621.43.013

The Influence of Exhaust and Scavenging Time Area Design on the Engine Performance in the Poppet Valve Uniflow Two-Cycle Engine*

(Part 1, Fundamental Equations and Preliminary Calculations)

By Hideo OKAMURA**

In order to study the optimum design conditions of exhaust and scavenging time areas in a poppet valve uniflow two stroke cycle engine, a series of fundamental equations and methods of estimating the engine performance were derived by developing generalized dimensionless expressions.

The possibility and accuracy of this calculation method could be checked by some calculations on a high speed three-cylinder diesel engine equipped with a mechanically driven blower.

1. Introduction

The designs of the exhaust and scavenging time areas have a great effect upon the engine performance in both turbocharged and non-turbocharged two stroke cycle engines of all types of scavenging systems. But very little has been studied about the poppet valve uniflow type of a two stroke cycle engine.

The object of this paper is, first, to present a general method of estimating the engine performance, and secondly, to show some influences of the exhaust and scavenging time area designs on the engine performance in poppet valve uniflow two stroke cycle engines equipped with mechanically driven blowers.

In part 1, the dimensionless representation for the main engine specifications, the exhaust valves and the scavenging ports, and also the fundamental equations and method of estimating engine performance are dealt with. Next the possibility and accuracy of this calculation method are checked by some preliminary calculations.

The influences of the design parameters of exhaust valves and scavenging ports are dealt with in Part 2, where some brief comparisons of the exhaust and scavenging time areas for poppet valve uniflow, opposed-piston uniflow and loop scavenging types of engine are also made.

2. Nomenclature

The main items of nomenclature are listed in Table 1.

3. Fundamental equations

The gas flow system of a poppet valve uniflow two stroke cycle engine including the silencer, manifolds, and pipes, can be schematically shown as in Fig. 1(a). Since as shown in Fig. 2, the fluctuations in the exhaust and scavenging gas flow systems for the usual high speed two cycle multi-cylinder engines are not prominent, the gas flow in these engines can be treated approximately as a steady flow. Figure 1(b) shows such a simplified gas flow system, where $\bar{A}_r$ and A_t are the equivalent orifices of the cylinder and exhaust systems, and in general, the intake flow resistance can be neglected. $\bar{A}_r$ depends upon the actual scavenging duration, $\theta_{SB} \sim \theta_{SO}$, and the exhaust and scavenging time areas, as shown in Fig. 3. The state variables such as G_0 , P_s , P_e , T_s , T_e , as shown in Fig. 1(b), depend not only upon P_0 , T_0 , $\bar{A}_r$ and A_t , but also upon certain other conditions, such as the specification and characteristics of the engine and the blower, the engine operating conditions (engine speed and fuel/air ratio), the characteristics of the scavenging system, the heat release pattern in the cylinder, and so forth.

Since these relations can be represented by the following fundamental equations, the state variables and the engine performance can be determined

* Received 21st April, 1970.

** Assistant Professor, Faculty of Science and Technology, Sophia University, Chiyoda-ku, Tokyo.

from these equations. Since a generalization of these problems was intended, dimensionless representation was adopted for all these equations.

3.1 Dimensionless expression for the main specifications of the engine

If we choose the cylinder diameter D for the characteristic length of an engine, generally, the main engine specifications can be determined by the following four dimensionless parameters: the number of cylinders Z , the stroke/bore ratio S/D , the ratio of the crank radius to the connecting rod length $\lambda=r/l$, and the compression ratio ϵ .

And if the piston area A_p and the displace-

ment volume V_{disp} are chosen for the characteristic area and the characteristic volume, then dimensionless expressions for the flow areas and the cylinder volumes can also be established.

3.2 The dimensionless scavenging port area

When we choose the area $A_0=\pi D\beta_s \sin \varphi_s$ shown in Fig. 4 for the theoretical area of the scavenging port, and denote the effective area and the flow coefficient by A_s and $\alpha_{cp}=A_s/A_0$, then the dimensionless scavenging port area A_s/A_p can be expressed as follows:

$$A_s/A_p = \alpha_{cp} \pi D \beta_s h \sin \varphi_s / (\pi D^2/4)$$

Table 1 Nomenclature

a : Constant related to scavenging efficiency	$V(\theta)$: cylinder volume at θ
A : reciprocal of mechanical equivalent ($=1/J$)	V_{disp} : displacement volume
A_e, A_s, A_t : effective area of flow Passage	ΔV_p : volume change due to piston motion
$\bar{A}_r$: reduced orifice of cylinder	$y(\theta), y_T$: valve lift at θ , total valve lift
b_e : fuel consumption	z : number of cylinders
c_1, c_2, c_3 : correction factors for the actual and apparent values	z_1, z_{sm} : dimensionless cylinder volume
c_p, c_v : specific heat at constant pressure and constant volume	α_{cp} : flow coefficient of cylinder ports
d : diameter of poppet valve	α_v : flow coefficient of poppet valves
D : diameter of cylinder	β_s : effective portion for cylinder port/inside circumference of cylinder
DRO, DRZ : delivery ratio	τ_0 : specific weight of air
G_0 : gas flow rate	ϵ : compression ratio
G_1, G_{10} : weights of actual and apparent charged fresh air,	ϵ_c, ϵ_e : effective compression, expansion ratio
G_t : weight of short-circuited fresh air	η_{adi} : adiabatic efficiency of blower
G_s : total gas weight in cylinder	η_c : combustion efficiency
h_u : heat addition per unit weight of cylinder gas	η_{mbt} : mechanical efficiency of blower
H_u : low heat value of fuel	η_s, η_{s0} : Scavenging efficiencies, actual and apparent,
i : Number of valves	η_{th}, η_{th0} : thermal efficiencies, actual and apparent
K : factor related to cam (θ_2/θ_1 in Fig. 5)	η_{tr} : apparent trapping efficiency
l : length of connecting rod	η_{v0} : apparent volumetric efficiency
L : Stoichiometric air-fuel ratio	θ : crank angle
n_c, n_e : exponents of compression, expansion stroke	θ_s : half of scavenging period
N, N_B : revolutions per minute of engine, blower	θ_T : half of exhaust period
P_e, P_0, P_s, P_z : pressures	κ : ratio of specific heat
P_{max} : maximum combustion pressure	λ : r/l
$P_{mi}, \Delta P_{mi}$: indicated mean effective pressure	λ_s, λ_{s0} : excess air ratios, actual and apparent
P_{me} : mean effective pressure	ρ : rate of explosion
P_{mbt} : blower-friction mean effective pressure	ρ_f : specific weight of fuel
P_{mf0}, P_{mf} : Mechanical-friction mean effective pressure	σ : cut-off ratio
Q_f : theoretical heat addition	φ_s : inclination of cylinder port
q : fuel flow rate	
r : crank radius	
S : stroke	
T_e, T_0, T_s, T_z : temperature	
v_{pis} : mean piston speed	

