534.113 : 621-233

Vibration of a Shaft Passing through a Critical Speed*

(1st Report, Experiments and Numerical Solutions)

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The vibrational problem which occurs during passage through a critical speed has been investigated with regard to the maximum deflection of shaft.

In this paper, we shall study the vibrational characteristics, such as the deflection of a shaft, the angular velocity of whirling motion and the phase angle between the unbalance and the whirling motion, under a uniform angular acceleration over the wide range of the angular velocity of a shaft.

From the numerical solutions and the experiments on a simple system with one disk, it is clarified that the amplitude of the whirling motion causes a beat after running through the critical speed and in case of acceleration the maximum (minimum) amplitude of beat occurs when the angular velocity of the whirling motion becomes maximum (minimum) and that the phase angle becomes $n\pi + \pi/2$ radians when the angular velocity of whirling is equal to the angular frequency corresponding to the critical speed of the system.

1. Introduction

The problem of transient vibration which occurs when a rotor passes through a critical speed at a uniform acceleration has been studied analytically mainly about how to evaluate the maximum deflection under certain damping and acceleration rate by adopting Lewis' method^{(1)~(6)}.

Shimoyama and Yamamoto⁽⁴⁾ carried out an experimental study on the transient vibration using a rotor-bearing system whose bearing pedestals have anisotropic rigidity. They ascertained that the shaft centre executes a transient ellipse motion whose locus varies with the change of the angular velocity of shaft.

Dimentberg⁽⁵⁾ dealt with the estimation of the maximum stress of shaft yielded when a rotor passes through the critical speed for two systems. One is a simply supported system and the other is a system supported by anisotropic spring. In each case the solution for the deflection of shaft was derived from the equations of motion but Dimentberg's solution contained Fresnel integrals. He carried out also a few experiments and compared them with the analytical results qualitatively.

Fernlund⁽⁶⁾ investigated on the transient vibration of a shaft with a disk. In the analysis he adopted a method by which the equations of motion are integrated directly by the aid of a digital computer, and he plotted the results for various rates of acceleration, various damping and various initial conditions. He also carried out experiments and compared the results with the calculated values but they are not in agreement.

In those papers, they do not make clear the characteristics of an angular velocity and the phase angle of the whirling motion of the shaft during running through the critical speed.

In this paper, we describe the experiments of vibration of a shaft under the condition of uniform acceleration and deceleration, in a system of one-disk rotor whose ends are supported on ball bearings. We also analyzed this problem by using the same method as Fernlund used and compared the results with the numerical solutions.

2. Nomenclature

$$a = \frac{d^2\phi}{dt^2}$$

c : damping coefficient

$D^* = c/(2m\omega_n)$: damping ratio

$e = \overline{SG}$: eccentricity of the disk

G : gravitational centre of the disk

k : bending stiffness of shaft

m : mass of disk

* Received 5th January, 1970.

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- O : origin of rectangular coordinate axes
 O' : statical equilibrium position of the geometrical centre S
 $r=\overline{O'S}$: displacement of the geometrical centre S
 S : geometrical centre of the disk
 t : time
 X : position of S in the X -direction
 $x=X$
 Y : position of S in the Y -direction
 $y=Y-mg/k$
 θ : phase angle between the directions of the unbalance and the deflection of shaft
 κ : ratio of the angular velocity of shaft when acceleration (or deceleration) starts to the natural circular frequency
 λ : nondimensional angular acceleration (deceleration) of the shaft [see Eq. (2)]
 $\rho=r/e$: nondimensional deflection of shaft
 $\tau=\omega_n t$: nondimensional time
 φ : phase angle of the whirling motion of shaft (see Fig. 5)
 $\dot{\varphi} = \frac{d\varphi}{d\tau} = \frac{d\varphi}{dt} / \omega_n$: nondimensional angular velocity of the whirling motion
 ψ : rotational angle which is measured from the axis (see Fig. 5)
 $\dot{\psi} = \frac{d\psi}{d\tau} = \frac{d\psi}{dt} / \omega_n$: nondimensional angular velocity of rotation
 $\omega_n = \sqrt{k/m}$: undamped natural circular frequency

3. Experimental apparatus and experimental procedure

Experimental apparatus is shown in Fig. 1. A rotating shaft which is supported on ball bearings carries a single disk midway between them. The shaft with a diameter of 17 mm has a span of 520 mm. The disk has a diameter of 120 mm, 50 mm in thickness and is 4.7 kg in weight. This rotor is driven by a 0.5 sp motor with variable speed and is given a speed varying from zero to 90 c/sec.

