Axial Impact of Low Carbon Mild Steel Rod*

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Behavior of viscoplastic material under dynamic load was considered for two cases; that is, a longitudinal impact of a rigid mass on a finite rod and a normal impact of a finite rod on a rigid wall. Constitutive equation of Bingham type was modified by taking strain-hardening effect into account. As governing equations of the problems were of nonlinear type with moving boundaries, the difference method was employed to solve them. Numerical results for distributions of particle velocities and of strains and their variations with respect to time differed even qualitatively from the results obtained by a theory without strain-hardening. Experiments on a soft mild steel containing 0.015% carbon were conducted. It was found that the numerical results and experiments for permanent strain distribution agreed well. Variations of strain distribution with respect to time during impact were observed by a high speed camera and numerical results were found to predict the behavior qualitatively.

1. Introduction

Behavior of a finite rod of strain-rate dependent material under dynamic axial load was analysed by Ting and Symonds using a Bingham model. This work was motivated rather to understand the essential features of a Bingham body under high-speed impact than to correlate the results with experiments. Linearizing the constitutive equation, they obtained analytical solutions to axial impact problems of semifinite and finite rods under various boundary conditions and showed variations of distributions of particle velocity, strain and stress with time and development of unloading boundary(1). Employing a non-linear constitutive equation, Ting investigated a free flight impact problem of a finite rod by making use of the method of Karman-Pohlhausen and obtained an approximate solution(2).

We have executed experiments of finite mass impact and free flight impact using a low carbon steel rod (0.015% Carbon) which is considered as a highly strain-rate dependent material and the final plastic strain distributions obtained are found to deviate somehow even qualitatively from the theoretical results of Ting-Symonds. Therefore, we take the effect of strain-hardening into account in the constitutive equation and the governing equations are solved numerically by difference scheme under appropriate boundary conditions. The numerical results thus obtained agree fairly well with the experimental results. As the final strain distributions of uniaxial impact based on one-dimensional theory with and without strain-rate dependency are found to give almost the same configuration(3), one may pretend a superiority of either theory in order to explain experimental results because both the theories neglect second order effects such as frictional constraint and lateral inertia. So we observe the variations of strain distribution with time during high-speed deformation by use of a high-speed camera having a capacity of 200,000 frames per second. According to the result, the processes of deformation of the material used in this experiment agree qualitatively with those obtained from the rate-dependent theory.

2. Theory

This paper treats impact problems of the types indicated in Fig. 1. A rigid body of mass $G$ strikes

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axially a cylindrical rod specimen of finite length with initial velocity \( V_0 \) as shown in Fig. 1(a). We call this type of problem "problem I", hereafter. Another type of problem, called "problem II" is a normal impact on a rigid wall by a cylindrical rod specimen of finite length with velocity \( V_0 \) as shown in Fig 1(b). Neglecting lateral inertia and lateral end constraint due to friction, we treat the problems as one-dimensional. As both types of problems differ only with regard to their boundary conditions, we make the discussions in parallel. Take the Lagrangian coordinate \( X \) along axis of the rod with origin \( X=0 \) at the struck end and \( X=L \) at the other end. \( V(X, T), \varepsilon(X, T) \) and \( \sigma(X, T) \) denote the particle velocity, nominal compressive strain and nominal compressive stress, respectively, where \( T \) is time. Uniform cross sectional area and density of the rod before deformation are denoted by \( A \) and \( \rho \). The equation of motion for an element of the rod is
\[
\frac{\partial \sigma}{\partial X} = -\rho \frac{\partial V}{\partial T}
\] (1)

The compatibility condition is
\[
\frac{\partial \varepsilon}{\partial T} = \frac{\partial V}{\partial X}
\] (2)

For a Bingham model, the rate of plastic flow is governed by the excess of stress \( \sigma - \sigma_0 \) over the static yield stress \( \sigma_0 \) required to initiate plastic flow, the model is rigid below the static yield stress and the strain-hardening is neglected. In this paper, however, the effect of strain-hardening is taken into account by a linear stress-strain curve the inclination of which is same for each strain-rate. Thus, the constitutive equation used here is
\[
\begin{align*}
\frac{\partial \varepsilon}{\partial T} &= D \left( \frac{\sigma - \sigma_0}{\sigma_0} - 1 \right) \varepsilon \quad \text{for } \sigma > \sigma_0 \\
\frac{\partial \varepsilon}{\partial T} &= 0 \quad \text{for } \sigma \leq \sigma_0
\end{align*}
\] (3)

where \( D, P \) and \( H \) are the material constants.

