⁾. In the former of these reports, the heat transfer coefficient of vertical rectangular ducts is utilized approximately as a substitute for that of fin arrays and the fin efficiency is obtained subject to a constant heat transfer coefficient; in the latter, the heat transfer coefficient of vertical parallel plates is utilized approximately and the fin efficiency is obtained taking the temperature dependency of heat transfer coefficient into consideration. Besides in both reports, the heat transfer from fin-edges is neglected. The validity of these approximations has never been verified quantitatively.

The present study has been carried out in

order to obtain a reasonable method for predicting the characteristics of nonisothermal-fin arrays with isothermal base-plates; it is developed by utilizing the natural-convection data on isothermal-fin arrays obtained in the author's previous reports^{(3)~(5)}. The fin efficiency is calculated taking account of the heat transfer from fin-edges and the temperature dependency of heat transfer coefficient, peculiar to natural convection. The numerical results obtained by the present method are presented in the form of design-charts. Furthermore, an experiment has been made on a fin array with the fin efficiency of about 80%; and the validity of the present method is verified from a comparison between the experimental values and the calculated ones.

2. Nomenclature

B : dimensionless parameter defined by Eq. (23)

C : dimensionless parameter defined by Eq. (20)

$D^* = \delta/H$

E : fin effectiveness defined by Eq. (13)

Gr : Grashof number based on $(\theta_0 - \theta_\infty)$

H : vertical height of fin

K : dimensionless thermal conductivity, λ_f/λ_∞

L : horizontal length of fin

$L^* = L/H$

N_u : average Nusselt number based on $\alpha_{(0)}$

* Received 3rd July, 1969.

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and λ_∞

P_r : Prandtl number

Q : rate of heat transfer

R_{aH} : Rayleigh number, $P_r G_{rH}$

R_{as}^* , $R_{a\delta}^*$: modified Rayleigh number, $P_r G_{rs} S/H$
and $P_r G_{r\delta} \delta/H$ respectively

S : fin-spacing

$S^* = S/H$

T : absolute temperature °K

x, z : Cartesian coordinates, see Fig. 1

$Z = z/L$

α : average heat transfer coefficient

δ : fin thickness

η : fin efficiency defined by Eq. (16)

θ : temperature °C

λ : thermal conductivity

ϕ : dimensionless temperature, $(\theta_z - \theta_\infty)/(\theta_0 - \theta_\infty)$

Subscripts

b : base plate

f : fin

H : dimensionless group is evaluated using H as the characteristic length

l : lower edge of fin

opt: optimum value

s : evaluated using S as the characteristic length

t : total value

u : upper edge of fin

v : vertical edge of fin

δ : evaluated using δ as the characteristic length

0: evaluated at a position of $Z=0$

1: at $Z=1$

Z : at $Z=Z$

∞ : ambient condition or condition of no extended surface, i.e., $S \rightarrow \infty$

Superscript

—: average value

In the present paper, all fluid properties are evaluated at the base-plate temperature.

3. Mathematical formulation

3.1 Assumptions

In fin arrays and heat dissipators for practical use⁽⁶⁾, fairly thick base-plates are generally adopted according to convenience of the production and the usage; hence the distribution of base-plate temperature tends to become uniform. From this standpoint, fin arrays with a vertical isothermal base-plate, as one of the basic types, will be treated in this report†. First, the following simplifying assumptions are made:

(1) The vertical distribution of fin-temperature is much more uniform than the horizontal one, and consequently the heat conduction in a fin can be treated approximately as a quasi-one-dimensional problem.

(2) The effect of radiative heat transfer on the fin-temperature distribution is negligibly small. According to the experiments of the present paper and Bartas et al.⁽⁸⁾, it would be practically safe to consider that Assumption (1) holds good under the premise of the isothermal base-plate. Assumption (2) is made on the basis of the fact that for an optimum-designed fin array the radiative component is, in general, considerably small compared with the convective one⁽²⁾⁽⁵⁾; under this assumption, the radiative component becomes separable and the present problem can be treated more simply.