Fig. 1 Experimental apparatus

The vibrations of shaft in two perpendicular directions are measured by electric capacity transducers. To measure the angular velocity of rotation and the phase angle, we attached a small strip of tape on the surface of the shaft near a bearing. This small tape is available to get a pulse which is generated when the tape passes on the transducer. The vibrations of shaft and the signal of pulse are recorded by an electro-magnetic oscillograph.

After balancing the rotor, a known unbalance is fixed to the disk. This rotor is driven at a constant speed near the critical speed (in the case of acceleration this constant speed is about 0.8~0.9 times the critical speed and in deceleration it is about 1.1~1.2 times). Then the rotor is accelerated (or decelerated) by changing the position of a knob on the motor as the rotor passes through the critical speed under the condition of a constant acceleration.

Typical examples of oscillograms obtained during acceleration and deceleration are shown in Figs. 2(a) and 2(b) respectively. In these oscillograms, each wave line lacks symmetry in each upper and lower half. It results from a characteristic of the measuring instruments. The actual whirling is a circular motion whose centre coincides with the static equilibrium position of the shaft centre.

In this paper, we apply the term of the critical speed to the rotational speed corresponding to the natural frequency of the system and do not apply it to the rotational speed at which the maximum deflection occurs in a transient motion as shown in Figs. 2(a) and 2(b).

4. Experimental results

In Figs. 3 and 4, we show the experimental

(a) Oscillogram of acceleration

(b) Oscillogram of deceleration

Fig. 2

results of the angular velocity of rotation $\dot{\psi}$, the shaft deflection r , the angular velocity of whirling $\dot{\phi}$ and the phase angle θ . Figure 3 represents the characteristics obtained under the condition that the acceleration rate is $2\pi \times 5.7$ rad/sec² and the critical speed is 41.9 c/sec. And in case of Fig. 4 the deceleration rate is $2\pi \times 4.5$ rad/sec² and its critical speed is 41.2 c/sec.

From Figs. 3(a) and 4(a) it is seen that the angular velocity of rotation $\dot{\psi}$ changes straightly with time. So this experiment satisfies the assumption of the uniform acceleration which is adopted in the analysis.

In Figs. 3(b) and 4(b), it is shown that the deflection of the shaft increases with time and reaches a maximum at a certain rotational speed which is greater than the critical speed in the case of acceleration, or is smaller than the critical

speed in the case of deceleration. Then the deflection, repeating beats, approaches a constant value.

In these figures, the characteristic of the angular velocity of whirling $\dot{\phi}$ is noteworthy. According to the change of $\dot{\phi}$, we can divide the whole range of rotational speeds into three regions. In the first region, $\dot{\phi}$ changes along with $\dot{\psi}$ while $\dot{\phi}$ varies from its initial value to the natural frequency. And in the second region, $\dot{\phi}$ varies up and down around the natural frequency and its period and amplitude are decreasing and increasing with time, respectively. In the third region, $\dot{\phi}$ varies about $\dot{\psi}$ with its decreasing period and amplitude with time and then approaches $\dot{\psi}$.

A transition point, where the vibrational phenomenon changes from the second region to the third one, comes near the critical speed in

case of small acceleration and large damping. This transition point appears at a node of beats as shown in Figs. 2(a), 2(b), 3(b) and 4(b). It may be difficult to discern the position of the transition point on an oscillogram because of small deflection at node. Generally, transient vibration considered here is not accomplished until these three regions finish.

Fernlund calculated for a half of the second region. But we have not seen any study which analyzes this problem over the range of rotational speeds treated in this report.