Initial and boundary conditions for problem I are
\[
\begin{align*}
V(0, 0) &= V_0 \\
V(X, 0) &= \varepsilon(X, 0) = 0 \\
\sigma(X, 0) &= \sigma_0 \\
G \delta V(0, T) / \delta T &= -A\sigma(0, T)
\end{align*}
\] (4)

\[
V(L, T) = 0
\] (5)

When an interface between plastically deforming region and unloaded region which starts from \( X=0 \) is at \( X=\zeta L \), the following equation takes the place of Eq. (5):
\[
(G + \rho \zeta L) \delta V(\zeta L, T) / \delta T = -A\sigma(\zeta L, T)
\] (6)

Conditions for problem II are
\[
\begin{align*}
V(X, 0) &= -V_0 \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (8) \\
V(0, T) &= 0 \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (9) \\
\rho(L(1-\zeta) \delta V(\zeta L, T) / \delta T &= \sigma(\zeta L, T) = \sigma_0 \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (10)
\end{align*}
\]

where \( X=\zeta L \) is an interface between plastically deforming region \( (0 \leq X < \zeta L) \) and rigid undeformed region \( (\zeta L < X \leq L) \) and the function \( \zeta(T) \) with \( \zeta(0)=0 \) is an unknown time dependent function which must be determined with the solution of the problem.

After these governing equations are written in nondimensional quantities, the difference equation schemes are to be formulated to obtain numerical results. The convergency of the difference scheme and instability due to round-off error are unknown beforehand, because these problems with floating boundaries are non-linear. Thus, computer programs used are checked for the cases to which Ting and Symonds gave analytical solutions. Otherwise, energy balance of the system is checked for general cases. For problem I, the initial kinetic energy of the rigid mass \((1/2)GV_0^2\) is partitioned during deformation by the kinetic energy of the mass \((1/2)G(V(0, T))^2\), kinetic energy due to particle velocity of the specimen \((1/2)\rho A \int_0^L V(X, T)^2 dX\) and energy of plastic deformation \(\int_0^L \sigma(X, T) \delta \varepsilon(X, T) / \delta T dX dT\) and finally is transformed entirely to energy of plastic deformation. Thus we have the following equation for energy balance.
\[
\begin{align*}
\frac{1}{2} GV_0^2 &= \frac{1}{2} G(V(0, T))^2 + \frac{1}{2} \rho A \int_0^L (V(X, T))^2 dX \\
&+ \int_0^L \int_0^\zeta \sigma(X, T) \delta \varepsilon(X, T) / \delta T dX dT
\end{align*}
\] (11)