Suffixes

$e, 0, s, z$: exhaust, atmospheric or standard condition, scavenging, cylinder
 EO, EC, SO, SC : exhaust open, exhaust close, scavenging open, scavenging close
 SB : actual scavenging start
 t : equivalent orifice for exhaust system.

$$= 4\alpha_{cp}\beta_s \sin\varphi_s (S/D) \{ (1/2)(\cos\theta_{s0} - \cos\theta) + (\lambda/8)(\cos 2\theta_{s0} - \cos 2\theta) \}$$

$$= f(\alpha_{cp}, \beta_s, \varphi_s, S/D, \lambda, \theta_{s0}, \theta) \dots\dots\dots (1)$$

A_s/A_p is a function of the crank angle θ but contains six dimensionless parameters α_{cp} , β_s , φ_s , S/D and θ_{s0} . In Fig. 4 the experimental values of α_{cp} are shown.

3.3 The dimensionless exhaust valve area

When we choose the Area $A_0 = \pi \cdot d \cdot y(\theta)$ shown in Fig. 4, as the theoretical area for a poppet valve, and denote the effective area, the number of valves and the flow coefficient by A_e , i , and $\alpha_v =$

$A_e/(i \cdot A_0)$, the dimensionless exhaust valve area A_e/A_p can be expressed as follows.

$$A_e/A_p = \alpha_v i \pi d y(\theta) / (\pi D^2/4)$$

$$= \alpha_v i (d/D)^2 \{ y_T/(d/4) \} \{ y(\theta)/y_T \} \dots\dots\dots (2)$$

where y_T denotes the total lift of the valve, and for most cases it can be assumed that $y_T/(d/4) \approx 1.0$. As for the valve lift curve $y(\theta)$, since specific analytical functions are preferred for the derivative curve so as to facilitate the prediction of the dynamical behavior of the valve gear linkage, the cams are usually denoted as constant-acceleration cam, sine-sine cam and so forth, in accordance with the second derivative of the cam lift curve: $\ddot{y}(\theta) = d^2 y/d\theta^2$. As shown in Fig. 5, $y(\theta)/y_T$ can be expressed as a function of θ/θ_T when K , and $\ddot{y}(\theta)$ are given; but θ_T must be determined by θ_{E0} and θ_{EC} . When the foregoing

Fig. 1 Gas flow in a two-stroke cycle engine

Fig. 2 P_s/P_0 , P_s/P_e in high speed two-stroke cycle engines

Fig. 3 Exhaust and scavenging time areas in poppet valve uniflow two-stroke cycle engines

Fig. 4 Theoretical areas and flow coefficients of scavenging ports and exhaust valve

Fig. 5 Parameters K , $\ddot{y}(\theta)$, y_T to determine cam profile

relations are considered, A_e/A_p can be rewritten as a function of θ which contains seven dimensionless parameters, α_v , $i(d/D)^2$, $\gamma_T/(d/4)$, K , $\bar{y}(\theta)$, θ_{EO} and θ_{EC} .

$$\begin{aligned} A_e/A_p &= \alpha_v i(d/D)^2 \{\gamma_T/(d/4)\} f\{K, \bar{y}(\theta), \theta_{EO}, \\ &\theta_{EC}, \theta\} = f\{\alpha_v, i(d/D)^2, \gamma_T/(d/4), K, \bar{y}(\theta), \\ &\theta_{EO}, \theta_{EC}, \theta\} \dots\dots\dots (2)' \end{aligned}$$

In Fig. 4, the experimental values of α_v are shown.