The variation of $\dot{\phi}$ has naturally influence on the characteristic of phase angle θ . In the case of acceleration, θ increases gradually from a phase angle of steady state in the first region. But in the second region, θ becomes $\pi/2$ radians (at this time

Fig. 3

the angular velocity of whirling coincides with the natural angular frequency according to calculations) and then when shaft deflection reaches its maximum, θ takes the value between $\pi/2$ and π radians. Further, in this region θ increases 2π radians per one cycle when $\dot{\phi}$ varies around the natural angular frequency. Because of the cycle getting shorter with time, the change of θ takes place rapidly. If the angular velocity of whirling is below the natural angular frequency, θ takes the value between $2n\pi - \pi/2$ and $2n\pi + \pi/2$ (n : integer) and if it is above the natural frequency, θ takes the value between $(2n+1)\pi - \pi/2$ and $(2n+1)\pi + \pi/2$ radians. This relationship between θ and $\dot{\phi}$ is same as that of steady state. In the third region,

θ ceases rapid growth and approaches $(2n+1)\pi$ radians.

In case of deceleration, the characteristic of phase angle has a similar process. From the beginning of deceleration, θ decreases gradually with time and reaches $\pi/2$ radians when $\dot{\phi}$ coincides with the natural angular frequency (first region). In the second region, θ decreases rapidly. And in the third region, θ approaches $-2n\pi$ radians.

Now, we consider the relationship between the angular velocity of whirling and the deflection of shaft. Under acceleration, in the second region, the maximum deflection of shaft can be found when the whirling speed $\dot{\phi}$ reaches the maximum, and the minimum deflection at

the minimum of $\dot{\phi}$. In the third region, the maximum and the minimum deflection occur at the minimum and maximum of $\dot{\phi}$ respectively. Under deceleration, these relationships are upset in each region.

We described some characteristics using the typical examples as shown in Figs. 3(b) and 4(b). We have obtained also similar results in another case where the critical speed is 40.7 c/sec and the acceleration rate is $2\pi \times 5.7$ rad/sec² and κ is equal to 0.885.

5. Analysis

We analyze this problem under the following assumptions. (1) A rotating shaft is supported simply at both ends and carries one disk at the middle of span. The mass of shaft is negligible. (2) The angular acceleration of shaft is constant with time.

$\omega_n = 2\pi \times 41.2$ rad/sec, $a = -2\pi \times 4.5$ rad/sec², $\kappa = 1.16$
(a) $\dot{\phi}$ -time curve during deceleration

$\omega_n = 2\pi \times 41.2$ rad/sec, $a = -2\pi \times 4.5$ rad/sec², $\kappa = 1.16$, $e = 4.5\mu$
(b) ρ , ψ , θ -time curves during deceleration in vertical direction

Fig. 4

Figure 5 shows the cross section of shaft at the position of disk. The equations of motion of the geometrical centre S may be written as

$$\left. \begin{aligned} m \frac{d^2 x}{dt^2} + c \left(\frac{dx}{dt} - e \frac{d\phi}{dt} \sin \phi \right) + kx \\ = me \left(\frac{d\phi}{dt} \right)^2 \cos \phi + me \left(\frac{d^2 \phi}{dt^2} \right) \sin \phi \\ m \frac{d^2 y}{dt^2} + c \left(\frac{dy}{dt} + e \frac{d\phi}{dt} \cos \phi \right) + ky \\ = me \left(\frac{d\phi}{dt} \right)^2 \sin \phi - me \left(\frac{d^2 \phi}{dt^2} \right) \cos \phi - mg \\ d^2 \phi / dt^2 = a \quad (a: \text{const.}) \end{aligned} \right\} \text{from the assumption (2).} \quad \dots\dots\dots (1)$$

Using the relations of $x=X$ and $y=Y-mg/k$ we obtain Eq. (2) after substituting $X=r \times \cos \varphi$, $Y=r \times \sin \varphi$ into Eq. (1).