3.2 Fin effectiveness

The respective heat-transfer rates from the component surfaces of a fin array, shown in Fig. 1, are as follows: The rate of heat transfer, Q_∞ , from a naked flat surface without fins is expressed as

$$Q_\infty = nH(1 + D^*/S^*)N_{us(\infty)}\lambda_\infty(\theta_0 - \theta_\infty) \dots\dots(1)$$

where n is the number of fins, and $N_{us(\infty)}$ the average Nusselt number of a vertical flat plate⁽⁹⁾ represented by

$$N_{us(\infty)} = 0.57R_{as}^{*1/4} \dots\dots\dots(2)$$

The rate of heat transfer, Q_b , from the base

Fig. 1 Illustration of vertical rectangular-fin array

† The condition of perfectly uniform heat-flux can not be realized because of the heat conduction through the thick base-plate. However, even so, the results for the isothermal base-plate are approximately applicable provided the representative temperature at $x=H/2$ is adopted⁽⁷⁾.

plate of a fin array is expressed as

$$Q_b = nHN_{u_{s(b)}}\lambda_\infty(\theta_0 - \theta_\infty) \quad (3)$$

where $N_{u_{s(b)}}$ is the average Nusselt number of the base-plate⁽³⁾ represented by

$$N_{u_{s(b)}} = \frac{Ra_s^*}{70} [1 - e^{-(57/Ra_s^*)^{2/3}}] \times [1 - e^{-10(57/4Ra_s^*)^{1/4}}]^{1/3} \quad (4)$$

The rate of heat transfer, Q_f , from the fin-flats may be approximated on the basis of Assumption (1) by

$$Q_f = 2nHL \int_0^1 \alpha_{f(Z)} (\theta_{f(Z)} - \theta_\infty) dZ \quad (5)$$

where $\theta_{f(Z)}$ is the fin temperature at a position Z and $\alpha_{f(Z)}$ the heat transfer coefficient of the fin-flat to a temperature of $\theta_{f(Z)}$.

Similarly, the following expression of the rate of heat transfer, Q_l , from the lower fin-edges is obtained by using the heat transfer coefficient of the lower edge, $\alpha_{l(Z)}$, to a temperature of $\theta_{f(Z)}$:

$$Q_l = n\delta L \int_0^1 \alpha_{l(Z)} (\theta_{f(Z)} - \theta_\infty) dZ \quad (6)$$

Here $\alpha_{f(Z)}$ and $\alpha_{l(Z)}$ can be evaluated respectively from the following empirical formulas obtained in the second⁽⁴⁾ and third⁽⁵⁾ reports:

$$N_{u_{s(f)}} = \frac{Ra_s^*}{24} [1 - e^{-(32.7/Ra_s^*)^{3/4} f_1}] \quad (7)$$

with

$$f_1 = 1 - 1/L^* Ra_H^{1/4}$$

$$N_{u_{s(l)}} = 1.60 \left(\frac{D^*}{S^* + D^*} \right)^{0.75} (Re_s Pr)^{0.4} \quad (8)$$

with

$$Re_s = G_{rs} \bar{U}$$

$$\frac{1}{\bar{U}} = 12 + \{1 - e^{-(250/Ra_s^* f_2)^{0.6}}\} \frac{Ra_s^* f_2}{7}$$

$$f_2 = 1 + 1.8/L^{*0.88}$$

Since the rate of heat transfer, Q_u , from the upper fin-edges is extremely small in general as shown in Fig. 4, it would be no practical error to neglect it. For reference, it is written below

$$Q_u = nHL^* N_{u_{s(u)}} \lambda_\infty (\bar{\theta}_f - \theta_\infty) \quad (9)$$

where $N_{u_{s(u)}}$ is the average Nusselt number of the upper edge⁽⁴⁾ represented by

$$N_{u_{s(u)}} = \frac{Ra_s^* f_3}{30} [1 - e^{-(17/Ra_s^* f_3)^{3/4}}] \left(\frac{D^*}{S^*} \right) \left/ \left(1 + \frac{D^*}{S^*} \right)^{1.4} \right. \quad (10)$$

with

$$f_3 = 1 + 1/3L^*$$

The rate of heat transfer, Q_v , from the vertical fin-edges may be approximated using the temperature $\theta_{f(1)}$ at the fin-tip ($Z=1$) as follows:

$$Q_v = nHN_{u_{s(v,1)}} \lambda_\infty (\theta_{f(1)} - \theta_\infty) \quad (11)$$