3.4 The dimensionless reduced area of the cylinder

As is commonly known, A_s and A_e in Fig. 1 can be brought down to a reduced area A_r under the assumption of a steady flow. For the incompressible gas flow and the compressible gas flow, A_r can be shown as follows:

Incompressible gas flow:

$$A_r = A_s/\sqrt{1+(A_s/A_e)^2} \dots\dots\dots (a)$$

Compressible gas flow:

$$A_r = A_s \psi\{(P_z/P_s)/\psi(P_e/P_s)\} \dots\dots\dots (b)$$

where $\psi(P_2/P_1)$ generally denotes that

$$\left. \begin{aligned} \text{for } \frac{P_2}{P_1} &> \left(\frac{2}{\kappa+1}\right)^{\kappa/(\kappa-1)}, \quad \psi\left(\frac{P_2}{P_1}\right) \\ &= \sqrt{\frac{2g\kappa}{\kappa-1} \left[\left(\frac{P_2}{P_1}\right)^{2/\kappa} - \left(\frac{P_2}{P_1}\right)^{(\kappa+1)/\kappa} \right]} \\ \text{for } \frac{P_2}{P_1} &\leq \left(\frac{2}{\kappa+1}\right)^{\kappa/(\kappa-1)}, \quad \psi\left(\frac{P_2}{P_1}\right) \\ &= \sqrt{g\kappa \left[\frac{2}{\kappa+1} \right]^{(\kappa+1)/(\kappa-1)}} \end{aligned} \right\} \dots\dots\dots (c)$$

Similarly the dimensionless reduced area A_r/A_p can be shown as follows:

for incompressible flow:

$$A_r/A_p = (A_s/A_p)/\sqrt{1+\{(A_s/A_p)/(A_e/A_p)\}^2} \dots\dots\dots (3)$$

for compressible flow:

$$A_r/A_p = (A_s/A_p) \psi(P_z/P_s)/\psi(P_e/P_s) \dots\dots\dots (4)$$

Since A_s/A_p and A_e/A_p have six and seven dimensionless parameters those can be determined independently, so A_r/A_p can be determined as far as these thirteen parameters are determined. Furthermore as shown in Fig. 16, A_r/A_p in a compressible gas flow is also affected by P_e/P_s in addition to A_s/A_p and A_e/A_p . Since the scavenging air is delivered only during the interval from θ_{SB} (where the cylinder pressure is reduced just to the scavenging pressure) to θ_{SC} , the following mean value of A_r/A_p should be considered for the dimensionless equivalent orifice.

$$\bar{A}_r/A_p = \int_{\theta_{SB}}^{\theta_{SC}} (A_r/A_p) d\theta / (\theta_{SC} - \theta_{SB}) \dots\dots\dots (5)$$

3.5 Estimation of the cylinder conditions during the exhaust and scavenging processes

If the fluctuations of P_s , P_e , T_s and T_e can be neglected, the pressure $P_z(\theta)$, temperature $T_z(\theta)$, and volume $V_z(\theta)$ of the cylinder at θ during the exhaust and scavenging processes can be derived by solving the following simultaneous equations.

$$\left. \begin{aligned} \bar{P}_z &= P_{z(\theta)} + (1/2) \Delta P_z \\ \bar{T}_z &= T_{z(\theta)} + (1/2) \Delta T_z \\ \bar{V}_z &= V_{z(\theta)} + (1/2) \Delta V_p \end{aligned} \right\} \dots\dots\dots (6)$$

$$\begin{aligned} \Delta P_z/P_{z(\theta)} &= \kappa \{-\Delta V_p + \Delta G_s R(\bar{T}_z/\bar{P}_z) \\ &- \Delta G_e R(\bar{T}_z/\bar{P}_z)\}/\bar{V}_z \dots\dots\dots (7) \end{aligned}$$

$$\begin{aligned} \Delta T_z/T_{z(\theta)} &= (\kappa-1) \Delta P_z/(\kappa \bar{P}_z) \\ &- \kappa R(\bar{T}_z - T_s) \Delta G_s/(\bar{P}_z \bar{V}_z) \dots\dots\dots (8) \end{aligned}$$

for $P_s > \bar{P}_z$

$$\Delta G_s = (\Delta\theta/6N)(\bar{A}_s/A_p) \psi(\bar{P}_z/P_s) A_p P_s / \sqrt{RT_s} \dots\dots\dots (9)$$

for $P_s < \bar{P}_z$

$$\Delta G_s = -(\Delta\theta/6N)(\bar{A}_s/A_p) \psi(P_s/\bar{P}_z) A_p \bar{P}_z / \sqrt{R\bar{T}_z} \dots\dots\dots (9)'$$

for $\bar{P}_z > P_e$

$$\Delta G_e = (\Delta\theta/6N)(\bar{A}_e/A_p) \psi(P_e/\bar{P}_z) A_p \bar{P}_z / \sqrt{R\bar{T}_z} \dots\dots\dots (10)$$

for $\bar{P}_z < P_e$

$$\Delta G_e = -(\Delta\theta/6N)(\bar{A}_e/A_p) \psi(\bar{P}_z/P_e) A_p P_e / \sqrt{RT_e} \dots\dots\dots (10)'$$

$$\begin{aligned} \Delta V_p &= -[(1/2)\{\cos(\theta + \Delta\theta) - \cos\theta\} \\ &+ (\lambda/8)\{\cos 2(\theta + \Delta\theta) - \cos 2\theta\}] V_{\text{disp}} \dots\dots\dots (11) \end{aligned}$$

As can be seen from these equations, for the increment $\Delta\theta$ during θ to $\theta + \Delta\theta$, we assumed that the mean values of $P_z(\theta)$, $T_z(\theta)$ and $V_z(\theta)$ are $\bar{P}_z$, $\bar{T}_z$ and $\bar{V}_z$, and the inflow of the scavenging air and the outflow of the cylinder gas are ΔG_s and ΔG_e , and the change of the cylinder volume due to the piston motion is ΔV_p . We also assumed that

(a) the instantaneous heat exchange between the scavenging air and the cylinder gas can be accomplished;

(b) there is no heat transfer between the cylinder contents and cylinder wall; and

(c) the gas constants of the scavenging air and of the exhaust gas are equal and constant.