Equation (11) remains valid even if the unloading boundary advances to \( X=\zeta L \). For problem II, kinetic energy of the specimen before impact is \((1/2)\rho ALV_0^2\) and at any instant after impact the total energy is composed of the kinetic energy of rigid region \((1/2)\rho AL(1-\zeta^2)(V(\zeta L, T))^2\), the kinetic energy of deforming region \((1/2)\rho A^{\zeta L} (V(X, T))^2 dX dX\) and the energy of plastic deformation \(\int_0^\zeta \int_0^\zeta \sigma(X, T) \delta \varepsilon(X, T) / \delta T dX dT\). Energy balance equation for this case is
\[
\begin{align*}
\frac{1}{2} \rho ALV_0^2 &= \frac{1}{2} \rho AL(1-\zeta^2)(V(\zeta L, T))^2 \\
&+ \frac{1}{2} \int_0^\zeta \int_0^\zeta (V(X, T))^2 dX dX \\
&+ \int_0^\zeta \int_0^\zeta \sigma(X, T) \delta \varepsilon(X, T) / \delta T dX dT
\end{align*}
\] (12)
Now we use nondimensional quantities after Ting and Symonds as follows:
\[ x = \frac{X}{L}, \quad t = \frac{a^2}{\rho \overline{D} L^3}, \quad k = \frac{G}{\rho \overline{A} L}, \]
\[ c_t^2 = \frac{\rho \overline{D} \overline{L}^2}{a^2}, \quad \gamma(x, t) = \frac{a \alpha x}{\rho \overline{D} \overline{L}^2}, \quad s(x, t) = \frac{a}{a_0} \]
for problem 1 and
\[ x = \frac{X}{L}, \quad t = \frac{a^2}{\rho \overline{L} \overline{V}_0}, \quad \alpha = \frac{V_0}{\overline{D} \overline{L}^2}, \]
\[ c_t^2 = \frac{\rho \overline{L} \overline{V}_0^2}{2a_0}, \quad \gamma(x, t) = \frac{a \alpha x}{\rho \overline{L} \overline{V}_0^2}, \quad s(x, t) = \frac{a}{a_0} \]
for problem II. Then, the governing equation for problem 1 becomes
\[ \frac{\partial\gamma}{\partial t} = -\frac{\partial\gamma}{\partial x}, \quad \gamma(x, t) = \frac{a \alpha x}{\rho \overline{D} \overline{L}^2} \]
\[ \frac{\partial^2 v}{\partial x^2} = \frac{P(-\partial v/\partial t)^{1-(1/P)}}{1+(\partial \gamma / \partial t)^{1/P}} - c_t^2 \frac{\partial \gamma}{\partial x} \quad \text{for } s > 1 \]
\[ \frac{\partial v}{\partial x} = 0 \quad \text{for } s \leq 1 \]

(15)

(16)

Initial and boundary conditions are
\[ v(0, 0) = v_e \]
\[ v(x, 0) = \gamma(x, 0) = 0 \]
\[ s(x, 0) = 1 \]
\[ k \frac{\partial v(0, t)}{\partial t} = 1 + \left( \frac{\partial \gamma}{\partial t} \right)^{1/P} + c_t^2 \eta \]
\[ v(1, t) = 0 \]

(17)

(18)

(19)

After unloading starts from the struck end, Eq. (18) is replaced by
\[ (k + \zeta) \frac{\partial v(x, t)}{\partial t} = 1 + c_t^2 \gamma(x, t) \]

(20)

Energy equation becomes
\[ \frac{1}{2} k v_x^2 = \frac{1}{2} k v(0, t)^2 + \frac{1}{2} \int_0^1 \{ \gamma(x, t) \}^{1+(1/P)} d x \]
\[ + \int_0^1 s(x, t) \frac{\partial \gamma(x, t)}{\partial t} d x \]

(21)

The governing equation for problem II is
\[ \frac{\partial \gamma}{\partial t} = \frac{\partial v}{\partial x} \]
\[ \frac{\partial^2 v}{\partial t} = \frac{a^{1/P}}{P} \frac{\partial^2 v}{\partial x^2} + \frac{1}{2} \frac{c_t^2 \gamma}{\partial x} \]
for \[ 0 \leq x < \zeta \]

(22)

(23)

Initial and boundary conditions are
\[ v(x, 0) = 1 \]
\[ v(0, t) = 0 \]

(24)

(25)

Let \( u(t) \) denote the velocity of rigid region, and we have
\[ \frac{\partial u}{\partial t} = \frac{1}{1 - \zeta} \]
\[ \zeta(0) = 0 \]

(26)

When the interface between plastic and rigid regions moves in negative \( x \)-direction, we use
\[ \frac{\partial u}{\partial t} = \frac{1}{1 + \zeta} \]

(27)

in place of the second equation of Eq. (26). Energy equation becomes
\[ 1 = (1 - \zeta) u_x^2 + \int_{x_0}^{x_1} s(x, t) \frac{\partial \gamma(x, t)}{\partial t} d x \]

(28)

3. Numerical results

Dividing \( x \)-axis in \( N \) equal spacings \( j x \) and \( t \)-axis in equal spacings \( \Delta t \), we represent the value of a function \( f(x, t) \) at the nodal point \( i j x \), \( j \Delta t \) by \( f_{i,j} \) and replace the governing equations by appropriate difference equations. In order to obtain stable numerical solutions of the difference equations by digital computer, the spacing ratio
\[ r = \frac{\Delta t}{(j x)^2} \]