The average Nusselt number $N_{u_{s(v,1)}}$ for the temperature $\theta_{f(1)}$ can be evaluated from the following empirical formula according to the second and third reports.

$$N_{u_{s(v,1)}} = 0.94 (Ra_s^* \phi_1)^{0.1} \times \left[1 + \left(\frac{D^*}{S^*} \right)^{2.5} \frac{Re_s'}{(Ra_s^* \phi_1)^{0.4}} \right] \quad (12)$$

with

$$Re_s' = 0.018 G_{rs} \phi_1 L^* \times [1 - e^{-50L^{*0.25}/(Ra_s^* \phi_1)^{0.5}}] f_4$$

$$f_4 = 1 - [1 - 0.61 (Ra_s^* \phi_1)^{0.15}] \times e^{-Ra_s^* \phi_1 L^{*0.5}}$$

Now, the fin effectiveness⁽¹⁰⁾ E will be defined as

$$E = (Q_f + Q_b + Q_l + Q_u + Q_v) / Q_\infty \quad (13)$$

Then, by denoting the rate of radiative heat transfer by Q_r , the total heat-transfer rate Q_t is represented by

$$Q_t = Q_r + Q_\infty E \quad (14)$$

This Q_r may be expressed as

$$Q_r = n(\delta\varepsilon + S\varepsilon_a)(H + 2L)\sigma(\bar{T}_f^4 - T_\infty^4) \quad (15)$$

where ε is the hemispherical emissivity of the heat transfer surfaces, ε_a the apparent emissivity⁽²⁾ of the open channel composing the fin array, σ the Stefan-Boltzmann constant. Further defining the fin efficiency η as

$$\eta = \frac{\int_0^1 (2H\alpha_{f(Z)} + \alpha_{l(Z)}\delta)(\theta_{f(Z)} - \theta_\infty) dZ}{(2H\alpha_{f(0)} + \alpha_{l(0)}\delta)(\theta_0 - \theta_\infty)} \quad (16)$$

the final expression for the fin effectiveness E becomes

$$E = [L^*\eta(2N_{u_{s(f)}}/S^* + N_{u_{s(l)}}) + N_{u_{s(b)}} + L^*\phi N_{u_{s(u)}} + \phi_{(1)} N_{u_{s(v)}}] / (1 + D^*/S^*) N_{u_{s(\infty)}} \quad (17)$$

By the way, since the factor $n(\delta\varepsilon + S\varepsilon_a)$ on the right-hand side of Eq. (15) is nearly constant in usual case, it would be safe to think that the performance of fin arrays depends primarily upon the fin effectiveness under Assumption (2). Therefore, this fin effectiveness alone will be discussed in the following sections.

3.3 Fin efficiency

Though many studies of the fin efficiency have been reported so far, the majority of them have been made under the assumption of constant heat transfer coefficient⁽¹²⁾. However, one often meets with practical cases where this assumption does not hold; accordingly, this assumption has been actively scrutinized in recent years^{(13)~(16)}.

In the present paper, an argument will be advanced by approximating the heat transfer coefficients $\alpha_{f(Z)}$ and $\alpha_{l(Z)}$ at a position Z by the following form with the dimensionless temperature ϕ :

† Details are described in reference (11).

$$\frac{2H\alpha_{f(z)} + \alpha_{l(z)}\delta}{2H\alpha_{f(0)} + \alpha_{l(0)}\delta} = \phi^m \quad (18)$$