In the above equations, $\bar{A}_s/A_p$, $\bar{A}_e/A_p$ denote the mean value of A_s/A_p , A_e/A_p during $\Delta\theta$.

If $P_z(\theta)$, $T_z(\theta)$, $V_z(\theta)$ for θ_{EO} and P_s , T_s , P_e , T_e , A_s/A_p , A_e/A_p are given, $P_z(\theta)$ and $T_z(\theta)$ can be estimated by the stepwise solution of the above equations. Evidently θ_{SB} can be obtained by finding the initial value of θ for which the con-

ditions $P_{z(0)} \leq P_s$ can be satisfied.

3.6 Relations between the engine operating conditions and the blower operating conditions

As is well known, the engine operating conditions of two stroke cycle engines can be defined by three parameters, namely by N , DRO and λ_{s0} . Since a steady flow is assumed, G_0 , shown in Fig. 1(b), can be expressed as follows:

$$G_0 = z\gamma_0 V_{disp} N DRO / 60 \quad \text{kg/sec} \quad \dots\dots\dots (d)$$

G_0 can also be expressed as follows for the operating conditions:

$$G_0 = z\bar{A}_r (\theta_{SC} - \theta_{SB}) / 360^\circ P_s \phi(P_e/P_s) / \sqrt{RT_s} \quad \text{kg/sec} \quad \dots\dots\dots (e)$$

And if no intercoolers are laid between the blower and the engine, T_s/T_0 can be expressed as follows:

$$T_s/T_0 = 1 + \{(P_s/P_0)^{(\kappa-1)/\kappa} - 1\} / \eta_{adi} \quad \dots\dots\dots (f)$$

Combining (d), (e), (f), and keeping in mind the relation $V_{disp} = A_p S$, the following expression can be obtained in dimensionless form.

$$\begin{aligned} & (\bar{A}_r/A_p)(\theta_{SC} - \theta_{SB}) / 360^\circ (P_s/P_0) \phi(P_e/P_s) \\ & / \sqrt{1 + \{(P_s/P_0)^{(\kappa-1)/\kappa} - 1\} / \eta_{adi}} \\ & = \gamma_0 DRO (S/D) (ND/60) \sqrt{RT_0/P_0} \quad \dots\dots\dots (12) \end{aligned}$$

If the characteristic curve for the blower and N_E corresponding to N are given, then the blower operating points can be found by stepwise checking of whether P_s/P_0 , η_{adi} , DRO (DRO can be obtained from N and G_0) corresponding to that point can satisfy the Eq. (12) along the $N_B = \text{constant}$ line. For this checking P_e must be assumed. Moreover P_e must also satisfy Eq. (28) which defines the steady flow across the equivalent orifice A_t (see Fig. 6)

3.7 Approximate expression for scavenging efficiency

As far as the⁽¹⁾ experimental results in model tests for various scavenging types are concerned, the

Fig. 6 An example of blower characteristics

scavenging efficiency η_s can be expressed by the following approximate relations:

$$\eta_s = 1 - \exp(-aDRZ) \quad \dots\dots\dots (g)$$

where DRZ denotes the air delivery ratio based upon the mean conditions of the cylinder during the scavenging period, and a denotes a constant, and where a depends prominently upon the scavenging type, and $a \approx 1.6 \sim 1.8$ and $a \approx 1.0$ are obtained for the average uniflow and the average loop scavenging types respectively.

Then η_s can be expressed as follows.

$$\eta_s = 1 - \exp[-aDRO(1/z_{sm})(P_0/P_{zm})(T_{zm}/T_0)] \quad \dots\dots\dots (h)$$

In general, the actual values of P_{zm} and T_{zm} for an engine in actual operation can not be estimated. If we assume P_s and T_s for the approximate values of P_{zm} and T_{zm} , and further assume a correction factor $c_1 = \eta_s/\eta_{s0}$, the following equations can be derived.

$$\begin{aligned} & \eta_{s0} = 1 - \exp\{-aDRO(1/z_{sm})(P_0/P_s)(T_s/T_0)\} \\ & \eta_s = c_1 \eta_{s0} \quad \dots\dots\dots (13) \end{aligned}$$

Here we take η_{s0} as an "apparent scavenging efficiency" and η_s as the "actual scavenging efficiency."

3.8 The charged fresh air and the volumetric efficiency

The charged fresh air at the beginning of compression at the point denoted by 1 in Fig. 7 can be expressed as follows:

$$G_1 = \gamma_0 V_1 (R_0/R_1) (P_1/P_0) (T_0/T_1) \eta_s \quad \text{kg} \quad \dots\dots\dots (i)$$

where V_1 denotes the cylinder volume at θ_{SC} or θ_{EC} whenever both exhaust valve and scavenging are completely closed. Since the actual values of R_1 , P_1 , T_1 and η_s in (i) can not be estimated, we assume an "apparent charged fresh air" G_{10} which is based upon the known values of R_0 , P_s ,

Fig. 7 Cylinder cycle

T_s and η_{s0} instead of R_1 , P_1 , T_1 and η_s . If a correction factor

$$c_2 = (R_0 P_1 T_s) / (R_1 P_s T_1)$$

can be assumed in addition to c_1 , the following equations can be obtained.

$$\begin{aligned} G_{10} &= \gamma_0 z_1 V_{\text{disp}} (P_s / P_0) (T_0 / T_s) \eta_{s0} \} \dots\dots\dots (14) \\ G_1 &= c_1 c_2 G_{10} \end{aligned}$$

Then an "apparent volumetric efficiency" η_{v0} , and an "apparent excess air ratio" λ_{s0} which are based upon G_{10} can be expressed as follows:

$$\eta_{v0} = G_{10} / (\gamma_0 V_{\text{disp}}) \dots\dots\dots (15)$$

$$\lambda_{s0} = G_{10} / (L_0 q \rho_f \times 10^{-6}) \dots\dots\dots (16)$$

where q denotes the fuel flow rate and ρ_f denotes the specific weight of the injected fuel.