(29)

is found to be
\[ r < \frac{1}{2} \left( \frac{v_{i+1,j} - v_{i+1,j}}{2 j x} \right)^{1-(1/P)} \]

(30)

by trial computations based on the value of \( r \) used in a problem of thermal diffusion. Unloading boundary \( \zeta(t) \) now at \( i j x \) is advanced to the neighboring point \((i+1)j x\) if it is satisfied
\[ |v_{i+1,j} - v_{i,j}| \leq \delta \]

(31)

for positive small number \( \delta \) which will be chosen appropriately. Thus in the deforming region, we have
\[ |v_{i+1,j} - v_{i,j}| > \delta \]

(32)

Putting this inequality into Eq. (30), we obtain
\[ r < \frac{1}{2} \left( \frac{\delta}{j x} \right)^{1-(1/P)} \]

(33)

which relates the values of \( \delta \) and \( r \). Accuracy of computation estimated from energy balance is found to be within 2% of final error for \( N=10 \).
\[ \delta = 0.001, \; r = 0.05 \] and within 1\% of final error for 
\[ N = 10, \; \delta = 0.0001, \; r = 0.01. \] In the following computations of the present paper, the values \( r \) are chosen such that the errors may remain within 2\%.

An example of the influence of hardening parameter \( c_1^2 \) on the final strain distribution is shown in Fig. 2. \( \gamma_f^g \) in this figure is the uniform final strain of a rigid-perfectly plastic rod which would absorb the initial kinetic energy of striking mass at \( s = 1 \), that is, \( \gamma_f^g = (1/2)kv_s^2 \) by standardizing \( (1/2)kv_s^2 = a_sA_x^gL \). The final strain distribution along the rod decreases monotonously from the struck end to the fixed end if the hardening is neglected. In case the hardening is considered, the final strain near the fixed end increases again because of the reflection of stress wave there. Although the linear theory \( P = 1 \) according to Ting and Symonds gave an almost uniform strain distribution along the rod (dotted line in the figure), the nonlinear theory \( P = 5 \) provides a considerably different strain distribution. Variations of particle velocity and strain distribution with time for problem I are shown in Fig. 3 (a) and (b) respectively. Solid curves in the figure are the cases which take the hardening effect into account and dotted curves are those which neglect the effect. The time \( t_s \) when the unloading starts at the struck end in this example of non-hardening case is \( t_s = 8.97 \). The velocity of the striking mass \( v(0, t) \) at this time \( t_s \) is about 2\% of its initial value. On the other hand, if the hardening is taken into account as \( c_1^2 = 0.02, \gamma_f^g = 1.04 \), that is, the unloading starts as early as \( v(0, t) \) is 80\% of \( v_0 \). Examples of the variations of rigid-plastic boundary \( \zeta(t) \), particle velocity and strain with position and time for problem II are shown in Fig. 4 (a), (b) and (c) respectively. It is found from the numerical results that the unloading starts also from impact end at \( t = 0.206 \) for this case of strain hardening and moves toward inside of the rod in contrast to the non-hardening case where the unloading starts always from the free end.

4. Experimental method and its results

Impact loader was composed of a high pressure nitrogen gas reservoir and a gun barrel which accelerated the striking mass or test specimen as shown in Fig. 5. Rigid wall was made of a chrome molybdenum steel block of which the surface was finished by sand paper after appropriate heat
treatment. In problem I, a rod specimen attached to rigid wall was struck by the striking mass accelerated by the impact loader. In problem II, a rod specimen was accelerated by the impact loader and hit against the rigid wall. Three kinds of striking masses of heat-treated high-speed steel SKH3 were used; their diameters were 18 mm and lengths were 30 mm, 50 mm and 100 mm respectively. Impact velocity was measured by a phototransistor and an electronic counter. Deformation process was observed by a high-speed camera having a capacity of 200,000 frames per second. Arrangement of the high-speed camera, the strobolight for it and a specimen was selected as shown in Fig. 6 after some trials to get as clear reflected images as possible. Diameters of the images of specimen thus obtained were measured under the universal projector of \( \times 20 \) magnifications. Axial strains were calculated from them assuming volume constancy. Specimens were an annealed low carbon mild steel of 0.015\% carbon having 12 mm diameter and lengths of 60 mm, 75 mm and 85 mm respectively for problem I. For problem II, the same material of 18 mm diameter and 50 mm and 100 mm lengths respectively was used as specimen. Fine circumferential lines were cut from the impact end by a spacing of 3 mm, because painted lines were scattered or deformed by the impact.