Then, the differential equation for one-dimensional heat conduction in a fin may be expressed as

$$\frac{d^2\phi}{dZ^2} = 2C\phi^{1+m} \quad (19)$$

where

$$C = \left(N_{u_{s(f)}} + N_{u_{\delta(l)}} \frac{S^*}{2} \right) \frac{L^{*2}}{S^* D^* K} \quad (20)$$

and K is the dimensionless thermal conductivity, defined by

$$K = \lambda_f / \lambda_\infty \quad (21)$$

The boundary conditions become

$$\left. \begin{aligned} \phi &= 1 & \text{at } Z=0 \\ -(\delta\phi/dZ)_{Z=1} &= B\phi_{(1)} & \text{at } Z=1 \end{aligned} \right\} \quad (22)$$

with

$$B = N_{u_{\delta(v,1)}} L^* / D^* K \quad (23)$$

The fin efficiency η and the dimensionless average temperature of fin $\bar{\phi}$ are evaluated respectively according to the following equations.

$$\eta = \int_0^1 \phi^{1+m} dZ \quad (24)$$

$$m = r + (1-2p) \left[1 - \frac{0.76S^*N_{u_{\delta(l)}} + 0.57e^{-(32.7/R_{a_s})^{3/4}f_1(1.5R_{a_s}^{*1/4} - 2S^*/L^*)}}{2N_{u_{s(f)}} + N_{u_{\delta(l)}}S^*} \right] \quad (28)$$

3.4 Procedure of computation

First, the fin efficiency η , dimensionless average temperature $\bar{\phi}$, and dimensionless temperature of fin-tip $\phi_{(1)}$ are computed for various values of parameters B , C , m . In this case, the value of ϕ is numerically solved by a finite difference method† and those of η and $\bar{\phi}$ are computed by Simpson's rule, with a step-size of 0.05 on a digital computer; it has been confirmed by comparison with the analytical solution for $B=0$ and $m=0$ that the accuracy of the present computations is satisfactory. From those computed values, the respective approximate expressions are derived within an error of about 1%.

Then, the values of fin effectiveness E are computed for numerous combinations (about 3 600 sets)

of the independent dimensionless parameters R_{a_H} , S^* , L^* , D^* , K by the use of those approximate expressions. In the computation, the following approximate values (applicable for air of 10 to 30°C) are adopted as the exponents r and p in Eq. (26):

Fig. 2 Fin efficiency η

† The numerical solutions of the differential Eq. (19) can be obtained, in a very short computing-time, as follows: First, Eq. (19) is solved with an appropriate temperature gradient at $Z=0$; then, from the obtained temperature distribution, the value of B is computed reversely according to the second expression of Eq. (22).

$$\bar{\phi} = \int_0^1 \phi dZ \quad (25)$$

By the way, the exponent m in Eq. (18) is determined as follows: First, the physical properties are approximated as

$$\left. \begin{aligned} \lambda_{\infty(z)} / \lambda_{\infty} &\doteq \phi^r \\ \nu_{(z)} / \nu &\doteq \phi^p \end{aligned} \right\} \quad (26)$$

Then, the left-hand side of Eq. (18) can be expressed by

$$\phi^r \left[\frac{2N_{u_{s(f,z)}} + N_{u_{\delta(l,z)}} S^*}{2N_{u_{s(f)}} + N_{u_{\delta(l)}} S^*} \right]$$

This group will be denoted by ξ . Here $N_{u_{s(f,z)}}$ and $N_{u_{\delta(l,z)}}$ are the average Nusselt numbers according to Eqs. (7) and (8) respectively, in which dimensionless groups $R_{a_s}^* \phi^{1-2p}$, $R_{a_\delta}^* \phi^{1-2p}$, $G_{r_s} \phi^{1-2p}$ are used instead of $R_{a_s}^*$, $R_{a_\delta}^*$, G_{r_s} . Thus, the exponent m applicable near $\phi=1$ can be obtained by putting

$$m = (\partial \xi / \partial \phi)_{\phi=1} \quad (27)$$

In the present case, it becomes

Fig. 3 Dimensionless average fin-temperature $\bar{\phi}$

$$\left. \begin{array}{l} r=0.08 \\ r=0.168 \end{array} \right\} \dots\dots\dots(29)$$