3.9 Thermal efficiency and mean indicated pressure of cylinder cycle

Let us distinguish the indicated mean pressure corresponding to area 1'-2-3'-3-4-1' from that corresponding to area 4-5-1-1'-4, and denote them as P_{mi} and ΔP_{mi} as shown in Fig. 7. As can be seen, ΔP_{mi} approximately corresponds to the work that would be done to the piston during the exhaust and scavenging processes. Otherwise P_{mi} depends greatly upon the heat release pattern and the heat transfer process in the cylinder, but no general and accurate method of estimating those complicated phenomena has yet been established.

For convenience, we assume that the theoretical heat addition of the injected fuel Q_f would be released according to $\eta_c Q_f = Q_v + Q_p$, as shown in Fig. 7. However $P_{\max}$ and η_c can be determined as a function of the mean piston speed v_{pis} and the apparent excess air ratio λ_{s0} . Furthermore we define the thermal efficiency as $\eta_{th} = (\text{indicated work } 1'-2-3'-3-4-1') / (\text{equivalent work of } Q_v + Q_p)$ as usual.

Since neither P_1 , T_1 , η_s , the actual excess air ratio λ nor the actual thermal efficiency $\eta_{th} = f(P_1, T_1, \eta_s, \lambda_s, \eta_c, P_{\max}, n_c, n_e, \varepsilon_c, \varepsilon_e)$ can be determined, we assume an "apparent thermal efficiency" η_{th0} which is based upon the known P_s , T_s , η_{s0} and λ_{s0} . Then $\eta_{th0} = f(P_s, T_s, \eta_{s0}, \lambda_{s0}, \eta_c, P_{\max}, n_c, n_e, \varepsilon_c, \varepsilon_e)$

can be written as follows for the compound cycle:

$$\eta_{th0} = \frac{AR \{ (T_3 - T_{3'}) + (T_3 - T_4) / (n_e - 1) - (T_2 - T_{1'}) / (n_c - 1) \}}{(c_v T_{3'} - c_v T_2) + (c_p T_3 - c_p T_{3'})}$$

for the constant volume cycle (that is $\sigma = 1$):

$$\eta_{th0} = AR [T_{3'} \{ 1 - (1/\varepsilon_e)^{n_e-1} \} / (n_e - 1) - (T_2 - T_{1'}) / (n_c - 1)] / (c_v T_{3'} - c_v T_2) \dots\dots\dots (17)$$

where $\varepsilon_c = V_1 / V_2$, $\varepsilon_e = V_4 / V_3$, $P_1 = P_s$

$$P_{1'} = P_1 (\varepsilon_c / \varepsilon_e)^{n_c}, P_2 = P_1 \varepsilon_c^{n_c}$$

$$P_{3'} = \rho P_2 = P_{\max} = P_3, P_4 = P_3 (\sigma / \varepsilon_e)^{n_e}$$

$$T_1 = T_s, T_{1'} = T_1 (\varepsilon_c / \varepsilon_e)^{n_c-1}, T_2 = T_1 \varepsilon_c^{n_c-1}$$

$$T_{3'} = \rho T_2, T_3 = \sigma T_{3'}, T_4 = T_3 (\sigma / \varepsilon_e)^{n_e-1}$$

$$\sigma = V_{3'} / V_3 = \{ h_u / (\rho c_v T_2) + (\kappa_g - 1) + 1 / \rho \} / \kappa_g$$

$$h_u = H_u \eta_{s0} \eta_c / (\eta_{s0} + \lambda_{s0} L_0)$$

..... (18)

If we assume a correction factor c_3 as $c_3 = \eta_{th} / \eta_{th0}$, the actual thermal efficiency can be expressed as follows:

$$\eta_{th} = c_3 \eta_{th0} \dots\dots\dots (19)$$

P_{mi} can be expressed as follows.

$$P_{mi} = H_u G_1 \eta_c \eta_{th} / (A \lambda_s L_0 V_{\text{disp}}) \dots\dots\dots (j)$$

Substituting Eqs. (14) and (19) for G_1 and η_{th0} , and considering the relations $\lambda_{s0} / \lambda_s = G_{10} / G_1 = 1 / (c_1 c_2)$, and $V_1 = z_1 V_{\text{disp}}$, P_{mi} can be rewritten as follows.

$$P_{mi} = \{ H_u \gamma_0 / (A \lambda_{s0} L_0) \} z_1 (P_s / P_0) (T_0 / T_s) \times \eta_{s0} \eta_{th0} c_3 \eta_c \dots\dots\dots (20)$$

Here all the terms except η_{th0} , c_3 and η_c can be estimated. Usually $P_{\max}$, n_c and n_e can be estimated from the indicator card for a particular engine or for a certain similar engine in order to estimate η_{th0} . However c_3 and η_c can not be estimated directly in that way.