$$\left. \begin{aligned} \ddot{\rho} + 2D^*(\dot{\rho} - \dot{\phi} \sin \theta) + (1 - \phi^2)\rho = \dot{\phi}^2 \cos \theta + \ddot{\phi} \sin \theta \\ \rho \dot{\phi} + 2D^*(\rho \dot{\phi} + \ddot{\phi} \cos \theta) + 2\dot{\rho} \dot{\phi} = \dot{\phi}^2 \sin \theta - \ddot{\phi} \cos \theta \\ \ddot{\phi} = \frac{1}{\omega_n^2} \frac{d^2 \phi}{dt^2} = \frac{\alpha}{\omega_n^2} = 2\lambda \end{aligned} \right\} \quad \dots\dots\dots (2)$$

A symbol of dot represents a derivative with respect to τ .

From Eq. (2) we obtain

Fig. 5

$\omega_n = 2\pi \times 41.9$ rad/sec, $\lambda = 0.00026$, $D^* = 0.008$, $\kappa = 0.867$, $e = 3\mu$
(a) ρ - ϕ curve during acceleration in vertical direction

$$\left. \begin{aligned} \ddot{\rho} = - (1 - \phi^2)\rho - 2D^*(\dot{\rho} - \dot{\phi} \sin \theta) \\ + \dot{\phi}^2 \cos \theta + \ddot{\phi} \sin \theta \\ \ddot{\phi} = - \frac{2\dot{\rho}\dot{\phi}}{\rho} - \frac{2D^*}{\rho} (\rho \dot{\phi} + \ddot{\phi} \cos \theta) \\ + \frac{\dot{\phi}^2}{\rho} \sin \theta - \frac{\ddot{\phi}}{\rho} \cos \theta \end{aligned} \right\} \quad \dots\dots\dots (3)$$

$$\ddot{\phi} = 2\lambda$$

and Fig. 5 gives $\varphi + \theta = \phi$.

First we calculate the initial values from the

$\omega_n = 2\pi \times 41.9$ rad/sec, $\lambda = 0.00026$, $D^* = 0.008$, $\kappa = 0.867$, $e = 3\mu$
(b) ϕ - ψ curve during acceleration in vertical direction

$\omega_n = 2\pi \times 41.9$ rad/sec, $\lambda = 0.00026$, $D^* = 0.008$, $\kappa = 0.867$, $e = 3\mu$
(c) θ - ϕ curve during acceleration in vertical direction

Fig. 6

following equations.

$$\rho = \rho_0 = \frac{\kappa^2}{\sqrt{(1-\kappa^2)^2 + (2\kappa D^*)^2}}, \quad \dot{\rho} = 0$$

$$\theta = \theta_0 = \tan^{-1} \frac{2\kappa D^*}{1-\kappa^2}$$

$$\psi = 0, \quad \dot{\psi} = \kappa \quad \psi = \lambda \tau^2 + \kappa \tau$$

$$\varphi = 0, \quad \dot{\varphi} = \kappa$$

And then $\ddot{\rho}$ and $\ddot{\varphi}$ are calculated by the aid of Eq. (3). The values of $\dot{\rho}$, $\dot{\varphi}$ for the new step are obtained as $\Delta\tau\ddot{\rho}$, $\Delta\tau\ddot{\varphi}$. Other quantities are calculated in the same way. Repeating this procedure we obtain curves of numerical solutions.

In calculation, we paid attention to an accumulation of errors. One example is Fig. 6 ($\lambda = 0.00026$, $\kappa = 0.867$, $D^* = 0.008$) which has a

$$\omega_n = 2\pi \times 41.2 \text{ rad/sec}, \lambda = -0.000213, D^* = 0.007, \kappa = 1.16, e = 4.5\mu$$

(a) ρ - ψ curve during deceleration in vertical direction

$$\omega_n = 2\pi \times 41.2 \text{ rad/sec}, \lambda = -0.000213, D^* = 0.007, \kappa = 1.16, e = 4.5\mu$$

(b) ψ - ψ curve during deceleration in vertical direction

width of unit step of $\Delta\tau = 0.05$ and the total number of steps is 20000. In Fig. 7 ($\lambda = -0.000213$, $\kappa = 1.16$, $D^* = 0.007$), $\Delta\tau$ is 0.01 and the total number of steps is 100000. In the calculation of Fernlund, $\Delta\tau$ was equal to unity and the total number of steps was 350 in his most cumbersome case ($\lambda = 0.001$, $\kappa = 0.90$, $D^* = 0.01$).