4. Results and discussion

4.1 Fin efficiency

Some of the fin efficiency η and the dimensionless average temperature $\bar{\phi}$ obtained in the above-mentioned way are shown in Figs. 2 and 3. It may be seen in the figures that both η and $\bar{\phi}$ decrease as the dimensionless parameters B and C increase and that the variational tendency of η with the exponent m is opposite to that of $\bar{\phi}$ with m . Even in case of no heat transfer from the fin-flat and the lower edge (i.e., $C=0$), η and $\bar{\phi}$ do not become unity because of the following relations.

$$\eta = \frac{(1+B)^{2+m}-1}{(2+m)B(1+B)^{1+m}} \dots\dots\dots(30)$$

$$\bar{\phi} = \frac{2+B}{2(1+B)} \dots\dots\dots(31)$$

4.2 General characteristics

Figure 4 shows the relation of the fin effectiveness E to the dimensionless fin-spacing S^* for $K=1.75 \times 10^3$, corresponding to a case of mild steel; it may be seen that the fin effectiveness depends largely on S^* . The curves E_f , E_b , E_v , E_l , and E_u in the figure represent the respective effectivenesses, defined in the same manner as Eq. (13), for the heat transfer from the fin-flats, base-plate, vertical edges, lower edges and upper edges. In this case, the working empirical formulas are extrapolated outside their applicable range so that the heat transfer coefficient of the vertical edge at $S^*=0$ may agree with that of a vertical flat plate and so that the heat transfer coefficients of the lower and upper edges at $S^*=0$ may agree with those of the lower and upper

Fig. 4 Typical relation of fin effectiveness E and others to dimensionless fin-spacing S^*

sides of a horizontal plate respectively. These extrapolations are indicated by dashed lines. Although Fig. 4 is a mere illustrative example, it suggests that neglect of heat transfer from other parts except the fin-flat may lead to a noticeable error.

Figure 5 shows the effect of the dimensionless thermal conductivity K on the fin effectiveness.

Figure 6 illustrates the effect of the dimensionless fin-thickness D^* on the fin effectiveness; it may be seen from the figure that the fin effectiveness has a local maximum value E_{opt} for a particular dimensionless fin-thickness and that the corresponding optimum fin-spacing S_{opt}^* ($=S_{opt}/H$) also depends upon D^* .

4.3 Optimum-design point

Since heat dissipators of this type are usually employed as parts attached to some

Fig. 5 Effect of dimensionless thermal conductivity K on fin effectiveness E

Fig. 6 Effect of dimensionless fin-thickness D^* on fin effectiveness E

equipment, their size, weight, and cost are determined so as to be in proper proportion to those of the equipment. Besides, the fin-thickness δ is limited by an industrial standard for size, the manufacturing process, and the mechanical strength of fin. It is, therefore, improbable that there exists the only combination of the optimum dimensions which is universally applicable to every case. Furthermore, the size of heat dissipators of natural-convection-type is apt to be larger than that of forced-convection-type. Consequently, compact-design is usually

preferred rather than economical-one. The following discussions on the optimum-design are made from this standpoint.

Figure 7 presents the relation of the local maximum of fin effectiveness E_{opt} to the dimensionless fin-thickness D^* for $K=4.17 \times 10^3$ corresponding to a case of brass. From the

Table 1 First approximation of the optimum fin-spacing

Ra_H	10^5	8×10^5	6.4×10^6	5.12×10^7	4.10×10^8
$S_{opt(1st)}^*$	0.1472	0.0875	0.0520	0.0309	0.0184

Fig. 7 Local maximum value of fin effectiveness E_{opt} ($K=4.17 \times 10^3$)

Fig. 8 Optimum fin-spacing S_{opt}^* ($K=4.17 \times 10^3$)

Fig. 9 Dimensionless average fin-temperature, $\bar{\phi}$, for the optimum fin-spacing ($K=4.17 \times 10^3$)

figure, one may see the aspect that the value of D^* giving the maximum fin-effectiveness $E_{\max}$ depends on the Rayleigh number Ra_H and the dimensionless fin-length L^* . The values of the optimum fin-spacing S_{opt}^* giving E_{opt} are plotted in Fig. 8; the dependency of S_{opt}^* on D^* and L^* is relatively slight.