As is well known, the closer the operating cycle approaches the constant volume cycle, the less is the influence of η_c upon η_{th0} . Since the operating cycle of a high speed diesel engine (especially for a direct injection type engine) is quite close to the constant volume cycle, and c_3 is still near unity, we can assume $c_3 \eta_c$ in place of η_c for calculating η_{th0} . Then we can assume as follows.

$$\eta_{th0} (P_{\max}, n_c, n_e, \dots, \eta_c, \dots) \approx \eta_{th0} (P_{\max}, n_c, n_e, \dots, c_3 \eta_c, \dots)$$

If the values of P_s , T_s , DRO , η_{s0} , $P_{\max}$, n_c , n_e and so forth are obtained experimentally, P_{mi} can be calculated by assuming $c_3 \eta_c$, and if P_{mi} for those operating conditions can be obtained from the indicator cards, the values of $c_3 \eta_c$ themselves can be estimated as shown in Fig. 8. For an experiment on a poppet valve uniflow two cycle diesel engine, it was found that $c_3 \eta_c$, $P_{\max}$ and $\rho = P_{\max} / P_2$ can be expressed as a function of $1/\lambda_{s0}$ and v_{pis} , and those relations are shown in Fig. 10.

If $P_2(\theta)$ and ΔV_p during the exhaust and scavenging processes are calculated by the methods mentioned in 3.5, ΔP_{mi} can be calculated as follows.

$$\Delta P_{mi} = \int_{V_{EO}}^{V_1} P_{z(0)} \Delta V_p / V_{disp} \dots\dots\dots (21)$$

3.10 Frictional loss due to blower and other mechanical frictions

Let us distinguish the frictional loss due to the blower from that due to other mechanical frictions, and denote them in the form of frictional mean pressure by P_{mbi} and P_{mf} . Substituting the state values under the operating points of the blower in the following expression, P_{mbi} can be easily calculated.

$$P_{mbi} = \{\kappa/(\kappa-1)\} P_0 DRO \{(P_s/P_0)^{(\kappa-1)/\kappa} - 1\} / (\eta_{adi}, \eta_{mbi}) \dots\dots\dots (22)$$

Generally P_{mf} depends not only upon the particular internal friction in engines, but also upon their accessories like cooling fans, air compressors, electric generators and so forth, and is also influenced by the viscosity of their engine oils. For multi-cylinder engines, accurate values of P_{mf} can only be obtained from accurate indicator cards for each cylinder, and very little is known about them.

For a six-cylinder gasoline engine, C.F. Taylor⁽²⁾ has confirmed that P_{mf} is usually influenced both by the mean gas pressure in cylinders and by the mean piston speed, and the author has also confirmed similar results in a single cylinder two stroke cycle diesel engine. The value of P_{mf} obtained by the Willan's line method or the motoring method may correspond to the value for $P_{mi} + \Delta P_{mi} = 0$, and we denote this by P_{mf0} . Since P_{mf} is the sum of this P_{mf0} and the other additional friction, and the latter is always influenced both by the mean gas pressure, namely, $P_{mi} + \Delta P_{mi}$, and the mean piston speed v_{pis} , we denote the latter as ΔP_{mf} . In experiments, C.F. Taylor and the author confirmed that P_{mf0} and ΔP_{mf} can be expressed as $P_{mf0} = (b_0 + m_0 v_{pis})$ and $\Delta P_{mf} = (b_1 +$

$$m_1 v_{pis})(P_{mi} + \Delta P_{mi}).$$

Thus P_{mf} can be generally expressed as follows:

$$P_{mf} = P_{mf0} + (b_1 + m_1 v_{pis})(P_{mi} + \Delta P_{mi}) \dots\dots\dots (23)$$

where b_0 , m_0 , b_1 and m_1 can be estimated experimentally for each engine.

3.11 Brake mean pressure and fuel consumption

If P_{mi} , P_{mi} , P_{mbi} and P_{mf} are calculated as above, the brake mean pressure P_{me} for the operating condition can be obtained as follows.

$$P_{me} = (P_{mi} + \Delta P_{mi}) - P_{mbi} - P_{mf} \dots\dots\dots (24)$$

And the fuel consumption b_e can be obtained from the corresponding P_{me} , P_{mi} , η_c (for convenience we assume $\eta_c \approx c_3 \eta_c$) and η_{th0} as follows.

$$b_e = \{632 \times 10^{-3} / (H_u c_3 \eta_c \eta_{th0})\} P_{mi} / P_{me} \dots\dots\dots (25)$$

Obviously the fuel flow per cycle per cylinder can be expressed as follows.

$$q = \{G_{10} / (\rho_f \lambda_{s0} L_0)\} \times 10^6 \text{ mm}^3/\text{cyl}/\text{c} \dots\dots\dots (26)$$

3.12 Equations to determine the exhaust gas flow

It is convenient to express the equivalent orifice A_t for the exhaust system in a dimensionless form as $A_t / (z A_p)$. Since a steady flow is also assumed for the exhaust gas flow, the gas flow through A_t in Fig. 1 (b) can be specially treated as similar to a gas flow through the turbine nozzles of a constant pressure type exhaust turbocharger for two stroke cycle turbocharged engines. As for the exhaust gas temperature in such a turbocharged engine, Nagao⁽³⁾ and Hirako have derived the following expressions: In these equations, the influence due to the kinetic energy of the blow-down gas and the additional work done by the blow-by gas are precisely taken into account. Combining these relations the exhaust gas flow through A_t must satisfy the following equations:

$$T_e = [T_4 \{1 - (c_p/c_{pz})(1 - 1/\kappa_z)(1 - P_e/P_4)\} + T_s (\Delta G_i/G_z)(c_p/c_{pz})] / \{1 + (\Delta G_i/G_z)(c_p/c_{pz})\} \dots\dots\dots (27)$$

$$G_0 \{1 + \eta_{tr} / (\lambda_{s0} L_0)\} = (A_t / z A_p) z A_p P_e \times \phi(P_0/P_e)(60/N) / \sqrt{RT_e} \dots\dots\dots (28)$$

where

$$\eta_{tr} = G_{10} / (\gamma_0 DRO V_{disp}) \dots\dots\dots (29)$$

$$\Delta G_i/G_z = \{\gamma_0 V_{disp}(1 - \eta_{tr}) DRO\} / G_0 \{1 + \eta_{tr} / (\lambda_{s0} L_0)\} \dots\dots\dots (30)$$

As can be seen, ΔG_i , G_z , G_0 denote the weight of the fresh charge that is blown through the cylinder during the scavenging process, the total exhaust gas, and the delivered total fresh charge, respectively, and P_4 and T_4 are already shown in Eq. (18).