6. Comparison between numerical solutions and experiments

In Figs. 6(a)~6(c) and Figs. 7(a)~7(c), the test results shown in Fig. 3 and Fig. 4 are compared with the numerical solutions, respectively.

We have obtained the nondimensional values of ρ , $\dot{\psi}$ and $\dot{\varphi}$ from the values of measurement in order to compare them with the numerical solutions. So we did this work, using the value of the known eccentricity of the disk and the critical speed which was obtained from the test.

A value of damping ratio was required to obtain a solution and it was better to measure the damping while the shaft rotates. But we used a suitable value for the damping ratio, referring to that which was obtained from an oscillogram of a damped free oscillation when a shaft did not rotate.

In Figs. 6(a) and 7(a) there is slight discrepancy between the experimental value and the calculated one, but it is found that the periodicity of beats is in good agreement. From Figs. 6(b) and 7(b) (the characteristic of the angular velocity of whirling) it is easily seen that the numerical solutions give the same result as mentioned in

$$\omega_n = 2\pi \times 41.2 \text{ rad/sec}, \lambda = -0.000213, D^* = 0.007, \kappa = 1.16, e = 4.5\mu$$

(c) θ - ψ curve during deceleration in vertical direction

Fig. 7

the section 4. In these figures, $\dot{\phi}$ takes negative values such as -6.27 , -2.99 and -0.53 . This means that the shaft performs a backward precession in a second but we couldn't ascertain whether this phenomenon really occurs or not. In Fig. 6(b), the transition point obtained by the experiment differs from that of calculation by one cycle. This discrepancy results mainly from the poor estimation of the damping ratio used

Table 1 Calculated values shown in Fig. 6 (acceleration)

	τ	$\dot{\phi}$	ϕ	ρ	θ	Characteristics
First region	0	0.867	0.867	3.02	2.8°	Acceleration starts
	256.0	1.000	0.987	22.7	55°	$\dot{\phi}=1.0$
	294.0	1.019	1.000	33.2	90°	$\theta=\pi/2$
Second region	344.5	1.041	1.011	40.7	148°	$\rho_{\max 1}$
	385.5	1.067	1.000	20.6	270°	$\theta=(3/2)\pi$
	403.5	1.077	0.956	12.4	369°	$\rho_{\min 1}, \dot{\phi}_{\min 1}$
	417.0	1.084	1.000	16.8	450°	$\theta=(5/2)\pi$
	434.5	1.093	1.025	21.6	524°	$\rho_{\max 2}, \dot{\phi}_{\text{nearly max.}}$
	459.0	1.106	1.001	11.9	630°	$\theta=(7/2)\pi$
	469.0	1.111	0.917	7.1	727°	$\rho_{\min 2}, \dot{\phi}_{\min 2}$
	479.5	1.116	1.000	10.6	810°	$\theta=(9/2)\pi$
	494.0	1.124	1.039	15.1	889°	$\rho_{\max 3}, \dot{\phi}_{\text{nearly max.}}$
	647.5	1.204	1.097	6.8	2336°	$\rho_{\max 7}$
	663.0	1.212	-6.274	0.08	2495°	$\rho_{\min 7}, \dot{\phi}_{\min 7}$
Third region	677.0	1.219	1.113	5.9	2696°	$\rho_{\max 8}$
	691.5	1.227	2.938	0.35	2705°	$\rho_{\min 8}, \dot{\phi}_{\max}$
	704.5	1.233	1.126	5.3	2695°	$\rho_{\max 9}, \dot{\phi}_{\text{nearly min.}}$
	718.5	1.241	1.961	0.70	2694°	$\rho_{\min 9}, \dot{\phi}_{\max}$

Table 2 Calculated values shown in Fig. 7 (deceleration)