By the way, the author has previously recommended the following equation for the first approximation, $S_{\text{opt}(1st)}^*$, of the optimum fin-spacing⁽²⁾:

$$S_{\text{opt}(1st)}^* = (47/Ra_H)^{1/4} \dots \dots \dots (32)$$

Table 1 presents a sample evaluation of $S_{\text{opt}(1st)}^*$ according to this equation; from a comparison with the exact values in Fig. 8, it may be seen that Eq. (32) gives a considerably better approximation of S_{opt}^* . For convenience of the evaluation of radiative heat-transfer rate, the dimensionless average temperatures of fin $\bar{\phi}$ are also plotted in Fig. 9. Figure 10 is a plot of the

Fig. 10 Maximum fin effectiveness $E_{\max}$

Fig. 11 Local maximum value of fin effectiveness E_{opt} for $L^*=0.4$

maximum fin-effectiveness $E_{\max}$; since the fin efficiency decreases generally as L^* and Ra_H are increased, the rate of increase of $E_{\max}$ also tends to decrease gradually. It will be added that the dashed lines in Figs. 7 through 10 indicate the region of $\phi_{(1)} < 0.5$ where the computed results are inapplicable in view of the accuracy of the present method, and that the value of $E_{\max}$ is unity at $L^*=0$ independently of Ra_H . Figures 11 through 15 illustrate the respective computed results for $K=1.75 \times 10^3$ and 1.25×10^4 (corresponding to a case of copper), and $L^*=0.4$; the similar tendencies to the above are observed.

On the basis of the above discussions, it is generally recommended to make Ra_H lower and

Fig. 12 Optimum fin-spacing S_{opt}^* for $L^*=0.4$

Fig. 13 Dimensionless average fin-temperature, $\bar{\phi}$, for the optimum fin-spacing ($L^*=0.4$)

Fig. 14 Maximum fin effectiveness $E_{\max}$

L^* larger for the compact-design of heat dissipators. On the other hand, in case where the values of R_{aH} and K are given: first, the fin-length L^* which makes a heat dissipator most compact can be easily obtained from the proper $E_{\max}-L^*$ chart; and then, it is advantageous to make the fin-thickness D^* smaller unless the performance is deteriorated; finally, the optimum fin-spacing S_{opt}^* is obtained from the proper $S_{\text{opt}}^*-D^*$ chart.

By the way, Welling et al.⁽¹⁷⁾, reporting on their experimental study of similar fin-arrays, say that there is an optimum fin-length for any fin-spacing, above or below which there is a reduction of the fin effectiveness. However, on the basis of the results of this series, in particular Figs. 10, 14, 15 of the present report, it is self-evident that the fin effectiveness E increases monotonically with the fin-length L^* and consequently such an optimum fin-length cannot exist.

Finally, it will be added that the exponent m in Eq. (18) is of the order of 0.3 to 0.6 in the above computations.

5. Experiment

5.1 Test fin-array and experimental procedure

In order to verify the present method of prediction, an experiment has been made on a fin array with the fin efficiency of about 80% and the isothermal base-plate. The test fin-array was made of brass and was composed of a test-section and two guard-sections; the total number of fins in the test-section was twelve on both sides of the base-plate. The measured principal dimensions of it are as follows:

Fig. 15 Maximum fin effectiveness $E_{\max}$

$$H = 140.0 \text{ mm}$$

$$L = 100.0 \text{ , i.e., } L^* = 0.714$$

$$S = 6.9 \text{ , } S^* = 0.0493$$

$$\delta = 1.0 \text{ , } D^* = 0.00714$$

The whole exposed surface of the test fin-array was uniformly anodized, and its hemispherical emissivity was 0.67 according to the measurement by the method described in the first report⁽⁸⁾. The fin-temperatures were measured with 0.1-mm-diam. copper-constantan thermocouples soft-soldered to a fin of the test-section at the following nine locations:

$$x = 5 \text{ mm and } z = 5, 50, 95 \text{ mm}$$

$$x = 60 \text{ mm and } z = 5, 50, 95 \text{ mm}$$

$$x = 135 \text{ mm and } z = 5, 50, 95 \text{ mm}$$

The other details of the construction and experimental procedure are omitted here, because they are the same as those described in the third report⁽⁵⁾.