Static compression test of the material used

Fig. 6 Arrangement for high speed photograph

Fig. 7 Nominal stress-strain relation of static compression test (plastic range is approximated by dotted line)
was made by the specimen having 12 mm diameter and 20 mm height to get static yield stress and strain hardening modulus. Strains up to 2% were measured by resistance strain gauge and strains over 2% were measured by micrometer after each unloading. A nominal stress-strain curve obtained is shown in Fig. 7. Approximating the plastic region by a dotted straight line, we obtain the static yield stress \( \sigma_y = 15.7 \text{ kg/mm}^2 \) and strain hardening modulus \( H = (\sigma - \sigma_y) / \varepsilon / \varepsilon = 12 \).

An example of high speed photographs for problem I is shown in Fig. 8 and strain distributions with time obtained from these photographs are shown in Fig. 9. Numbers in the figure are frame numbers from the start of deformation. Figures 10 and 11 are photographs and strain distributions with time respectively for problem II.

5. Comparison of experimental and numerical results

Material constants of the specimen used are as follows: \( \sigma_y = 15.7 \text{ kg/mm}^2 \), \( H = 12 \), \( D = 40 \text{ sec}^{-1} \), \( P = 5 \), \( \rho = 7.5 \times 10^{-4} \text{ kg, sec}^2 \text{/cm}^4 \), where values of \( D \) and \( P \) are obtained from Manjoine's data. These values of material constants together with values of \( G, A, L \) and \( V_0 \) for each experiment are used to give numerical results and comparison of these values with experimental results is made in the following. In Fig. 12, final strain distribution of an experiment for problem I is compared with two numerical values with and without strain hardening. As the result of non-hardening theory represents a monotonously decreasing distribution of final strain from the impact end, this theory cannot give experimental result even qualitatively. It can be seen that the result of the theory with strain hardening explains the strain distribution much better. Slight decrease of the final strain at the fixed end may be an effect of frictional constraint there which is neglected in the theory. Figure 13 shows a final strain distribution of problem I for three cases with different impact velocities and otherwise same conditions. Circles and solid lines are experimental and numerical results respectively. Figure 14 shows an example of comparison of experimental strain distribution and corresponding numerical values with and without strain hardening for problem II. Improvement of the calculated values due to the consideration of strain hardening effect is much better for this case than for problem I. Figure 15 shows final strain distributions of problem II for three cases with different impact velocities and otherwise same conditions.

It can be seen from Figs. 3(b) and 9 for problem I and Figs. 4(c)
Fig. 9 Experimental values of strain distributions with time

Fig. 10 High speed photographs for free flight impact test

Fig. 11 Experimental values of strain distributions with time

Numbers are frame numbers after the start of deformation, an interval of consecutive frames is 7.1 micro seconds. Specimen: 18 mm diameter and 100 mm length. Impact velocity: 97.8 m/sec

and 11 for problem II that although calculated values of variations of strain distributions with time during deformation coincide qualitatively with experimental observations, they do not agree well quantitatively. Theoretical result anticipates rather rapid progress of deformation in the incipient period, but actual behavior does not follow this pattern.

6. Conclusions

(1) A finite mass impact problem and a free flight impact problem of a finite rod of strain-rate dependent material are solved numerically by assuming the constitutive equation of Bingham type with linear strain hardening whose modulus for each strain-rate is supposed to be equal to that of static one. Most remarkable differences of the results from those for non-hardening material are: the final strain distribution which has the maximum value at the impact end decreases gradually and increases again near the fixed end for a finite mass impact problem and the unloading may start also from the impact end for a free flight impact problem and the final strain distribution for this problem gives much lower values at impact end and slightly higher values inside the rod in contrast to the non-
hardening case.

(2) Numerical values based on material constants obtained from static tests and Manjoine's data agree well for final strain distributions with experimental results.

(3) Variations of strain distribution with time obtained from numerical calculations agree qualitatively with those observed by a high speed camera.

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