	τ	$\dot{\phi}$	ϕ	ρ	θ	Characteristics
First region	0	1.16	1.16	3.9	177°	Deceleration starts
	373.0	1.00	1.01	25.4	124°	$\dot{\phi}=1.0$
	415.5	0.982	1.00	35.4	90°	$\theta=\pi/2$
Second region	455.0	0.965	0.991	41.7	37°	$\rho_{\max 1}$
	516.5	0.939	1.000	20.6	-90°	$\theta=-\pi/2$
	537.5	0.930	1.035	12.5	-190°	$\rho_{\min 1}$
	552.5	0.923	1.000	16.0	-270°	$\theta=-(3/2)\pi$
	571.5	0.915	0.979	19.8	-340°	$\rho_{\max 2}, \dot{\phi}_{\min}$
	599.0	0.904	1.000	11.3	-450°	$\theta=-(5/2)\pi$
	611.5	0.898	1.061	7.0	-544°	$\rho_{\min 2}, \dot{\phi}_{\max}$
	622.0	0.894	0.999	9.7	-630°	$\theta=-(7/2)\pi$
	638.0	0.887	0.968	13.0	-707°	$\rho_{\max 3}, \dot{\phi}_{\min}$
	658.0	0.878	1.000	7.1	-810°	$\theta=-(9/2)\pi$
	667.5	0.875	1.096	4.3	-902°	$\rho_{\min 3}, \dot{\phi}_{\max}$
	857.0	0.794	2.016	0.34	-2689°	$\rho_{\min 8}, \dot{\phi}_{\max}$
	871.5	0.788	0.901	3.49	-2873°	$\rho_{\max 9}, \dot{\phi}_{\text{nearly min.}}$
	886.5	0.781	3.487	0.13	-3037°	$\rho_{\min 9}, \dot{\phi}_{\max}$
	900.5	0.775	0.889	3.06	-3256°	$\rho_{\max 10}$
Third region	914.5	0.769	-2.991	0.07	-3276°	$\rho_{\min 10}, \dot{\phi}_{\min}$
	928.0	0.764	0.876	2.67	-3238°	$\rho_{\max 11}, \dot{\phi}_{\text{nearly max.}}$
	941.0	0.758	-0.527	0.20	-3255°	$\rho_{\min 11}, \dot{\phi}_{\min}$

in calculation. This discrepancy causes a difference of phase angle (2π radians) between the test result and the calculated one as shown in Fig. 6(c). Excepting this, the test results of the angular velocity of whirling and the phase angle are in good agreement with the calculated results.

Next we will consider the relationship among angular velocity of rotation $\dot{\psi}$, shaft deflection r , angular velocity of whirling $\dot{\phi}$ and phase angle θ . To show these relations, we give two tables which represent the calculated values according to the notable angular velocity of $\dot{\psi}$. Tables 1 and 2 represent the calculated values in the cases of Fig. 6 (acceleration) and Fig. 7 (deceleration) respectively.

From these tables, it is found that the phase angle θ in each case takes the value of $2n\pi \pm \pi/2$ (n : integer) when $\dot{\phi}$ is equal to unity. Under acceleration, r in the second region is maximum when $\dot{\phi}$ becomes maximum and is minimum when $\dot{\phi}$ becomes minimum. And r in the third region is maximum when $\dot{\phi}$ becomes minimum and is minimum when $\dot{\phi}$ becomes maximum. Under deceleration these relationships are reversed.

We have one more example ($\lambda=0.00065$, $\kappa=0.952$, $D^*=0.0096$, $e=9\mu$, $\omega_n=2\pi \times 40.8$ rad/sec horizontal direction) and in this case both the experimental results and the numerical solutions are in good agreement.

7. Conclusions

It is found that the transient vibration of a rotational shaft which passes through its critical speed at a uniform acceleration may be divided into three regions.

In the first region, the angular velocity of whirling motion varies along with the angular velocity of shaft. The deflection of shaft increases gradually with time and the phase angle increases in case of acceleration or decreases in case of deceleration.