5.2 Comparison with calculated values

Figure 16 presents a typical distribution of measured fin-temperatures. It may be seen from this figure that the primary temperature-reduction arises in the horizontal direction and consequently Assumption (1) in section 3.1 holds approximately. The average fin-temperatures $\bar{\theta}_f$ obtained from these distribution curves are plotted in Fig. 17 (a) in the non-dimensionalized form, with the calculated values (solid curve). Although the calculated values are slightly greater than the measured ones, the difference is only 3% at most. Figure 17 (b) shows the relation of the total heat-transfer rate Q_t to the temperature-difference $(\bar{\theta}_0 - \theta_\infty)$; the calculated values

Fig. 16 Typical fin-temperature distributions of a test fin-array

according to Eq. (14) agree fairly well with the measured ones. From this, the validity of the present method has been verified.

By the way, the predicted fin effectiveness according to Bartas and Shinn's method⁽¹⁾ stated in Introduction is usually apt to be greater than the true one in the case of low fin efficiency; e.g., in the case of $K=1.75 \times 10^3$, $R_{aH}=5.12 \times 10^7$, $D^*=0.026$, $L^*=1.0$, and $S^*=0.034$,

according to the author's method: $E=16.9$

according to Bartas and Shinn's method: $E=18.6$ Besides in consideration of the heat transfer from fin-edges neglected in their method and of the heat transfer characteristics of Fig. 13 of the third report⁽⁵⁾, it is self-evident that the predicted values according to their method are contrariwise smaller than the true ones near the optimum-design point in the case of thick fins.

6. Conclusions

The results obtained in the present study are summarized as follows:

(1) The fin-temperature ϕ , its average value $\bar{\phi}$, and fin efficiency η are obtained by numerically solving the one-dimensional heat conduction equation on the assumption that the heat transfer

(a)

(b)

Fig. 17 Relations of dimensionless average fin-temperature $\bar{\phi}$ and total heat-transfer rate Q_t to temperature difference $(\bar{\theta}_0 - \theta_\infty)$ (as for the thermal conductivity of brass, the recommended value of Eckert et al.⁽¹⁸⁾ is used.)

coefficients for the fin-flat and lower fin-edge are directly proportional to ϕ^m and heat is transferred from the vertical fin-edge, or fin-tip too. Generally, η decreases with an increase in the exponent m and in the respective dimensionless parameters for the fin-flat, lower fin-edge, and vertical fin-edge. On the other hand, $\bar{\phi}$ increases slightly with m , but decreases with an increase in the other parameters as well as η .

(2) The heat-transfer characteristics of fin arrays depend primarily upon the fin effectiveness E , which is most remarkably affected by the dimensionless fin-spacing $S^*(=S/H)$. The optimum fin-spacing S_{opt}^* giving the local maximum of fin effectiveness E_{opt} depends primarily upon the Rayleigh number R_{aH} , and the value of S_{opt}^* increases as R_{aH} decreases.

(3) In general, E_{opt} increases monotonically with the dimensionless fin-length $L^*(=L/H)$, dimensionless thermal conductivity $K(=\lambda_f/\lambda_\infty)$ and R_{aH} . On the other hand, the fin-thickness δ is closely connected with the fin efficiency and the heat transfer from the fin-edges; consequently, with an increase in the dimensionless fin-thickness $D^*(=\delta/H)$, E_{opt} first increases rapidly, attains a maximum, and then decreases gradually due to choking by the bulk of fins.

(4) In order to verify the present method, an experiment has been made on a fin array with the fin efficiency of about 80%; good agreement between theoretical prediction and experiment has been obtained. Furthermore, it is pointed out that it will be hardly possible by Bartas and Shinn's method to obtain the exact fin effectiveness in the case of either thick or thin fins.

Acknowledgment

The author wishes to express his gratitude to Prof. Emeritus T. Tsubouchi, Tōhoku University, for his guidance and encouragement during this work. He also wishes to express his thanks to Prof. S. Shimodaira, Research Institute for Iron, Steel and Other Metals of Tōhoku University, who has anodized the test fin-array, and to Miss S. Chiba, assistant official of the Institute, for her assistance in computations and execution of the experiment. The numerical computations have been carried out on the Digital Computer NEAC-2230 in the Computation Center, Tōhoku University.