Fig. 8 $c_3 \eta_c$ assumed from indicator cards

4. Method of calculation

It can be seen that, provided the data such as are listed in Table 2 can be given, the engine performance under a steady operating condition can be calculated by solving the fundamental equations which are derived in the previous section. But since direct solution for the engine performance, namely for P_{me} and b_e , can not be obtained algebraically, an iterational trial and error method on a digital computer is the only practical means. A flow chart for a calculation method is shown in Fig. 9.

Table 2 Data for estimating engine performance

No.	Contents of data
①	$D, S/D, \lambda, \varepsilon, z$
②	$\alpha_{cp}, \beta_s, \sin \varphi_s, \lambda, S/D, \theta_s$
③	$\alpha_v, i(d/D)^2, y_T/(d/4), \dot{y}(\theta), K, \theta_{EO}, \theta_{EC}$
④	$G=f(N_B, P_s/P_0, \eta_{adi}, \eta_{mbi})$
⑤	A_t/zA_p
⑥	$\eta_s=1-\exp(-aDRZ)$
⑦	$P_{max}=f(1/\lambda_{s0}, v_{pis}), c_3\eta_c=f(1/\lambda_{s0}, v_{pis})$
⑧	$P_{mf}=f(v_{pis}, P_{mi}+\Delta P_{mi})$

Fig. 9 Flow chart for estimating engine performance

In this calculation the processes of calculating A_s/A_p , A_e/A_p , $\bar{A}_r/A_p$, estimating the operating point of the blower, the cylinder cycle, $P_z(\theta)$ and $T_z(\theta)$ for the exhaust and scavenging processes and so forth are arranged as individual subprograms. Stepwise calculations are always started with the assumption of exhaust pressure P_e and the actual scavenging start angle θ_{SB} , and are iterated until appropriate values of those P_e/P_0 and θ_{SB} are obtained which can simultaneously satisfy the above fundamental equations. Practically a few percent of error was permitted for establishing the solutions.

5. Preliminary calculation

In order to check the possibility and accuracy of this calculation, a preliminary calculation was made for a commercial three-cylinder high speed two stroke cycle diesel engine for vehicle use.

5.1 Preliminary calculation for a vehicle diesel engine

The engine used for a preliminary calculation is a familiar water-cooled, direct-injection type of diesel engine. Before starting the calculation, the flow coefficients for the scavenging ports and the exhaust valves, namely α_{cp} and α_v were obtained by a flow test on a single-cylinder engine of the same specifications, and the relations for $P_{max}=f(1/\lambda_{s0}, v_{pis})$ and $c_3\eta_c=f(1/\lambda_{s0}, v_{pis})$, as shown in Fig. 10, were also estimated from indicator cards of this single-cylinder engines. These indicator cards were obtained for various combinations of N and q , but the injection timing was set such that

Fig. 10 Experimental values for $C_3\eta_c$, P_{max} and ρ

the maximum output could be obtained for each particular combination of N and q . Furthermore the mechanical friction expressed by equation (23), and the characteristic curve of the Roots-type blower are obtained for this particular engine.

The data used for this calculation are listed as follows:

- ① $D=110\phi$, $S/D=1.1818$, $\lambda=1/4$, $\varepsilon=16$, $z=3$
- ② $\beta_s \sin \varphi_s=0.625$, $\theta_s=47^\circ$, α_{cp} is shown Fig. 4, λ , S/D are already shown above 1.
- ③ $i(d/D)^2=0.2644$, $y_t/(d/4)=1.0$, $\theta_{EO}=94^\circ_{\text{aTDC}}$, $\theta_{EC}=229^\circ_{\text{aTDC}}$, K , $\dot{y}(\theta)$, $y(\theta)$ are shown in Fig. 11, α_v is shown in Fig. 4.
- ④ $G_0=f(N_B, P_s/P_0, \eta_{adi})$ is shown in Fig. 6, $N_B/N=2.05$, $\eta_{mbi}=0.9$.
- ⑤ $A_t/(zA_p)=1/15$, (this is assumed from the experimental values for G_0 , λ_{s0} , P_e , T_e).
- ⑥ $\eta_{s0}=1-\exp(-1.6DRZ)$.
- ⑦ $P_{\max}=f(1/\lambda_{s0}, v_{pis})$, $C_3\eta_c=f(1/\lambda_{s0}, v_{pis})$ and $\rho=f(1/\lambda_{s0}, v_{pis})$ are shown in Fig. 10.
- ⑧ $P_{mf}=P_{mf0}+(0.0413+0.00714v_{pis})(P_{mi}+\Delta P_{mi})$, $P_{mf0}=0.55+0.173v_{pis}$. The experimental values for P_{mf0} are shown in Fig. 12.