In the second region, the angular velocity of whirling varies up and down around the natural frequency of the system and the deflection of shaft reaches the maximum value and then beats. In the case of acceleration the maximum deflections of shaft during this beat occur when the angular velocity of whirling becomes maximum and the minimum deflection of shaft occurs at the minimum whirling speed. But in the case of deceleration, these relationships are reversed. The phase angle changes by 2π radians according to one cycle of the beat and

takes the values of $(n\pi + \pi/2)$ radians when the whirling speed coincides with the natural frequency.

In the third region, the angular velocity of whirling approaches with time to the angular velocity of shaft and is varying about it. The phase angle stops the rapid change and becomes equal to $(2n+1)\pi$ radians (acceleration) or to $2n\pi$ radians (deceleration). The deflection of shaft continues to beat in this region but the maximum deflection occurs when the angular velocity of whirling becomes minimum and the minimum deflection occurs when the angular velocity of whirling is maximum in the case of acceleration. In the case of deceleration, this relationship is reversed.

At the transition point between the second region and the third one, there is the rotational speed at which the angular velocity of whirling takes a negative value. Further, the angular

velocity at which the transition occurs becomes closer to the critical speed if damping ratio becomes larger and acceleration rate becomes smaller.

The authors wish to express their gratitude to H. Murai and S. Hatake for their assistance in performing the experiments and the calculations.

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Discussion

Y. KOBORI (Hitachi Ltd.):

(1) What is the definition of critical speed? What is the difference between critical speed and peak frequency?

(2) The authors arrange the calculated results and the experimental ones by using the natural frequency. In arranging the results, however, it may be better to use the critical speed (peak frequency) which is obtained under infinitesimal acceleration instead of the undamped natural frequency of the system, I think. What do you think about it?

Editorial Director:

(3) The authors stated that in the study of Fernlund, his experimental results differed from numerical solutions. The authors, however, treated the same system and adopted the same analytical method as Fernlund used. I would like to know the difference between the two.

(4) What do the authors think about analyzing this problem by using the asymptotic method, assuming a quasi linear system with a slowly varying parameter?

Authors' closure

(1) Usually the term of critical speed is used in the steady state vibration and it means a rotational speed where a system is in resonance. In this paper, we apply the term of critical speed in the meaning mentioned above. Therefore, in

the transient vibration, the critical speed differs from a peak frequency at which the maximum deflection occurs.

(2) In this report, we arranged the calculated values by using the undamped natural frequency and in arranging the experimental values we use the critical speed at which the maximum deflection occurs under constant rotational speed, considering that the critical speed is close enough to the undamped natural frequency, $\frac{1}{2\pi} \sqrt{k/m}$.

(3) The difference between Fernlund's paper and the authors' lies in their treatment of this problem. Fernlund shows how the acceleration rate and the damping force affect the maximum deflection by numerical solutions. Fernlund, naturally from his standpoint, does not treat this problem over a wide range of rotational speeds uses in our experiments nor mentions the relationship between the deflection and the phase angle and the angular velocity of whirling.

On the other hand, the authors try to clarify the vibration phenomena which occur during passage through a critical speed, especially the relationship between the deflection and the phase angle and the angular velocity of whirling.

There is one more difference, that is, experimental results and numerical solutions are not

in good agreement in Fernlund's paper, but both are in good agreement in our study though they used the same analytical method. We do not know the reason. We think there is no large difference between Fernlund's analytical results and authors' one except the difference of a unit length of calculation step. But in experiments, Fernlund said in his paper, it was difficult to get uniform acceleration and his system was disturbed by the secondary critical speed when acceleration began.

(4) We think a numerical integration using the Runge-Kutta method is the best way available at present to explain the experimental results. We would like to mention difficulty to use the

asymptotic method in analyzing this problem. There is an assumption in the asymptotic method, that is, parameters of the system vary gradually with time. However, the parameters $\dot{\phi}$ and θ vary rapidly near the transition point and do not satisfy the assumption. This is the first reason.

The second reason is that in the asymptotic method there is not clear distinction between $\dot{\phi}$ and ϕ .

The numerical calculation, however, takes a lot of time and is not suitable to grasp the physical meaning of the problem. Then we are studying to obtain an approximate solution to this problem.