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Discussion

Editorial Director:

(1) On what experimental ground is the assumption of an isothermal base-plate based? How does the fact that calculated values agree with experimental ones within an error of 2% relate to the results of the author's previous reports on isothermal-fin arrays?

(2) Has the validity of Assumption (1), one-dimensional distribution of fin temperature, been verified? Explain the variational tendency of $\theta_f - \theta_\infty$ with x in Fig. 16. On the basis of the facts that the effect of S/H is seen in Fig. 4 and parameter H is included in B and C , the discussor supposes that there are some limitations on L/H -value. How does the author think about it?

(3) It is seen in Figs. 10, 14 and 15 that the maximum fin-effectiveness $E_{\max}$ increases monotonically with L for a particular H -value. Can this tendency be still expected in case of $L/H > 1.0$? The discussor supposes that the assumption of isothermal base-plate does not hold for $L/H > 1.0$ and furthermore that Assumption (1) does not hold for a large H -value even in case of $L/H < 1.0$. How does the author think about these points?

Author's closure

(1) It seems to the author that although base-plates of various temperature conditions are adopted in fin arrays[†] for practical use, the results obtained for isothermal base-plates are approximately applicable by the use of a proper average base-temperature. For example, the base-plate of uniform heat-flux cannot be realized; however, even so, the results for the isothermal base-plate are approximately applicable provided

the base-temperature at $x=H/2$ is adopted according to the experimental result of Sobel et al.⁽⁷⁾ and theoretical one of Engel et al. for parallel-plates with uniform heat flux. From this standpoint, the isothermal base-plate which gives the most basic temperature condition among base-plates for practical use and is easy to analyze has been treated in this report. However, for fear that one may feel such a doubt as to be pointed out, some expressions of the relevant part will be rewritten.

As for the latter half of Question (1), such good agreement means that one can predict the performance of nonisothermal-fin arrays within a practically slight error by the use of the results for isothermal-fin arrays as the present method.

(2) (i) Strictly speaking, the assumption of one-dimensional distribution of fin temperature does not hold. However, on the basis of the present experiment (Fig. 16), of the discussion^{††} on Bartas and Shinn's experiment and of the comparison of the average temperatures in Fig. 17(a), it would be practically safe to make this assumption for the purpose of discussing the final performance subject to the isothermal base-plate.

(ii) The temperature gradient in the x -direction increases with the distance, z , from the base-plate in the range of small z ($z < 0.5L$ in the case of Fig. 16); however, in the range of large z , the temperature gradient hardly increases or, in some cases⁽⁵⁾, decreases with an increase in z .

(3) (i) Even if $L/H > 1$, the computed

[†] For example, see p. 1203 of reference (6).

^{††} Author's closure in discussion of reference (2).

value of $E_{\max}$ according to Eq. (17) seems to increase monotonically to an asymptotic value of $L^* \rightarrow \infty$. For reference, the author will add that the accuracy of the present method of prediction is quantitatively unwarranted for such a large L as makes $\phi_{(1)} < 0.5$, and furthermore that fins of $L^* > 1$ are seldom adopted from the standpoint of the mechanical strength for practical use.

(ii) On the basis of Closure (1), the isothermal condition of base-plate has been struck out from the assumptions and taken up as a premise. Further in the author's opinion, the isothermal condition of base-plate seems to

again become realizable in the case of an extremely large L^* -value. Closure (2) (ii) also supports this opinion.

(iii) Since the isothermal condition of base-plate is taken up as a premise, Assumption (1) holds for $L^* \ll 1$ without reservation. As for the intermediate regime of $L/H \doteq 1$, please see Closure (2). By the way, since the laminar natural convection data on isothermal-fin arrays obtained in the author's previous reports are utilized in the present report, the upper limit of H -value is of the order of $R_{aH} \doteq 4 \times 10^8$ as shown in Figs. 7 through 15.