In this calculation, A_r/A_p and $\bar{A}_r$ were calculated assuming an incompressible gas flow, $n_c=1.4$, $n_e=1.3$ for the cylinder cycle. The specific heat for air was also assumed in place of

Fig. 11 $\dot{y}(\theta)$, $y(\theta)/y_T$ for calculations

Fig. 12 P_{mf0} obtained from motoring method and Willan's line method

the specific heat of the cylinder gas. In calculating the specific heat, the influence of the gas temperature was precisely taken into account, but the influence of the fuel/air ratio and so forth was neglected for simplicity.

As for the calculation of ΔP_{mi} , $P_z(\theta)$ and $T_z(\theta)$ for the duration of $\theta_{EO} \sim \theta_{SB}$ were only calculated strictly in solving the simultaneous Eqs. (6), (7), (8), (9) or (9'), (10) or (10'), (11), but for the duration of $\theta_{SB} \sim \theta_1$, $P_z(\theta)=P_s$ was assumed. As can be seen from Fig. 7, since ΔP_{mi} depends mainly upon the work done during the interval $\theta_{EO} \sim \theta_{SB}$ and is only slightly affected by the work during the interval $\theta_{SB} \sim \theta_1$, the above simplifications might be justified.

As was mentioned above, these calculations must start with assuming the values for P_e/P_0 and θ_{SB} , and then proceed to the assumptions of blower operating conditions, scavenging efficiency, trapped fresh charge, operating cycle in the cylinder and so forth; the author iterates these stepwise assumptions until Eqs. (12) and (28') can be satisfied within an error of less than 2.5%.

The calculated results of P_s/P_0 , P_e/P_0 , DRZ , η_{s0} , η_{v0} , η_{th0} , q , $P_{mi}+\Delta P_{mi}$, P_{mbi} , P_{mf} , P_{me} and b_e for an operating condition assumed as $P_0=10332 \text{ kg/m}^2$, $T_0=288^\circ\text{K}$, $\lambda_{s0}=1.5$, $N=750 \sim 2000 \text{ rpm}$ ($ND=8220 \sim 22000 \text{ cm/min}$) are shown in Fig.

Fig. 13 Calculated engine performance for $\lambda_{s0}=1.5$

13. Similar results for $\lambda_{s0}=2.5$ are also obtained. Since space is limited, only some results are shown in Fig. 15.

5.2 Comparison between preliminary calculations and experimental results

Some experimental results for an engine very similar to the calculated example are shown in Fig. 14. Comparison between those experimental results and the calculated values for P_s/P_0 , P_e/P_0 , T_e , DRO and P_{me} corresponding to $\lambda_{s0}=1.5$ and 2.5 are obtained from Fig. 13 and Fig. 14, and shown in Fig. 15. In Fig. 15, the calculated values for P_s/P_0 and T_e were a little higher than the experimental values, but good agreement was obtained between the calculated and the experimental values for P_s/P_0 , DRO and P_{me} .

The disagreements for P_s/P_0 , P_e/P_0 , DRO , and T_e seemed to be caused first by errors in estimation of $\bar{A}_r/A_p$. In these calculations we assumed an incompressible gas flow, but as was shown in Fig. 16, the values of $\bar{A}_r/A_p$ for a compressible and for an incompressible gas flow are a little different. $\bar{A}_r/A_p$ in the case of a compressible gas flow is always larger than that in an incompressible gas flow, and also it increases slightly

with the pressure ratio P_s/P_0 . Hence the higher the engine speed is (namely, the larger P_s/P_0 is), the more $\bar{A}_r/A_p$ in an assumed incompressible gas flow might deviate from the actual $\bar{A}_r/A_p$ for their operating conditions.

Furthermore, errors such as were permitted for the iterative processes and in approximating the gas as air and so forth also caused the above disagreement. Hence a better agreement may be expected from an improved estimation of $\bar{A}_r/A_p$.

Fig. 14 Experimental values for DRO , P_s/P_0 , P_e/P_0 , T_e and P_{me} .

Fig. 15 Comparison of calculated and experimental values for engine performance

Fig. 16 $\bar{A}_r/A_p$ assumed for compressible and for incompressible gas flow

cause a serious error in the estimation of the engine performance. As for the estimations of P_{mbi} and P_{mf} , provided complete experimental data are obtained, no trouble can occur.

When some experimental errors and a little difference of engine conditions between the single-cylinder engine for experimental use and the three-cylinder engine for commercial use are taken into consideration, it can be concluded that the disagreement in Fig. 15 can be considered as sufficiently small, and the accuracy of those estimations can be regarded practically as sufficiently good.

In Fig. 20, some differences in engine performance which might be brought about by the flow resistance of the exhaust muffler are shown for $A_t/(zA_p)=1/5$, $1/10$, and $1/25$. Good agreement between the calculated value and the experimental value is also confirmed.

6. Conclusions

(1) In order to study the optimum design conditions of exhaust and scavenging time areas in

a poppet valve uniflow two stroke cycle engine, a series of fundamental equations and the methods of estimating the engine performance were derived, where generalized dimensionless expression were developed.

(2) The above fundamental equations and calculation methods were applied beforehand to a high speed three-cylinder diesel engine equipped with a mechanically driven blower. Some good agreement between the calculated and the experimental values was obtained and the possibility of estimating the influence of exhaust and scavenging time area design on the engine performance was confirmed.

References

- (1) List, H., *Der Ladungswechsel der Verbrennungskraftmaschine, Zweiter Teil, Der Zweitakt*, (1950), Springer.
- (2) Taylor, C.F., *The Internal Combustion Engine in Theory and Practice*, Vol. 1 (1961), MIT Press.
- (3) Nagao, F. and Hirako, Y., *Trans. Japan Soc. Mech Engrs.* (in Japanese), Vol. 24, No. 148 (1958), p. 998